AMERICAN ACADEMY *of* ACTUARIES

Report from the American Academy of Actuaries'
Economic Scenario Work Group
To

The National Association of Insurance Commissioners'
Life Risk Based Capital Working Group
and
Life and Health Actuarial Task Force

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Economic Scenario Work Group

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Economic Scenario Work Group Report

Summary

The American Academy of Actuaries' Economic Scenario Work Group (ESWG) was asked to develop an interest rate generator and calibration criteria so companies could use either Academy-generated scenarios or their own internal model that calibrates to the Academy scenarios for regulatory reserve and capital calculations. This report will summarize the Stochastic Log Volatility (SLV) interest rate scenario generator model developed by the ESWG and describe the recommended calibration criteria. This report builds from and replaces all earlier reports from the ESWG. All references to "Academy" work products within this report refer to the specific committees and work groups that developed these specific reports and generators.

The ESWG received its direction from the Standards for Stochastic Methods Work Group (SSMWG) Report, September 2006. The charge to the ESWG was stated as:

- The ESWG will provide a prescribed generator containing updated parameters. This report includes an interest rate generator, a set of 10,000 Treasury curve scenarios using September 30, 2008 as the starting date and a scenario picking tool.
- The ESWG will generate calibration criteria so companies can use their own generator or a subset of the base 10,000 Academy scenarios. This report includes calibration criteria to allow use of company generated scenarios, along with a tool that helps evaluate the requirements of the calibration criteria. As recommended in the SSMWG's report, the provided scenario sets will not use pre-selected criteria designed to stress test specific blocks of business. More specifically, the ESWG does not support specific subsets of scenarios as were utilized in C3 Phase I RBC capital calculations. These scenario sets were designed and tested to apply to specific blocks of annuity business and are not appropriate when testing a variety of products with different risk characteristics.

The goal of the Academy is to develop an integrated debt-equity generator that can be used for all reserve and capital calculations. We have not yet accomplished this goal, but this report provides an interest rate generator and scenarios based on updated parameters, along with calibration criteria to specify generators acceptable for regulatory calculations.

The ESWG, along with the Academy's Life Capital Adequacy Subcommittee, recommends that the SLV generator replace other interest rate generators currently in use for all regulatory purposes. To the extent a company is exposed to other financial market risks, such as equity, credit and currency risks, these risks will also need to be modeled by the actuary, as well as any correlations between these risks. Note that the equity scenarios provided by the Academy for C-3 Phase II RBC requirements continue to be appropriate.

The focus of the ESWG's testing were the results in the tails of the stochastic distribution. Concentrating on tail results is appropriate for identifying relatively weakly capitalized companies and calculating policy reserves for regulatory purposes. A few other relevant comments about this generator and scenarios include the following:

- Use of randomly generated movements of interest rates can also reduce the likelihood of products designed around the scenarios and highlight unanticipated interactions between interest rates and policyholder behavior.
- The Academy SLV generator will adapt well to changing economic environments, allowing consistency from year to year in interest rate scenario selection.
- The Academy consulted individuals with expertise in interest rate scenario generation and using stochastic scenarios in practical situations. These experts included financial engineers and actuaries with experience across a broad range of practice, including software vending.
- The resulting scenarios from the Academy SLV model were compared to the scenarios produced by other generators, including commercially available software. By comparing relevant statistics about the distribution

of rates, including dispersion and volatility, the ESWG concluded that the Academy generator produces reasonable scenarios and furthermore, the calibration criteria requirements satisfy an appropriate balance between (a) being so onerous as to require the Academy generator be used in almost all circumstances and (b) so loose as to allow potentially inappropriate company-generated scenarios.

Special thanks go to the following volunteers. These are listed to help the NAIC better understand the input and battle testing that the Academy generator has already undergone prior to its release.

- Geoff Hancock built the current Academy generator, documented it and designed it so that it is easy to use. He converted it to an Excel format to increase its transparency.
- Dave Kester developed user-friendly features like the statistics calculator

History

The direction provided by the Standards for Stochastic Methods Work Group was accepted by the NAIC and formed the basis of the ESWG's work.

The NAIC [LHATF and LRBC] received reports describing the proposed Academy interest rate scenario generator at the Summer 2007 NAIC meeting and again at the Fall 2007 NAIC meeting. No comments were received from the NAIC or individual regulators regarding either of these reports. There have been only slight adjustments to the generator since then. Technical aspects of the generator are included in Appendices I, II and III.

The Stochastic Log Volatility (SLV) model is consistent with the previous interest rate model approved by the NAIC during the C-3 Phase I project that was completed in 1999. The ESWG has spent the past year considering calibration criteria to allow company-generated interest rate scenarios. An Academy document describing the reasons for and against using company-generated scenarios has been included as Appendix IV for reference.

The ESWG has researched other regulatory regimes to understand the range of practices. We found some information specific to Canada and the United Kingdom, where peer review and transparency are key features. There is little specific guidance from either banking or insurance regulatory bodies, with the Basel Accords in particular requiring defense of the assumption chosen rather than a prescriptive requirement. Stochastic interest rate scenarios are generally considered "best practice" in the financial services industry for valuing the cash flows from contingent claims. No references to deterministic scenarios used for principle-based approaches to reserves or capital were found. In the information that was found, companies are encouraged to use the same models for valuation that is used to manage the business.

Scenario generators can never accurately model the uncertainty that will actually occur in the future, especially over long time horizons. Using historical data, the SLV model reflects potential future variability consistent with the volatility and other characteristics (e.g., curve shape, correlated movements across the maturity spectrum, clustering, etc.) of observed interest rates. The SLV model is one of many that could be chosen. It has been found to produce reasonable real world interest rate scenarios. Alternative generators are not limited to the SLV, although their output will need to satisfy the Academy calibration criteria. Since model characteristics vary, tolerances have been added to allow consideration of alternative generators. Since there are multiple aspects of the calibration criteria, the actuary will apply judgment to determine if the company-generated scenarios satisfy enough of the requirements. This judgment will be based on the risk profile of the products being modeled.

