

AAA Economic Scenario Generator: Supporting Statistics and Sensitivity Analysis

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Agenda for this Session

- Background on the AAA Generator
- Evaluation of the Baseline AAA Generator
 - Statistical Description
 - Mean Reversion Parameter
- Statistical Illustrations of AAA Generator
- Sensitivity Testing of Parameters
- Appendix: ESIWG Update to LHATF, June 2009



Background on the Academy Generator

- AAA interest rate generator released in December, 2008
- ESWG chose to continue with Stochastic Log Volatility model used for C-3 Phase I adopted in 1999
 - Different types of generators were evaluated, but AAA decided to continue use of the SLV generator. Comparable results were obtained for different types of generators (e.g. double mean reverting, etc...)
 - Generator design and parameter choices based on intended use of the generator – calculation of long term liabilities and associated capital
- Refreshed some parameters using Treasury data from 1953–2008 with the most historical data available
 - Believe it is important to use a historical period long enough to cover business and credit cycles
 - Selecting a particular historical period as justification for the direction of future rates can create bias in the generator, as many elements influence rates in a selected time frame (e.g. Fed actions)



Background on the Academy Generator

- Soft cap of 18% limits the maximum long rate (reduces maximum rates with minimal impact on overall results)
- Yield curve interpolation uses historical curves
- Established processes (formulas) for automatically updating Mean Reversion Parameter (MRP) for target long interest rate
 - Long rate is the 20-year Treasury rate
 - Recommended MRP is 5.50%; C-3 Phase I MRP is 6.55%



....a list of all the parameters....

Parameter	Value	Name	Description
τ_1	Formula	Tau1	Target for the long interest rate process, expressed as a nominal semi-annual yield
β_1	0.00509	Beta1	Mean reversion strength for the long rate process
θ	1	Theta	Exponent for spread volatility factor
τ_2	0.01	Tau2	Target spread between nominal long and short rates
β_2	0.02685	Beta2	Mean reversion strength for the spread process
σ_2	0.04148	Sigma2	Volatility parameter for the spread process
τ_3	0.0287	Tau3	Target volatility for the long rate volatility process
β_3	0.04001	Beta3	Mean reversion strength for the log volatility process
σ_3	0.11489	Sigma3	Volatility of the log volatility process for the long rate
$\rho(1,2)$	-0.19197	Correl12	Correlation between the log long rate and nominal spread processes
$\rho(1,3)$	0	Correl13	Correlation between the log long rate and log volatility processes
$\rho(2,3)$	0	Correl23	Correlation between the nominal spread and log volatility processes
ψ	0.25164	Psi	Steepness adjustment
ϕ	0.0002	Phi	Spread tilting parameter
	0.004	Minr2	Threshold lower bound for nominal short maturity rate
	0.0115	Minr1	Minimum nominal long maturity rate (before random innovation)
	0.18	Maxr1	Maximum nominal long maturity rate (before random innovation)
κ	0.25	Kappa	Short / Long ratio when nominal short rate falls below the threshold lower bound
${}_1\sigma_0$	0.0287	InitialVol	Initial volatility of the log volatility process

