Appendix H

Loading Validation

In an effort to validate the relationship between various mortality loads, such as the nonsmoker load being generally larger than the smoker load when expressed as a percent but smaller when expressed in dollars per $1,000, the Academy Task Force explored the purposes of the load as well as a hypothetical load based on a number of unknowns.

Purposes of the Load

There are a number of reasons for the load and the purpose of each gives some indication of the kind of margin that should be added.

1. **Confidence** -- The mortality represented by a valuation mortality table should be sufficient to cover the mortality underlying the study upon which the valuation table is based. If the study is based on limited data, a significant margin may be needed in order to assure that the resulting table covers all potential underlying mortality levels. The 1990-95 SOA mortality study was based on a significant amount of data, so the amount of load needed to produce sufficient confidence for the 2001 CSO Table is negligible.

2. **Company Variation** – The mortality represented by a valuation mortality table should be sufficient to cover the mortality expected to be experienced by most of the companies that will use the table. In the Academy Task Force's work, we have assumed that the table should be sufficient to cover the experience of 71 percent of the companies that will use it (this corresponds to the 15 percent overall load that the LHATF directed the Academy Task Force to use in the development of the 2001 CSO Table).

   The Academy Task Force spent considerable time attempting to evaluate the size of the margin necessary for company variation. Ideally, the Academy Task Force would have looked at the experience of the various companies that contributed to the study on a cell-by-cell basis and designed a table such that it covered the results of a given percentage of those companies. Unfortunately, while the aggregate amount of data the Academy Task Force had for the study was quite large, the amount of data available for individual companies for specific cells was, in many cases, quite small or even non-existent. The Academy Task Force was able to get an idea of the overall variation by company, but it did not believe that there was enough data to get a similar feel for groupings of the data.

The Academy Task Force discussed how company experience might vary from one company to the next. For the most part, one might guess that these differences are due to differences in underwriting, either the level of underwriting or the acceptance level. Within a particular company, different levels of underwriting are frequently expressed as percentages of standard. For example, a substandard class is typically expressed as a percentage of the standard class. Based on this logic, it would appear that the margin due to company variation should be expressed as a percentage of the base rate.

3. **Random Fluctuation** -- The table should be sufficient to cover random fluctuations in mortality experience that are expected in the mortality of the companies that will use it. In some respects, covering random fluctuation is the complement of the confidence issue. The confidence issue deals with random fluctuation in the industry experience underlying the table due to limitations in the industry experience. The random fluctuation issue deals with the variability in results expected by a particular company because the company covers a limited number of insureds.

   The margin necessary for random fluctuation will not be a flat percentage of the mortality rate. In a binomial distribution, the standard deviation of the distribution is easily calculated. In general, since the standard deviation is a square root function, the standard deviation becomes a smaller percentage of the probability as the probability becomes higher. In other words, as the mortality rate increases there is less need for margin. This indicates that the margin for random fluctuation is not a percentage of the base rate, but rather a decreasing percentage of that rate as the rate goes up.

   Some examples of this are shown in Table H-1. Columns 4 and 5 show the actual percentage load in the 2001 CSO Table (obtained by dividing the 2001 CSO Table by the 2001 VBT and subtracting one) for nonsmokers and smokers, respectively. Columns 6 and 7, show the ratio of the standard deviation to the mean for a binomial distribution with 10,000 lives exposed and mortality based on the 2001 VBT, for nonsmokers and smokers, respectively. That is,

$$\text{Ratios in columns 6 and 7} = \frac{\sigma}{\mu} = \frac{\sqrt{n \cdot q \cdot (1-q)}}{n \cdot q} = \sqrt{\frac{(1-q)}{n \cdot q}}$$

   where n = number of lives and q = 2001 VBT mortality rate. This gives us a theoretical construct indicating the percentage load required in the 2001 CSO Table to provide for mortality deviating up to one standard deviation from the expected.