Definitions are as follows:

- Short rate: 1-year Treasury rate
- Long rate: 20-year Treasury rate
- Spread: Long rate minus short rate

Generator Updates

The revised SLV model contains several updates that have been implemented since the last ESWG report:

- Refreshed the parameterization using monthly Treasury data from 1953-2008;

- Established processes (formulas) to automatically update the Mean Reversion Parameter (MRP) for the target long interest rate;
- Prepared a Microsoft Excel "generator" for broad distribution to the industry;
- Documented the model, data sources, key decisions and parameters; and
- Developed a "scenario picking" tool and statistics calculator so that practitioners can use subsets of the base scenario set or company-generated scenarios.

The primary assumption and methodological differences from the original C-3 Phase I model are:

- A "soft cap" of 18% for the long rate limits the maximum rate with minimal impact on overall results. "Soft" means the limit is applied before the random shock (i.e., long maturity interest rates can exceed 18% due to random noise);
- The volatility of the long (20y) rate – which is a stochastic process (modeled as log volatility) – varies monthly; it only changed annually in the original C-3 Phase I model;
- The mean reversion parameter for the long rate has been updated to reflect additional history and to give more weight to recent history relative to the previous model. The current MRP is 5.50%, rounded to the near 25 basis points, down from the previous 6.55%; see Appendix III for the rationale;
- A methodology to automatically update the MRP. The recommended update frequency is annual; and
- The "curve completion" methodology (to obtain the entire Treasury curve, not just the simulated short and long rates) is now based on relationships observed in history, not a parametric formula. This provides a very rich set of curve shapes while ensuring consistent movements across maturities. It is easy to understand and also faster to execute in a simulation.

Calibration Criteria

Calibration criteria have been established to govern the principle-based approaches for reserve and capital calculations. The goals were:

- Subject to the recommended calibration requirements, companies will be able to use their own interest rate generators;
- Allow suitably parameterized models with similar characteristics to the Academy model to be deemed acceptable for regulatory calculations;
- Criteria should be dynamic and not require frequent revision by the Academy; and
- Standards will include qualitative and quantitative requirements, with the qualitative requirements built around documentation and peer review. The Academy generator is built in Excel, allowing full transparency and documentation of the methods.

The Academy has also provided a statistics calculator (and summary template) so that companies can determine if a given set of scenarios (however generated, subject to formatting requirements) passes the calibration tests as determined by the base set of 10,000 Academy scenarios (with the same start date).

The Academy model will generate monthly interest rate scenarios (complete Treasury yield curves) to a maximum 100 year projection horizon. The projection period depends on the actuary's judgment and knowledge of the mix of products and assets. Calibration requirements extend to 30 years or the end of the projection period if shorter.

Importantly, it is the actual scenarios that are judged, not the parameters that define the model. If desired, the user may specify an output frequency other than monthly.

Calibration criteria reflect the dispersion of the scenarios based on different types of models, some of which incorporate the initial forward curve (the SLV does not).
The calibration criteria include:

- By definition, 10,000 scenarios generated by the SLV model are assumed to be calibrated. Subsets of this scenario set must pass the calibration criteria;
- A definition of "acceptable tolerances" around the SLV statistics will ensure that the calibration criteria remain dynamic and relevant in the future; and
- Use of the following statistics:
  - Distribution results at the 5% and 95% point-in-time percentiles for the long rate and short rate at the 1- , 5- , 10- , and 30-year horizons.
  - Distribution results at the 5% and 95% cumulative percentiles for the spread across the 30-year horizon.
  - Because volatility parameters are key drivers, tolerances will be 20% (0.80) at the 1-year time horizon. At all other time horizons, the factor will tighten to 10% (0.90).

In developing the calibration criteria, efforts were focused on the scenarios in the tails of the distribution and the results over the thirty year horizon. The spread statistic is very dependent on the initial yield curve, so rather than compare point-in-time statistics this test is cumulative across all curves in all scenarios (i.e., monthly curves across 30 years and 10,000 scenarios would provide 12 × 30 × 10,000 = 3.6 million data points). This reflects issues that can arise when the initial yield curve is at an extreme, either inverted or very steep. By the 30th year, models are expected to produce similar tail results.

Actuaries must document their reasoning for the scenario set chosen. The scenarios need not strictly satisfy all calibration points, but the actuary should be satisfied with the calibration and be able to explain and justify material differences from the Academy generator. In particular, the actuary should be mindful of which tail most affects the business being valued. If results are less dependent on the right (left) tail for all products under consideration (e.g., a return of premium guarantee would primarily depend on the low interest rate scenarios in the left tail, an enhanced benefit equal to a percentage of the gain would be most sensitive to the right tail, etc.), it is not absolutely necessary to meet the right (left) calibration points.

The ESWG considered a variety of tolerance levels around the calibration criteria. A tighter banding will tend to force companies to use the Academy-generated scenarios or the SLV model. While this would encourage regulatory consistency, companies selecting their own generator may be forced to choose between a set of well-understood scenarios that they have worked with in the past to manage their business versus a set of new scenarios that may be less familiar. A wider tolerance band will promote greater use of company generators, allowing greater consistency with a company's internal risk management process. Principle-Based Approaches encourage companies to develop internal models that best reflect their own blocks of business and company-specific risks. The ESWG supports broader calibration tolerances as long as the actuary is required to document and support the choice of scenarios. The actuary should qualitatively document enough information about a company generated model that an objective qualified actuary could understand the approach taken.

Probability Measures

In general, there are two probability measures for simulating investment returns. The *Q*-measure, or "risk neutral" distribution, is a convenient framework used for pricing securities and is predicated on the concept of replication under a 'no arbitrage' environment. Under the *Q*-measure, risk is hedged (hence, securities are expected to earn the risk-free rate) and derivatives (options) can be priced using their expected discounted cash flows. The *Q*-measure is crucial to option pricing. The *Q*-measure is relevant only to pricing ("fair market value" determination) and replication (a fundamental concept in hedging).

Attempts to project values ("true outcomes") for a risky portfolio are generally based on an appropriate (and unfortunately subjective) "real world" probability model. This is the so-called physical measure, or *P*-measure. The "real world" model should be used for all cash flow projections, consistent with the risk preferences of the market. This is the basis for the valuation of required capital. However, the risk neutral measure is relevant if the company's risk management strategy involves the purchase or sale of derivative securities or other financial instruments in the capital markets. In particular, if the simulation involves buying or trading derivatives, these need to be priced using a risk neutral model embedded in the simulation.