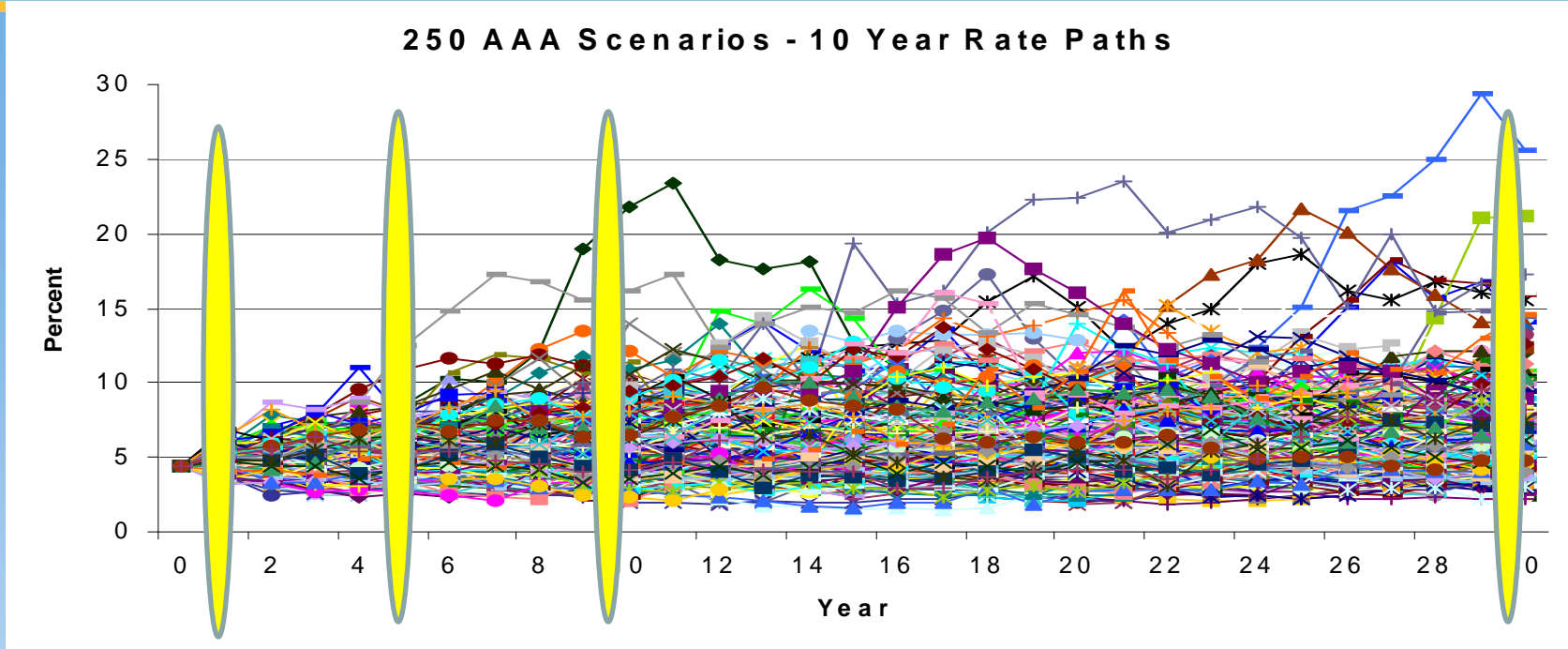


Evaluation of the Academy Generator: Objectives

- **Dispersion of results across scenarios was a key factor in evaluating the generator and parametric choice.**
 - Dispersion of results *across scenarios* is a standard method for evaluating generators.
 - Evaluating the dispersion or path of results *within a scenario* would not provide sufficient data points to be credible and characterizing a generator by a path of results would be very similar to specifying deterministic scenarios.
- **Particular attention was given to the tail scenarios.**
 - Recall that tail scenarios are captured in the reserve and capital calculations in two ways: through the tail scenarios modeled in the generator process and use of CTE risk metric in establishing reserves/capital.



Illustration of Scenario Paths



1. Initial yield curve is input into generator
2. Rates on the yield curve are projected (monthly for 30 years)
3. The projection of the 1yr, 5yr, 10yr, and 30yr rates is important
4. A statistical distribution of rates at particular points in time is constructed to evaluate the robustness of the generator.