Table H-1

Comparison of Load in Select & Ultimate 2001 CSO Table To Ratio of Binomial Standard Deviation to Binomial Mean For Nonsmokers and Smokers (Binomial Distribution Assumes 10,000 Lives)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Gender	Issue Age	Dur	2001 CSO % Load		Binomial σ/μ Ratio	
			Nonsmoker	Smoker	Nonsmoker	Smoker
Male	35	1	71.0%	39.7%	56.8%	39.8%
		5	38.4	20.8	37.0	25.5
		10	30.0	16.0	27.7	18.8
		15	21.8	11.7	20.1	13.6
		20	19.2	10.5	15.9	10.8
		25	16.2	9.8	12.2	8.7
	55	1	65.5%	31.9%	29.0%	18.7%
		5	30.2	15.9	17.0	11.3
		10	16.7	9.8	10.5	7.3
		15	13.2	8.7	7.6	5.6
		20	11.0	8.2	5.6	4.4
		25	9.7	8.2	4.2	3.5
Female	35	1	95.2%	60.5%	69.0%	51.3%
		5	57.8	33.7	47.1	33.5
		10	36.4	20.1	31.8	22.1
		15	25.5	14.2	22.8	15.6
		20	19.5	10.9	17.0	11.6
		25	16.0	9.6	13.1	9.2
	55	1	74.2%	33.8%	32.8%	20.4%
		5	33.8	16.7	19.3	12.4
		10	21.7	12.2	13.0	8.8
		15	15.3	9.8	9.1	6.5
		20	12.4	9.1	6.7	5.2
		25	11.6	8.8	5.3	4.1

Table H-1 demonstrates that, for a binomial distribution, the ratio of the standard deviation to the mean decreases as the underlying mortality rate increases. Consistent with this observation, the percentage load in the 2001 CSO Table was constructed to decrease as the mortality increases.

4. **Unknown Variation** -- A valuation mortality table must cover not only expected results, but also a range of unexpected results. Reserves are intended to assure that a sufficient amount of money is retained in the early years to pay claims in the later years, but the mortality experience of the future is not known. A margin should be added so that the table will be sufficient most of the time. Examples of the kind of unknown variation that could occur include one-time events, such as a flu epidemic, as well as changes in overall mortality levels that might occur due to things like AIDS or changes in general health conditions.

The margin for unknown variation is the largest uncertainty. By its very nature, it is unknown. In conversations about potential mortality aberrations, actuaries typically talk in terms of extra deaths per thousand. On the other hand, it is not inconceivable that changes in mortality levels could happen that could simply increase all mortality rates by a particular percentage.

The Academy Task Force's chosen formula has margins that increase in absolute terms, but decrease in percentage terms, as age increases. The margins also increase in absolute terms as the expectation of life decreases.

Development of a Hypothetical Loading Formula

Based on the four purposes of loading discussed above, a hypothetical loading formula was developed. The Academy Task Force did not use this formula in the development of the 2001 CSO Table, but it did use this formula to validate the relationship in loads between smokers and nonsmokers.

The hypothetical loading formula is as follows:

$$q_{[x]+t}^{Loaded} = \left[q_{[x]+t}^{VBT} \cdot (1 + m_1) + m_3\right] \cdot (1 + m_2)$$

where: $q_{[x]+t}^{Loaded}$ = hypothetically loaded mortality rate

$q_{[x]+t}^{VBT}$ = mortality rate from 2001 Valuation Basic Table

m_1 = loading factor that relates to company variation

m_2 = loading factor that relates to random fluctuation

m_3 = loading factor that relates to unknown variation

The m_2 values are based on the binomial standard deviation over the binomial mean for 100,000 lives with the binomial q based on the underlying VBT rate. If the m_1 value was set at 9% and the m_3 value was set at 0.10 extra deaths per 1,000 , hypothetically loaded mortality rates are produced that are reasonably close to the 2001 CSO Table rates. The hypothetical mortality rates are within 3 percent of the 2001 CSO Table's mortality rates in 36 of the 48 cells shown in Table H-2 on the following page, within 5 percent in 40 of the 48 cells, and within 10 percent in 45 of the 48 cells.