The scenarios created by the SLV model are real world, *P*-measure, scenarios. Importantly, since the SLV model is parameterized to history and only loosely based on current (recent) conditions, it does not produce scenarios that are arbitrage free in respect of the starting yield curve. As such, the SLV scenarios are not appropriate for market pricing of financial instruments (assets or liabilities); they are intended for real world cashflow projections.

Number of Scenarios

The ESWG is not recommending a specific minimum for the number of scenarios. The actuary will choose the scenario set, which might have differing projection periods by product line and asset mix combination. The opining actuary will need to defend the choices made, much as she will for other assumptions and aggregation methods. The same scenario generator must be used across aggregated results.

Mean Reversion Parameter Details

A methodology has been developed to allow the mean reversion parameter (MRP) to adjust over time to better reflect current expectations. The long rate will revert to a simple average of two results, the long rate over the past 50 years (600-month median, adjusted down by 25 basis points) and the long rate over the past 36 months (mean). It is rounded to the nearest 25 basis points to minimize frequent changes in this parameter, but still allow changes to occur if the underlying trend in the long rate persists. This MRP is currently (October 2008) 5.50% and has been steady at this rate since June 2005.

Using a combination of the long-term median rate and a more recent mean to set the MRP has shown a steady, but slow, movement toward current rates over the period since C-3 Phase I was adopted. The MRP since the beginning of 1999 under this formula would have been:

- 6.50% from January 1999 – January 2000
- 6.25% from February 2000-August 2002
- 6.00% from September 2002 – June 2003
- 5.75% from July 2003 – May 2005
- 5.50% since June 2005 (unrounded MRP at October 2008 is 5.56%)

During this period interest rates were in a general state of decline. If interest rates had been steady since 2000, the MRP would have remained at 6.50%. If interest rates would have risen instead of declining the MRP would have slowly increased. An automated, formulaic, approach to updates allows the rate to slowly drift in the direction of the recent movement in interest rates rather than periodically being adjusted with large discontinuities at that time. It should be noted that within the scenario generator, the MRP, once set at the beginning of the projection, is constant for all scenarios and throughout the projection period.

The ESWG has considered several options for updating these parameters. One option is to have no automatic adjustments for the mean reversion parameter. The NAIC would update on an ad hoc basis. This methodology has the advantage of being subject to review before action, but could also be forgotten or politicized, which could delay the implementation of an important development. For example, the C-3 Phase I model has used this ad hoc approach since adoption in 1999 and has not been revisited (the MRP in that model is 6.55%).

Another option is to recalculate the parameters once each year with data updated through the previous year-end. Calculations with projection dates starting with the following March would use these updated parameters.

A third option would be to update more frequently (i.e., quarterly or monthly), using data up to the valuation date (or through the previous quarter).

The ESWG recommends automatic annual updates, with the complete historical monthly data set of Treasury curves used for interpolation (i.e., completion of the full maturity spectrum), while the MRP uses the most recent 600 months of data.

A set of company-generated or SLV subset scenarios must calibrate to the 10,000 Academy SLV scenarios each time a valuation is performed.

The scenarios will be defined by a yield curve reflecting the U.S. Treasury curve at (or as close as possible to) the valuation (start) date. Specifically, it will be defined by the ending curve on the last date available during the period (e.g., the year-end start curve would be the December 31 curve unless the markets were closed that day).

The ESWG has developed a generator for U.S. Treasury interest rates only. We have not addressed the question of how to calibrate an economic scenario generator that combines stochastic processes for both equity and interest rates. For now, it is assumed that each will calibrate independently and no calibration is required for correlation factors. The ESWG did not address interest rate spreads reflecting credit and liquidity, which is the variable driving the recent credit crisis. The recent historical period has not gone outside the Academy generator's range of outcomes for U.S. Treasury rates.

Calibration Details

In order to meet calibration requirements, the scenarios used must meet the following requirements:

- For the short rate and long rate, point in time statistics at 1-, 5-, 10-, and 30-year horizons:
  - Left Tail (low interest rates):
    5^{th} percentile rate ≤ Academy 5^{th} percentile rate + Max(A, B × Academy 5^{th} percentile rate)
  - Right Tail (high interest rates):
    95^{th} percentile rate ≥ Academy 95^{th} percentile rate – Max(A, B × Academy 95^{th} percentile rate)
  - For the 1-year horizon: A = 1.00% and B = 20%
  - For the 5-, 10-, and 30-year horizons: A = 0.50% and B = 10%
- For the spread, cumulative statistics for the 30-year horizon:
  - Left Tail (low spread):
    5^{th} percentile spread ≤ Academy 5^{th} percentile spread + 0.50%
  - Right Tail (high spread):
    95^{th} percentile spread ≥ Academy 95^{th} percentile spread – 0.50%
- All tests must be considered (point-in-time statistics at four time horizons for long and short rates, 30-year cumulative statistic for the spread, with tail statistics considered for both the 5% and 95% levels).
- The Academy percentiles referred to above reflect the 10,000 scenarios created by the SLV interest rate generator provided by the American Academy of Actuaries using the same starting yield curve.