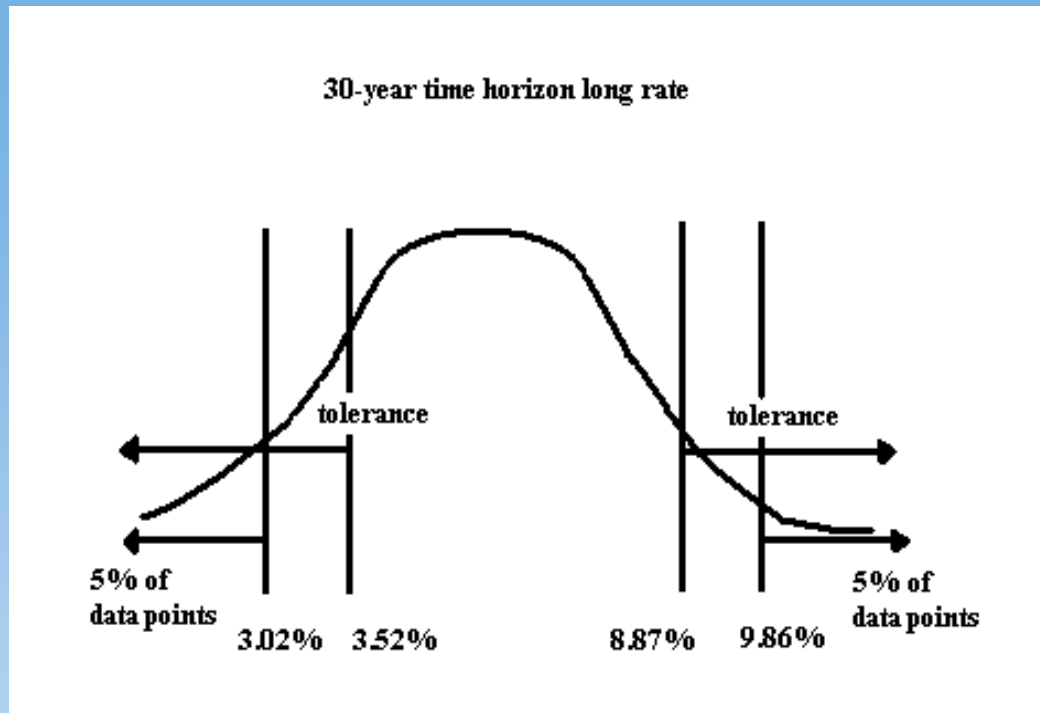


Evaluation of the Generator: Statistical Framework

- For the short rate and long rate, point in time statistics at 1-, 5-, 10-, and 30-year horizons:
 - Left Tail (low interest rates):
 - 5th percentile rate \leq Academy 5th percentile rate + $\text{Max}(A, B \times \text{Academy 5}^{\text{th}} \text{ percentile rate})$
 - Right Tail (high interest rates):
 - 95th percentile rate \geq Academy 95th percentile rate – $\text{Max}(A, B \times \text{Academy 95}^{\text{th}} \text{ percentile rate})$
 - For the 1-year horizon: $A = 1.00\%$ and $B = 20\%$
 - For the 5-, 10-, and 30-year horizons: $A = 0.50\%$ and $B = 10\%$
- For the spread, cumulative statistics for the 30-year horizon:
 - Left Tail (low spread):
 - 5th percentile spread \leq Academy 5th percentile spread + 0.50%
 - Right Tail (high spread):
 - 95th percentile spread \geq Academy 95th percentile spread – 0.50%
- All tests must be considered (point-in-time statistics at four time horizons for long and short rates, 30-year cumulative statistic for the spread, with tail statistics considered for both the 5% and 95% levels).
- The Academy percentiles referred to above reflect the 10,000 scenarios created by the SLV interest rate generator provided by the American Academy of Actuaries using the same starting yield curve.



Evaluation of the Generator: Illustration of the Statistical Framework



MEAN REVERSION PARAMETER (MRP)

- MRP is based on the long rate. Academy generator includes 2 changes to MRP:
 - Change to MRP value from 6.55% to a rounded value of 5.50%. Change based on shift from completely historical perspective to a combined historical perspective and prospective view driven by an analysis of Federal Reserve Bank behaviors and objectives.
 - Reversion of the long rate to a simple average of the median long rate over the past 50 years (600-month median adjusted down by 25 bps) and the average over the past 36 months (as of the measurement date).
- Academy generator also includes a process for automatically updating the MRP based on recent experience.



Statistical Illustrations: Baseline Scenarios

Statistics as of Time Horizon 10 year

	Short Rate (1 Yr)	Long Rate (20 Yr)	Spread (20YR - 1YR)
Min	0.37%	1.24%	-3.07%
0.01	0.70%	1.88%	-1.19%
0.05	1.37%	2.42%	-0.38%
0.1	1.70%	2.66%	-0.11%
0.15	1.97%	2.89%	0.10%
Median	3.00%	3.83%	0.76%
0.9	5.15%	5.41%	1.64%
0.95	5.96%	6.15%	1.94%
0.99	7.93%	7.74%	2.57%
Max	17.80%	17.75%	3.55%
Avg	3.24%	4.01%	0.77%
Stdev	1.53%	1.27%	0.74%



Statistical Illustrations: Sensitivity Testing of the One Year Rate at the Ten Year Horizon