Table H-2

Results of Hypothetical Loading Formula

										Difference	
M/F	NS/SM	x	t	$q^{VBT}_{[x]+t}$	m_1	m_2	m_3	$q^{Loaded}_{[x]+t}$	$q^{CSO}_{[x]+t}$	#	%
M	NS	35	1	.00031	.09	.180	.0001	.00052	.00053	-.00001	-3%
			5	.00073	.09	.117	.0001	.00100	.00101	-.00001	-1%
			10	.00130	.09	.088	.0001	.00165	.00169	-.00004	-2%
			15	.00248	.09	.063	.0001	.00298	.00302	-.00004	-1%
			20	.00396	.09	.050	.0001	.00464	.00472	-.00008	-2%
			25	.00668	.09	.039	.0001	.00767	.00776	-.00009	-1%
		55	1	.00119	.09	.092	.0001	.00153	.00197	-.00044	-23%
			5	.00344	.09	.054	.0001	.00406	.00448	-.00042	-9%
			10	.00904	.09	.033	.0001	.01028	.01055	-.00027	-3%
			15	.01685	.09	.024	.0001	.01891	.01908	-.00017	-1%
			20	.03067	.09	.018	.0001	.03413	.03405	.00008	0%
			25	.05486	.09	.013	.0001	.06068	.06016	.00052	1%
	SM	35	1	.00063	.09	.126	.0001	.00089	.00088	.00001	1%
			5	.00154	.09	.081	.0001	.00192	.00186	.00006	3%
			10	.00282	.09	.059	.0001	.00336	.00327	.00009	3%
			15	.00539	.09	.043	.0001	.00623	.00602	.00021	4%
			20	.00848	.09	.034	.0001	.00966	.00937	.00029	3%
			25	.01306	.09	.027	.0001	.01472	.01434	.00038	3%
		55	1	.00285	.09	.059	.0001	.00340	.00376	-.00036	-10%
			5	.00773	.09	.036	.0001	.00883	.00896	-.00013	-1%
			10	.01853	.09	.023	.0001	.02076	.02034	.00042	2%
			15	.03083	.09	.018	.0001	.03431	.03350	.00081	2%
			20	.04863	.09	.014	.0001	.05385	.05264	.00121	2%
			25	.07584	.09	.011	.0001	.08368	.08205	.00163	2%
F	NS	35	1	.00021	.09	.218	.0001	.00040	.00041	-.00001	-2%
			5	.00045	.09	.149	.0001	.00068	.00071	-.00003	-4%
			10	.00099	.09	.100	.0001	.00130	.00135	-.00005	-4%
			15	.00192	.09	.072	.0001	.00235	.00241	-.00006	-2%
			20	.00344	.09	.054	.0001	.00406	.00411	-.00005	-1%
			25	.00583	.09	.041	.0001	.00672	.00676	-.00004	-1%
		55	1	.00093	.09	.104	.0001	.00123	.00162	-.00039	-24%
			5	.00269	.09	.061	.0001	.00322	.00360	-.00038	-11%
			10	.00589	.09	.041	.0001	.00679	.00717	-.00038	-5%
			15	.01201	.09	.029	.0001	.01357	.01385	-.00028	-2%
			20	.02154	.09	.021	.0001	.02407	.02421	-.00014	-1%
			25	.03453	.09	.017	.0001	.03838	.03852	-.00014	0%
	SM	35	1	.00038	.09	.162	.0001	.00060	.00061	-.00001	-2%
			5	.00089	.09	.106	.0001	.00118	.00119	-.00001	-1%
			10	.00204	.09	.070	.0001	.00249	.00245	.00004	1%
			15	.00409	.09	.049	.0001	.00478	.00467	.00011	2%
			20	.00743	.09	.037	.0001	.00850	.00824	.00026	3%
			25	.01177	.09	.029	.0001	.01330	.01290	.00040	3%
		55	1	.00240	.09	.064	.0001	.00289	.00321	-.00032	-10%
			5	.00646	.09	.039	.0001	.00742	.00754	-.00012	-2%
			10	.01267	.09	.028	.0001	.01430	.01422	.00008	1%
			15	.02296	.09	.021	.0001	.02565	.02521	.00044	2%
			20	.03619	.09	.016	.0001	.04018	.03950	.00068	2%
			25	.05614	.09	.013	.0001	.06209	.06110	.00099	2%

Conclusion

Based on this analysis, it is clear that a number of acceptable margins could be developed, and that there is no one, appropriate margin. Some purposes of a load indicate that a percentage of the base rate is appropriate, while other purposes show that a flat amount may make more sense. Based on this analysis, the Academy Task Force developed a loading formula that increases in dollars per $1,000 as the underlying mortality rate increases, but decreases when expressed as a percent as the underlying mortality rate increases.