Appendix I

Description of the Stochastic Log Volatility (SLV) Model

The SLV model simulates the following three (3) correlated stochastic processes in discrete monthly time:

1.	${}_1i_t$	The natural logarithm of the long maturity interest rate
2.	α_t	The nominal spread between the long and short maturity rates
3.	ν_t	The natural logarithm of the volatility of the long maturity rate process

The SLV discrete time equations are:

$$
\begin{aligned}
{}_1i_t &= Max\left\langle {}_1\lambda_L, Min\left[{}_1\lambda_U, (1-\beta_1)\cdot {}_1i_{t-1} + \beta_1 \cdot \ln\tau_1 + \psi\cdot\left({}_2\tau_t - \alpha_{t-1}\right)\right]\right\rangle + {}_1\sigma_t \cdot {}_1Z_t \\
\alpha_t &= (1-\beta_2)\cdot\alpha_{t-1} + \beta_2 \cdot {}_2\tau_t + \phi\cdot\left({}_1i_{t-1} - \ln\tau_1\right) + \sigma_2 \cdot {}_2Z_t \cdot \left({}_1r_{t-1}\right)^{\theta} \\
\nu_t &= (1-\beta_3)\cdot\nu_{t-1} + \beta_3 \cdot \ln\tau_3 + \sigma_3 \cdot {}_3Z_t
\end{aligned}
$$

where

$$
\begin{aligned}
{}_1i_t &= \ln\left({}_1r_t\right) \\
{}_1\lambda_U &= \ln\left({}_1r_{Max}\right) \\
{}_1\lambda_L &= \ln\left({}_1r_{Min}\right) \\
{}_2r_t &= \exp\left({}_1i_t\right) - \alpha_t \\
\text{If } {}_2r_t &< {}_2r_{Min}, \text{ then } {}_2r_t = \kappa \cdot {}_1r_t \\
{}_1\sigma_t &= \exp\left(\nu_t\right) \\
{}_1Z_t, {}_2Z_t, {}_3Z_t &\sim N(0,1) \text{ with constant correlation matrix } \rho
\end{aligned}
$$

The parameters for the monthly SLV model are:

Parameter	Value	Name	Description
τ_1	Formula	**Tau1**	Target for the long interest rate process, expressed as a nominal semi-annual yield
β_1	0.00509	**Beta1**	Mean reversion strength for the long rate process
θ	1	**Theta**	Exponent for spread volatility factor
τ_2	0.01	**Tau2**	Target spread between nominal long and short rates
β_2	0.02685	**Beta2**	Mean reversion strength for the spread process
σ_2	0.04148	**Sigma2**	Volatility parameter for the spread process
τ_3	0.0287	**Tau3**	Target volatility for the long rate volatility process
β_3	0.04001	**Beta3**	Mean reversion strength for the log volatility process
σ_3	0.11489	**Sigma3**	Volatility of the log volatility process for the long rate
$\rho(1,2)$	-0.19197	**Correl12**	Correlation between the log long rate and nominal spread processes
$\rho(1,3)$	0	**Correl13**	Correlation between the log long rate and log volatility processes
$\rho(2,3)$	0	**Correl23**	Correlation between the nominal spread and log volatility processes
ψ	0.25164	**Psi**	Steepness adjustment
ϕ	0.0002	**Phi**	Spread tilting parameter
${}_2r_{Min}$	0.004	**Minr2**	Threshold lower bound for nominal short maturity rate
${}_1r_{Min}$	0.0115	**Minr1**	Minimum nominal long maturity rate (before random innovation)
${}_1r_{Max}$	0.18	**Maxr1**	Maximum nominal long maturity rate (before random innovation)
κ	0.25	**Kappa**	Short / Long ratio when nominal short rate falls below the threshold lower bound
${}_1\sigma_0$	0.0287	**InitialVol**	Initial volatility of the log volatility process

Notes on the SLV model and parameters:

- The model simulates log long rates. Hence, the nominal long rate is: ${}_1r_t = exp\left({}_1i_t\right)$
- τ_1 is the target nominal long maturity rate and is often called the "mean reversion parameter" (MRP) for the long rate. It is determined by formula as described in Appendix III.
- The total volatility of the spread process is $\sigma_2 \cdot {}_2Z_t \cdot \left({}_1r_{t-1}\right)^{\theta}$. The parameter θ allows the volatility of the spread process to vary with the level of nominal long interest rates. For example, $\theta = 1$ allows the volatility to vary linearly with the level of rates (i.e., higher spread volatility when interest rates are higher).
- ψ is the "steepness adjustment" for the log long rate. When ${}_2\tau_t > \alpha_{t-1}$ (i.e., the spread in the prior month is less than the long-term target spread), the log long rate (and hence the nominal long interest rate) will "tilt" upwards, thereby increasing the likelihood of spreads increasing. ψ can be interpreted as an indirect way of varying the mean reversion strength of the spread process due to variability in the nominal spread itself.
- ϕ is the "tilting parameter" for the nominal spread. When ${}_1i_{t-1} > \ln \tau_1$ (i.e., the log long rate in the prior month exceeds the target log rate), the spread will "tilt" upwards, thereby increasing the likelihood of increasing spreads. ϕ can be interpreted as an indirect way of varying the mean reversion strength of the spread process due to variability in the log long rate.
- ${}_1\lambda_L$ and ${}_1\lambda_U$ are respectively the "soft" lower and upper bounds for the log long rate process. Both of these bounds are "soft" in that they are applied before the random innovation; hence, simulated values can "break through" the bounds due to random noise. This form of constraint produces much more natural results. Lower and upper bounds were deemed appropriate for the following reasons:
  - Although modeling log rates guarantees positive nominal values, a lower bound avoids extraordinarily low long Treasury bond yields (e.g., less than 0.75%).
  - Since the model simulates log volatility and log long rates, the conversion to a nominal long rate effectively requires a double exponentiation. With enough random trials, extremely high (i.e., > 50%) interest rates would eventually result. As such, an upper bound is needed to avoid a "runaway" process.
- While the nominal long rate will always be positive, without an adjustment there would be no guarantee that nominal short rates would remain non-negative (since spreads can be large even in a low interest rate environment). Accordingly, whenever the nominal short rate ${}_2r_t$ is less than ${}_2r_{Min} = 0.4\%$, it is reassigned to the value $\kappa \cdot {}_1r_t$. In other words, the short rate is set equal to a constant fraction κ of the long rate.