	Baseline	MRP+1%	MRP-1%	mean reversion off	mean reversion off, wider caps
Min	0.37%	0.40%	0.33%	0.28%	0.20%
0.01	0.70%	0.87%	0.52%	0.64%	0.62%
0.05	1.37%	1.64%	1.07%	1.12%	1.12%
0.1	1.70%	1.98%	1.37%	1.46%	1.46%
Median	3.00%	3.43%	2.52%	3.04%	3.04%
0.9	5.15%	5.81%	4.41%	5.92%	5.92%
0.95	5.96%	6.73%	5.13%	7.15%	7.15%
0.99	7.93%	8.90%	6.87%	10.85%	10.85%
Max	17.80%	18.28%	17.18%	23.51%	36.58%
Avg	3.24%	3.70%	2.74%	3.48%	3.48%
Stdev	1.53%	1.68%	1.36%	2.04%	2.07%
Skew	2.113	1.945	2.388	2.047	2.421
Kurt	11.605	9.279	15.639	8.460	15.394



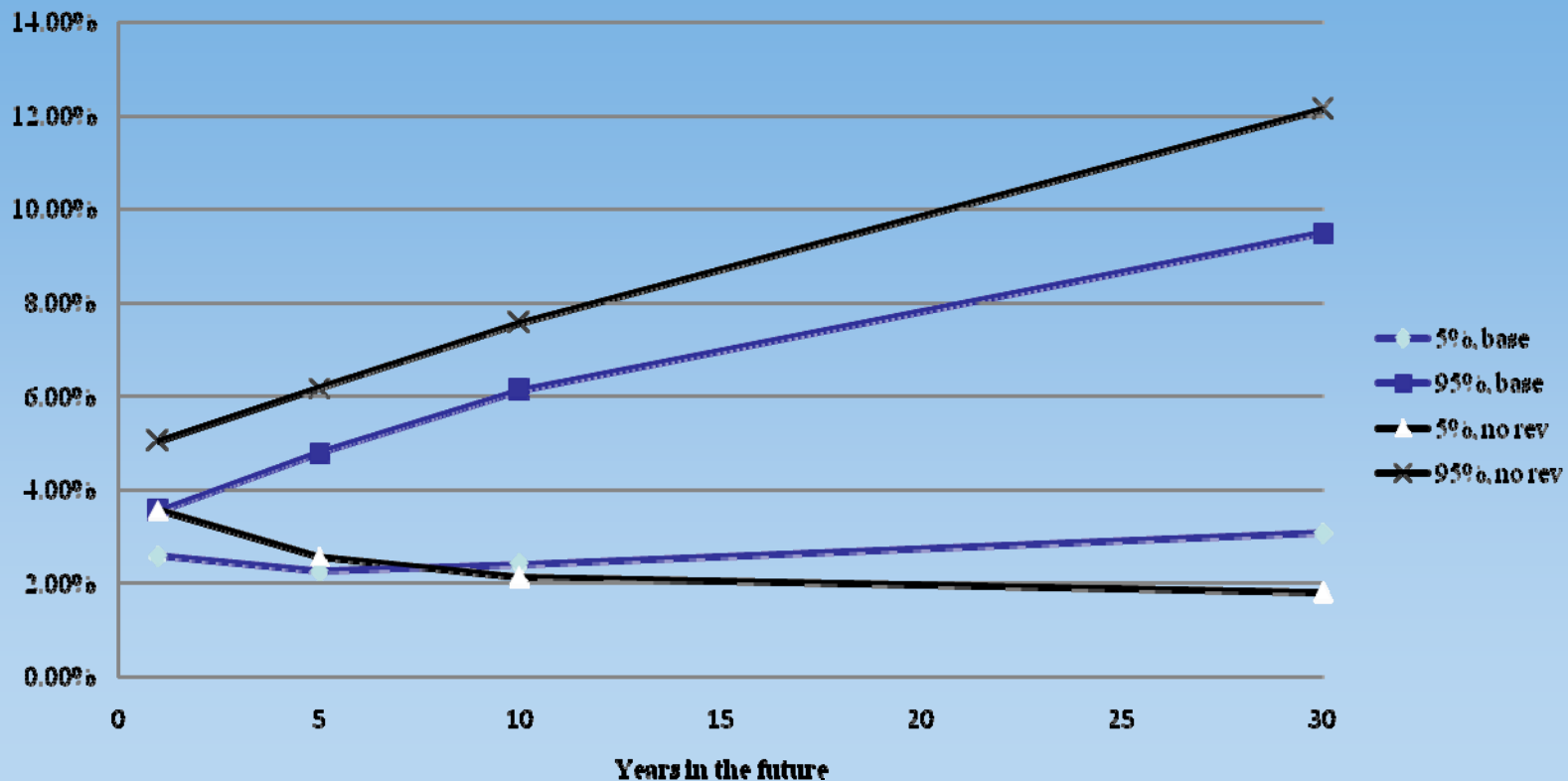
Statistical Illustrations: Sensitivity Testing of the Twenty Year Rate at the Ten Year Horizon