Sample Output as of September 30, 2008

Starting curve

Date	0.25y UST	0.5y UST	1y UST	2y UST	3y UST	5y UST	7y UST	10y UST	20y UST	30y UST
2008-Sep-30	0.92%	1.60%	1.78%	2.00%	2.28%	2.98%	3.38%	3.85%	4.43%	4.31%

Statistics as of Time Horizon 1 year

	Simulated Rates		
	Short (1y)	Long (20y)	Spread
Min	0.40%	2.54%	-0.11%
0.01	0.63%	3.34%	1.16%
0.05	0.96%	3.65%	1.56%
0.1	1.22%	3.78%	1.77%
Median	2.17%	4.31%	2.40%
0.9	3.19%	4.90%	2.88%
0.95	3.54%	5.11%	3.02%
0.99	4.23%	5.59%	3.29%
Max	5.76%	6.82%	4.11%
Avg	2.20%	4.33%	2.36%
Stdev	0.78%	0.45%	0.44%
Skew	0.357	0.401	-0.481
Kurt	0.208	0.803	0.555
Dispersion	1.196	0.340	0.605

Statistics as of Time Horizon 5 years

	Simulated Rates		
	Short (1y)	Long (20y)	Spread
Min	0.40%	1.55%	-2.72%
0.01	0.67%	2.43%	-0.30%
0.05	1.19%	2.92%	0.35%
0.1	1.57%	3.19%	0.68%
Median	3.07%	4.28%	1.76%
0.9	5.07%	5.73%	2.65%
0.95	5.83%	6.31%	2.83%
0.99	7.55%	7.82%	3.19%
Max	13.41%	13.28%	4.62%
Avg	3.24%	4.40%	1.70%
Stdev	1.44%	1.08%	0.77%
Skew	0.903	1.078	-0.439
Kurt	1.798	3.117	0.116
Dispersion	1.510	0.793	1.411

Statistics as of Time Horizon 10 years

	Simulated Rates		
	Short (1y)	**Long (20y)**	**Spread**
Min	0.32%	1.20%	-5.36%
0.01	0.86%	2.22%	-0.83%
0.05	1.46%	2.82%	-0.11%
0.1	1.86%	3.16%	0.24%
Median	3.56%	4.55%	1.37%
0.9	6.19%	6.64%	2.47%
0.95	7.32%	7.53%	2.70%
0.99	10.21%	9.91%	3.12%
Max	20.13%	17.93%	6.21%
Avg	3.88%	4.79%	1.35%
Stdev	1.89%	1.54%	0.87%
Skew	1.524	1.483	-0.275
Kurt	5.062	4.840	0.172
Dispersion	1.648	1.033	2.058

Statistics as of Time Horizon 30 years

	Simulated Rates				History 1953.04-2008.09		
	Short (1y)	**Long (20y)**	**Spread**		**Short (1y)**	**Long (20y)**	**Spread**
Min	0.37%	1.35%	-9.39%	Min	0.82%	2.57%	-3.33%
0.01	0.91%	2.38%	-1.45%	0.01	1.09%	2.65%	-1.83%
0.05	1.67%	3.02%	-0.50%	0.05	1.51%	3.08%	-0.98%
0.1	2.07%	3.41%	-0.11%	0.1	2.23%	3.78%	-0.34%
Median	4.16%	5.22%	1.07%	Median	5.31%	6.15%	0.91%
0.9	7.94%	8.46%	2.28%	0.9	9.35%	10.51%	2.68%
0.95	9.63%	9.86%	2.61%	0.95	11.57%	12.04%	3.38%
0.99	13.71%	13.49%	3.27%	0.99	14.87%	13.86%	3.80%
Max	21.80%	19.04%	9.51%	Max	16.72%	15.13%	4.08%
Avg	4.69%	5.66%	1.06%	Avg	5.62%	6.59%	0.97%
Stdev	2.58%	2.21%	0.97%	Stdev	2.97%	2.63%	1.23%
Skew	1.639	1.528	-0.225	Skew	1.041	0.886	0.172
Kurt	4.490	3.669	1.420	Kurt	1.382	0.510	0.177
Dispersion	1.915	1.309	2.928		1.895	1.456	4.788

Appendix II

Completion of the Yield Curve

The SLV model simulates long (20 year) and short (1 year) Treasury rates. The rest of the yield curve is based on the relationships from the "best fit" historic curve using the data set from 1953 to present (data set gets bigger each time it is updated) according to the following steps:

1. A "sample" mid-term (7-year) rate is derived based on a deterministic formula (derived from history):

$$ {}_{mid}r_t = (1-\phi)\times {}_{mid}r_{t-1} + \phi \times [\,{}_2r_t + \omega_t + \chi_t \cdot ({}_1r_t - {}_2r_t)] $$

   where:

   ϕ = 0.6487417 is the mean reversion strength

   ω_t = 0.0009903 if ${}_1r_t \geq {}_2r_t$ (i.e., long rate ≥ short rate), otherwise 0.0007359

   χ_t = 0.6845636 if ${}_1r_t \geq {}_2r_t$ (i.e., long rate ≥ short rate), otherwise 0.7453071

2. The curve that produces the lowest weighted sum of squared deviations is the "best fit" curve. For a given simulated scenario/duration node, the 1-year, sample 7-year and 20-year rates are compared to history with weights = 40, 20, 40 respectively.

3. The simulated (i.e., final) 7-year rate is estimated from the historic best fit curve; that is, the "sample rate" is only used to determine the "best fit" curve.

4. The interpolation (extrapolation) for key maturities is described below. The following notation is used:

   - ${}_mr_t$ is the nominal interest rate of maturity "*m*" (in years); this notation is different from that used elsewhere in this document (e.g., ${}_1r_t$ indicates the 1-year maturity rate at time *t*, not the 20-year maturity rate). For greater clarity, ${}_1r_t$ and ${}_{20}r_t$ respectively represent the simulated 1-year and 20-year Treasury yields at time *t* for a given scenario.

   - The subscript *hyc* indicates the value from the "best fit" historic curve

   For maturities < 1 year (i.e., for *m* = 0.25 and 0.50):

$$ {}_mr_t = {}_1r_t \times \left(\frac{{}_mr_{hyc}}{{}_1r_{hyc}} \right) $$

   For maturities > 20 years (i.e., for *m* = 30):

$$ {}_mr_t = {}_{20}r_t \times \left(\frac{{}_mr_{hyc}}{{}_{20}r_{hyc}} \right) $$

   For intervening maturities (i.e., m = 2, 3, 5, 7 and 10):

$$ {}_{m}r_{t} = \Omega_{m} \times {}_{1}r_{t} \cdot \left(\frac{{}_{m}r_{hyc}}{{}_{1}r_{hyc}} \right) + \left(1 - \Omega_{m}\right) {}_{20}r_{t} \cdot \left(\frac{{}_{m}r_{hyc}}{{}_{20}r_{hyc}} \right) $$

where Ω_m is an interpolation factor based on the maturities:

$$ \Omega_{m} = \left(\frac{20 - m}{20 - 1} \right) $$

Appendix III

The Mean Reversion Parameter for the Long Interest Rate Process

Background

When the Academy's C3WG established the Mean Reversion Parameter (MRP) during the late 1990s using Maximum Likelihood Estimation (MLE) to fit the stochastic variance model, the MRP was set at 6.55%. This compares to an average (median) of 6.94% (6.99%) for the GS20 for the period 1953 to 1995. The current average (median) for the period 1953 to 2005 is 6.71% (6.38%). Other factors in the model along with the MLE optimization, primarily the steepness adjustment, biased the MRP to be slightly lower than the long-term average or median.

Development of the MRP

- The MRP (target τ_1 for long rate) is $0.5 \times (M - 25 \text{ bps}) + 0.5 \times A$, rounded to nearest 25 bps, where

  M = median 20-year yield over most recent 600 months
  A = arithmetic average 20-year yield over preceding 36 months

Recommendation of the ESWG

The ESWG is recommending a modification of the model's MRP for the Long Rate (LR) from 6.55% to an "unrounded" value of 5.56%, and a rounded value of 5.50%. The basis for this change is a shift in perspective, from a completely historical viewpoint, to a prospective view driven by an analysis of Federal Reserve Bank behaviors and objectives. While the MRP recommendation for today's environment is 5.50%, the ESWG believes that, if long-term economic and market expectations were to change substantially in the future, then the MRP recommendation would have to be reconsidered. These expectations include inflation, real growth, market liquidity and other risk preferences.

Furthermore, the ESWG recommends that the long rate revert to a simple average of the median long rate over the past 50 years (600-month median adjusted down by 25 bps) and the average over the past 36 months (as of the measurement date). It is rounded to the nearest 25 basis points to minimize frequent updates to this parameter, while still allowing changes to occur if the underlying trend in the long rate persists. The ESWG suggests that this calculation be made at the end of each calendar year and held steady for the following calendar year.

This view is a compromise of the competing views presented below.

1. The MRP should be based on history.

Support for this view is based on practical considerations, as it contends that models based on expectations are very complicated, and difficult to calibrate and obtain a consensus on. These challenges are inconsistent with the resources available to the Academy that are needed to maintain such models on a regular basis. On the other hand, a model based entirely on history is objective and easy to parameterize.

However, most who support the use of history to measure the MRP accept that we can't simply use the average of known history, as this would produce an MRP that is higher than 6% at the current date. Thus, some degree of subjectivity is still likely to be required, and those who hold this view would peg the MRP lower, at a level in the 5.7% – 5.8% range.

Another major difficulty with this approach is that it is highly dependent on the selected historical period. Depending on the specific historical period chosen, the results could be materially inconsistent with how the economy is being managed going forward and with market expectations about future interest rates. This distribution of historical long rates simply does not have enough data points to have stabilized.

In selecting the historical dataset, if one chooses the period of known data since 1953 but we exclude the 1970s and 1980s because it reflects an era of "easy money," we get the following average values:

Historical period	GS20[1]	CPI
1953–2005	6.71%	3.81%
1953–1970 & 1990–2005	5.23%	2.44%

As we discuss below, a 2.44% inflation assumption is consistent with current economist expectations that are measured in a survey by the Philadelphia Federal Reserve Bank. Thus, a 3.81% inflation rate would probably not be tolerated by the Federal Reserve.

2. The MRP should be based on current Federal Reserve expectations of future rates.

The Federal Reserve is charged with maintaining "full" employment and stable prices. There have been periods of time when Fed policy was compromised by emphasizing one objective over the other. An example of this is the "easy money" policy of the 1970s, which was initiated to maintain high employment. However, this did not lead to full employment and "stagflation" was the result. The economy and market settled into a phase of expecting inflation and this self-fulfilling expectation countered any employment benefits from an "easy money" policy. The Federal Reserve has set goals stating that low inflation, in the range of 1.5–2.0%, is the best policy to preserve a healthy growing economy and high employment. This view was held in the Volcker/Greenspan era of the 1980s and 1990s and has continued under current Fed Chairman Benjamin Bernanke's leadership.

As an illustration, this view would set the MRP at approximately 4.90%, based on historical quarterly information over the last three years, using a building block approach.

Inflation Expectations	2.30%
TIPS Yield (20-yr)	2.20%
Risk Premium	0.40%
GS20 Expectation	4.90%

In the table above, the source for inflation expectations is the Philadelphia Fed survey of economic forecasters, which measures the ability of the Federal Reserve to control inflation. Note the unofficial Fed target for inflation is 1.50% to 2.00%. The TIPS and GS20 yields are from the Federal Reserve website. The risk premium, reflecting the uncertainty in inflation expectations, is set to the residual to arrive at the total GS20 Expectation.

The TIPS yield is a real interest rate and can also be viewed as a component of expected GDP real growth. If a combination of factors were to materialize, such as the Fed attaining its inflation target, while at the same time conveying an expectation of a more stable inflationary environment, along with a fall of GDP into a slow growth longer term trend, then the GS20 yield could drop considerably. For example, under this scenario, economic forecasters may expect inflation to be 1.80%, and the risk premium and TIPS yield may drop to 0.20% and 1.00%, respectively. Adding these three components, we obtain a 3.00% GS20 yield. However, under a scenario of low inflation, GDP is likely to grow faster and the TIPS yield would move higher. Of course, the opposite can happen and the GS20 could rise to 6.00% or more.

Although more accurate, the challenge with relying on Federal Reserve policy is that it is subjective and maintenance of the Academy model parameters is higher.

3. The MRP should be set consistent with current market expectations.

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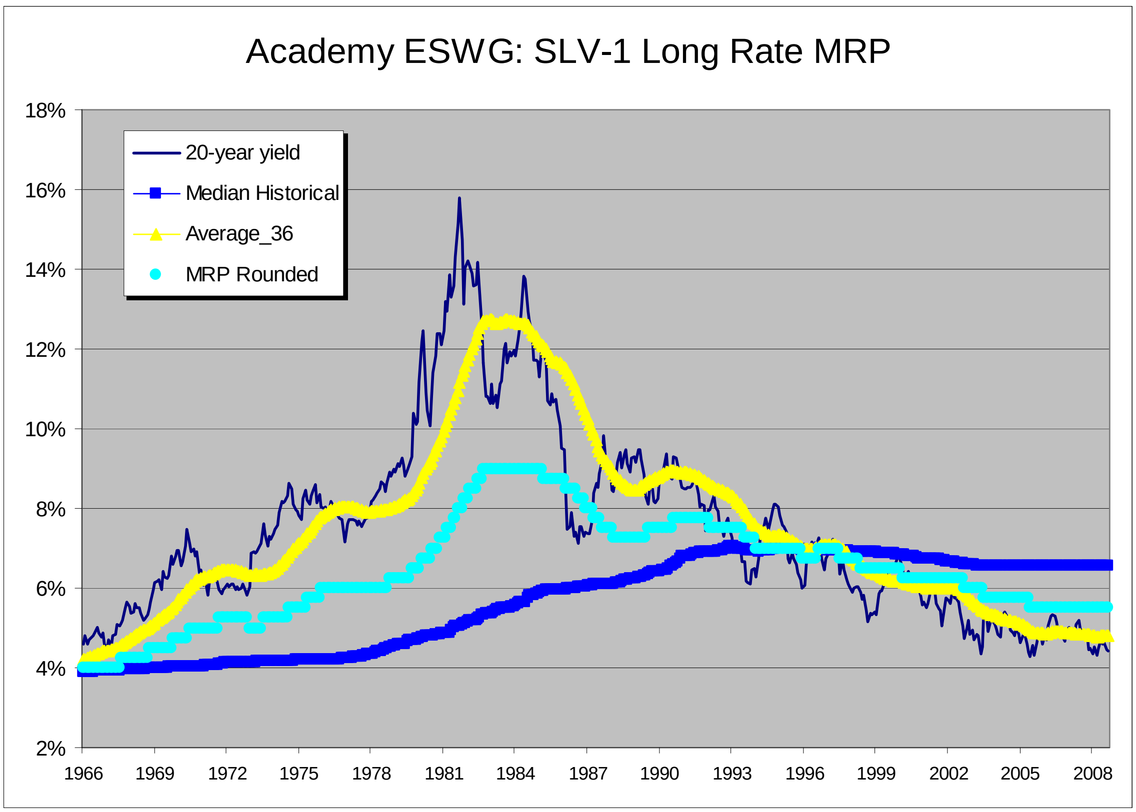
1 U.S. government securities/Treasury constant maturities/Nominal. This analysis has previously been presented to NAIC using this historical period and has not materially changed.

This approach is based on the simple proposition that market interest rates reflect a blend of pessimism and optimism concerning the market. For example, if market participants are pessimistic and see interest rates rising, they are inclined to sell bonds. If they see rates falling they will typically buy bonds. That is, pricing in the market will settle into equilibrium between pessimists and optimists. Furthermore, this approach is based on the assumption that Federal Reserve policy is inherently reflected in current market prices and interest rates.

This method involves selecting a recent historical period to measure market interest rates. The longer the period selected, the more likely idiosyncrasies in the market will tend to cancel each other out. However, a shorter period will tend to favor outdated temporary inflationary expectations and other market expectations about the future. The appropriate period is probably in the three-to-five-year range. If a three-year period ending in 2005 is selected, the average GS20 yield is 4.88%. If a five-year period is selected, the average is 5.14%. Therefore, a number in the 4.9%–5.1% range would be consistent with market expectations.

The chart below shows various components of the MRP formulation. Note that the line labeled Median Historical includes fewer than 600 data points when 50 years of data was not available prior to 2003.



Appendix IV

August 12, 2008

Larry Bruning, Chair, Life and Health Actuarial Task Force
Leslie Jones, Vice Chair, Life and Health Actuarial Task Force
National Association of Insurance Commissioners
c/o John Engelhardt

Re: Company Generated Scenarios in VM-20

Dear Mr. Bruning and Ms. Jones:

The American Academy of Actuaries'[2] Life Financial Soundness/Risk Management Committee (also known as the SVL2 Steering Committee) believes that all principle-based approaches should allow for the option to use company-generated stochastic scenarios that meet required calibration criteria (i.e., for both separate account fund performance and interest rate scenarios). In order to accomplish this, the SVL2 Steering Committee recommends that the NAIC Life and Health Actuarial Task Force should reject proposed amendment VM-20_080329_004 (Proposed Amendment) which seeks to limit the source of stochastic scenarios to those developed by regulators (i.e., pre-packaged scenarios) in the *Requirements for Principle-Based Reserves for Life Products* (VM-20). The SVL2 Steering Committee believes the language in the March 29, 2008 exposure of VM-20 remains appropriate and will allow companies to better apply the objectives of a principle-based approach (PBA) in the development of reserves for life products.

The use of company-generated stochastic scenarios supported by appropriate calibration criteria will allow companies to better meet the concepts of a principle-based approach both now and in the long-term. In particular, companies will more effectively capture the risks associated with their specific contracts and will do so in a way that "utilizes risk analysis and risk management techniques to quantify the risks." By using company-generated stochastic scenarios, companies will be "guided by the evolving practice and expanding knowledge in the measurement and management of risk" and will be allowed to incorporate "assumptions, risk analysis methods and models and management techniques that are consistent with those utilized within the company's overall risk assessment process."[3] For example, the calibration criteria for equities in C-3 Phase II contain not just gross wealth ratios (also known as calibration points), but also include guidance and requirements on the development of scenarios, including calibration of funds other than the S&P 500, the correlation of fund returns, the use of random number generators, the number of scenarios, efficiency in estimation, frequency of projection, time horizon, and the appropriateness of using the optional pre-packaged scenarios. The criteria also require the equity scenarios to be available in electronic format to facilitate review.

The sub-committees and work groups of the SVL2 Steering Committee have been very involved in the development of the provisions allowing for the optional use of company-generated stochastic scenarios, which are contained in the current exposure of VM-20 and in other PBA proposals. They have also been involved in the development of the calibration criteria that support these provisions. For example:

1. The provisions and calibration criteria are based on those contained in C-3 Phase II and proposed Actuarial Guideline VACARVM, which was the culmination of a three-year process, and included many discussions with, and much feedback from, members of LHATF.
2. The Academy's Economic Scenario Work Group (ESWG) has been working for over two years to develop guidelines that allow companies the option to use alternative interest rate scenario generators and is in the process of developing a recommendation for required calibration criteria.

[2] The American Academy of Actuaries' mission is to serve the public on behalf of the U.S. actuarial profession. The Academy assists public policymakers on all levels by providing leadership, objective expertise, and actuarial advice on risk and financial security issues. The Academy also sets qualification, practice, and professionalism standards for actuaries in the United States.
[3] All the quotes in this paragraph are from the 5/31/08 exposure of VM-00.

To prohibit the use of company-generated stochastic scenarios would be going backwards, not forward.

The development of these provisions recognizes the fact that no single model is superior to others, that the science of stochastic modeling has and continues to evolve, and that principle-based approaches should encourage, and not hinder, that evolution. The SVL2 Steering Committee is not aware of any other jurisdictions that require companies to use only specific scenarios approved by the regulator in advance.

While the SVL2 Steering Committee believes the use of pre-packaged scenarios has its place in a PBA, we are concerned that by only allowing the use of a pre-packaged set of scenarios, the Proposed Amendment would require companies to rely on scenarios generated by a single (one set fits all) source. As with the application of the current reserve and risk-based capital systems to certain products, it is unlikely that these scenarios will work for all companies in all situations, and thus will increase the likelihood that material risks will be missed or misstated with reserves either under- or over-estimated. Here are some practical considerations:

1. The Academy's Life Capital Adequacy Subcommittee's June 2005 Report on C-3 Phase II stated, "The calibration criteria should permit a wide range of reasonable and popular models, provided such models are suitably parameterized. It is the calibration itself that establishes the 'common ground' and helps to narrow the range of acceptable practice without dictating the specific form of the model or parameters. Indeed, the shortcomings of simpler models can often be overcome by accepting certain compromises and adjusting the parameters."

2. The pre-packaged scenarios will not include scenarios for all indices to which companies map their funds and their assets. Asset classes affected by foreign exchange rates are not considered, nor are alternative asset classes. This will result in inaccurate modeling when scenarios for a given index are not available, necessitating a role for the NAIC in updating the scenarios.

3. The pre-packaged scenario sets are not all correlated and an approach to add correlation has not been developed. The use of required scenarios that lack correlation will be particularly ineffective for companies with hedging programs in that they would be required to use models that are inconsistent with those used to implement the strategy.

4. Companies that employ hedging strategies often need to incorporate risk-neutral scenarios into their models. In order to value a dynamic hedging strategy (e.g., one supporting VUL guaranteed benefits), risk-neutral scenarios will likely be needed at future points within each real world scenario. As of each valuation date, a set of risk-neutral scenarios will be needed that validate to the current market value of the hedge assets. As the model projection rolls the hedge assets forward under a particular scenario, new risk-neutral scenarios will be needed at future points in time to revalue the hedge assets. The scenario generators used to develop the pre-packaged scenarios do not currently produce risk-neutral interest rate scenarios.

5. Models improve when there is a transparent flow of information. For example:

   a. In the UK, the FSA has shared some of what it has learned in the review of financial firms' filings as its approach is to have the actuary defend assumptions through peer review.

   b. The ESWG is currently analyzing calibration requirements for interest rate scenarios by soliciting a variety of interest rate generators and comparing them against the distribution of results from the 10,000 generated scenarios consistent with those that were presented to the NAIC in 2007.

   Such approaches help both vendors and companies to incorporate new best practices into models and improve their risk management tools. This iterative improvement process becomes very difficult in an environment where companies are required to use pre-packaged scenarios.

6. Insurers provide a market for risk to be shared. Modelers provide a market for risk management tools. Allowing a variety of generators will encourage improvements to best practices by modelers, and this in turn improves the management of risk. By requiring only pre-packaged scenarios, this improvement process will be hindered. By allowing companies to generate their own scenarios with the appropriate calibration criteria, modelers will be encouraged to adapt and better understand the economic environment. While these models should not be considered predictive they are very instructive. These improvements help the user to better plan strategically for potential future scenarios. It's also important to consider that

there is quite an advantage to have consistency between scenarios used to manage the business and those used for regulatory purposes.

One of the reasons given for desiring the exclusive use of a single set of scenarios is that it would assist in the review process, providing consistency between companies, assisting in checking for compliance, and helping in the comparison of the resulting reserves. The SVL2 Steering Committee understands the importance of these concerns, but believes they should not hinder the utilization and development of better tools and that they can be addressed by making the review process more uniform and streamlined. A standard reporting template with required illustrative tests, for example, could help further support the transparency of the calibration criteria. This could include elements of the deterministic reserve calculation and/or the deterministic scenarios associated with the stochastic exclusion test. Qualitative and even additional quantitative elements to the calibration criteria could also be added if needed. The SVL2 Steering Committee is willing to work with members of LHATF to address these issues.

In conclusion, the SVL2 Steering Committee believes that company-generated stochastic scenarios (i.e., for both separate account fund performance and interest rate scenarios) are not only a valuable part of a PBA, but are a necessary part of a principle-based approach and that their use should be encouraged rather than hindered. We are concerned that removing this aspect of the modeling process could result in a model that does not move forward to a more accurate reserve determination since it potentially overstates or understates the impacts of certain risks. The SVL2 Steering Committee does understand the complexities involved in these proposals and the affect this may have on the review process and is ready and willing to assist in addressing these concerns while allowing for the highest quality models.

Sincerely,

Donna Claire, Chair
American Academy of Actuaries' Life Financial Soundness and Risk Management Committee