	Baseline	MRP+1%	MRP-1%	mean reversion off	mean reversion off, wider caps
Min	1.24%	1.35%	1.18%	1.03%	0.76%
0.01	1.88%	2.06%	1.68%	1.59%	1.58%
0.05	2.42%	2.65%	2.18%	2.15%	2.15%
0.1	2.66%	2.90%	2.39%	2.48%	2.48%
Median	3.83%	4.21%	3.43%	3.97%	3.97%
0.9	5.41%	5.96%	4.82%	6.47%	6.47%
0.95	6.15%	6.77%	5.48%	7.59%	7.59%
0.99	7.74%	8.54%	6.84%	10.72%	10.72%
Max	17.75%	17.92%	17.52%	18.67%	37.10%
Avg	4.01%	4.40%	3.59%	4.31%	4.31%
Stdev	1.27%	1.39%	1.15%	1.81%	1.84%
Skew	2.218	1.941	2.676	1.854	2.348
Kurt	15.415	11.096	23.166	6.750	16.640
Dispersion	0.930	0.936	0.920	1.371	1.371



Sensitivity Testing of Parameters

Effect of Removing Mean Reversion Distribution of Generated 20yr treasury rate



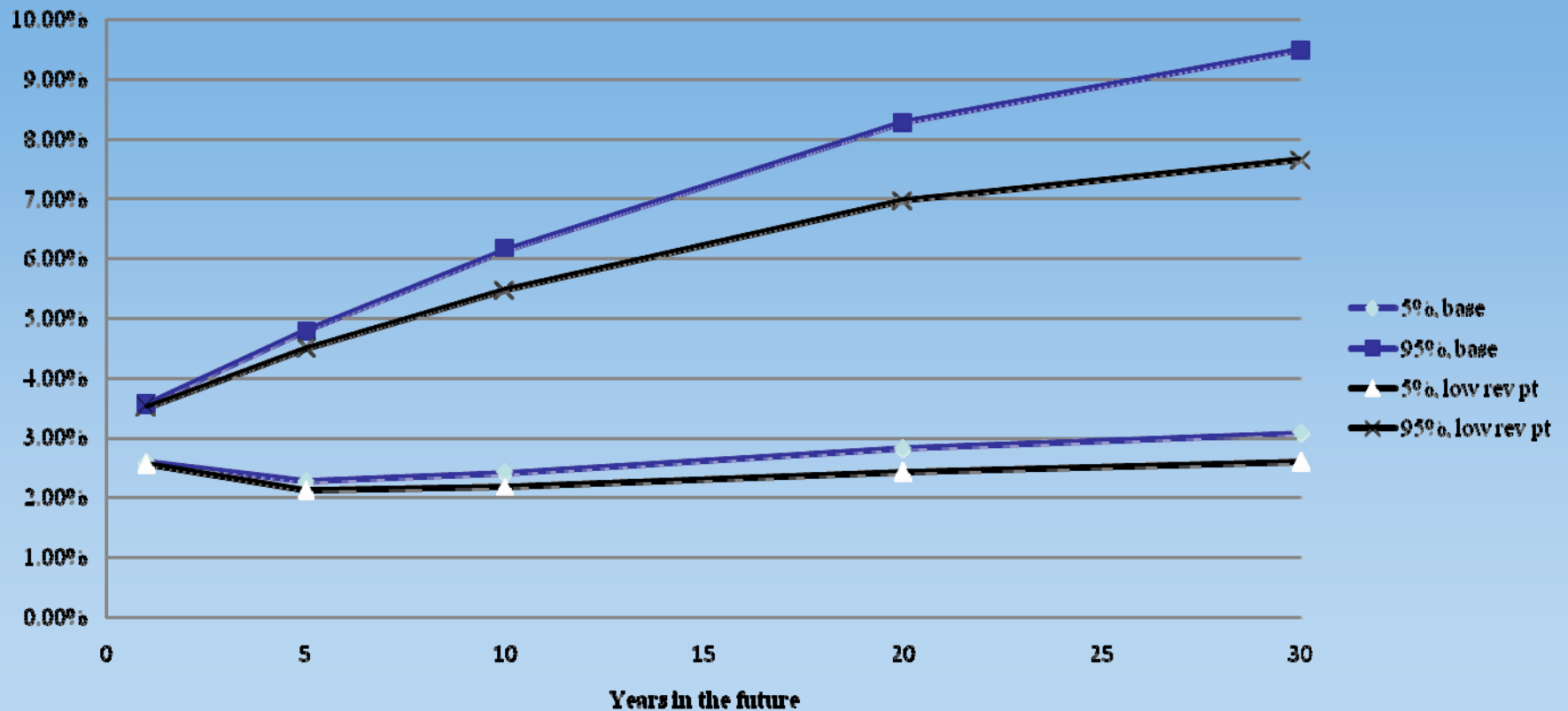
Effect of Removing MRP

- In the short run, the distribution is lower because there is no upward attraction to the mean reversion point, which in this case is higher than the starting level of interest rates.
- In the long run, the distribution is wider, with more “very high” and more “very low” interest rates.
- The effect of removing the MRP is relatively insignificant over periods less than 30 years since the strength of the MRP is fairly weak in the base case.



Sensitivity Testing of MRP

Effect of 1% Lower Mean Reversion Point Distribution of Generated 20yr Treasury Rate

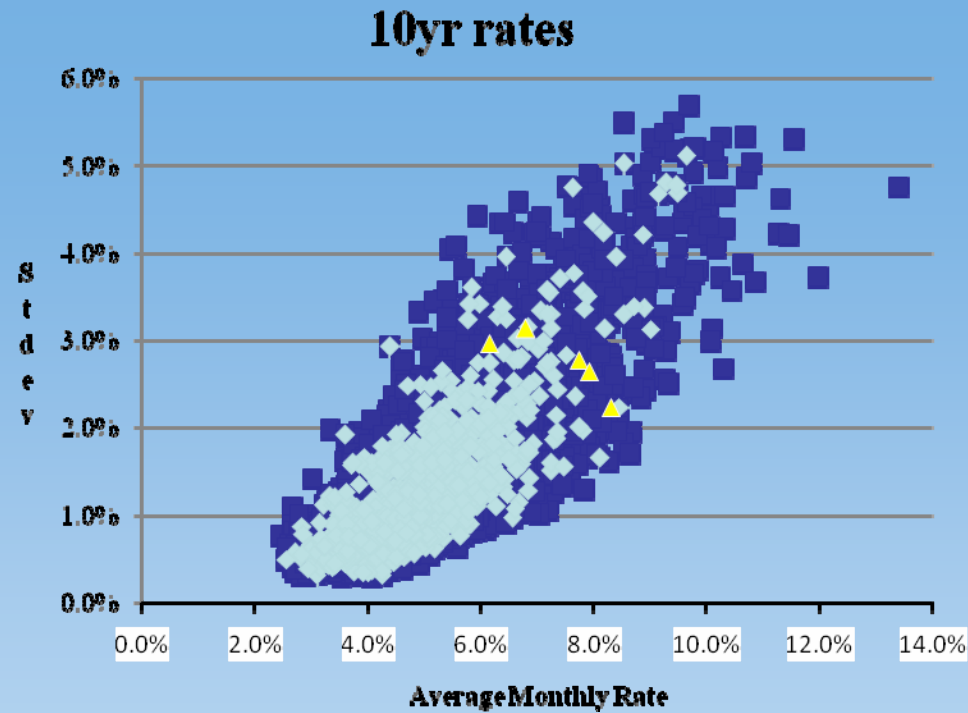


Effect of 1% Lower Mean Reversion Point

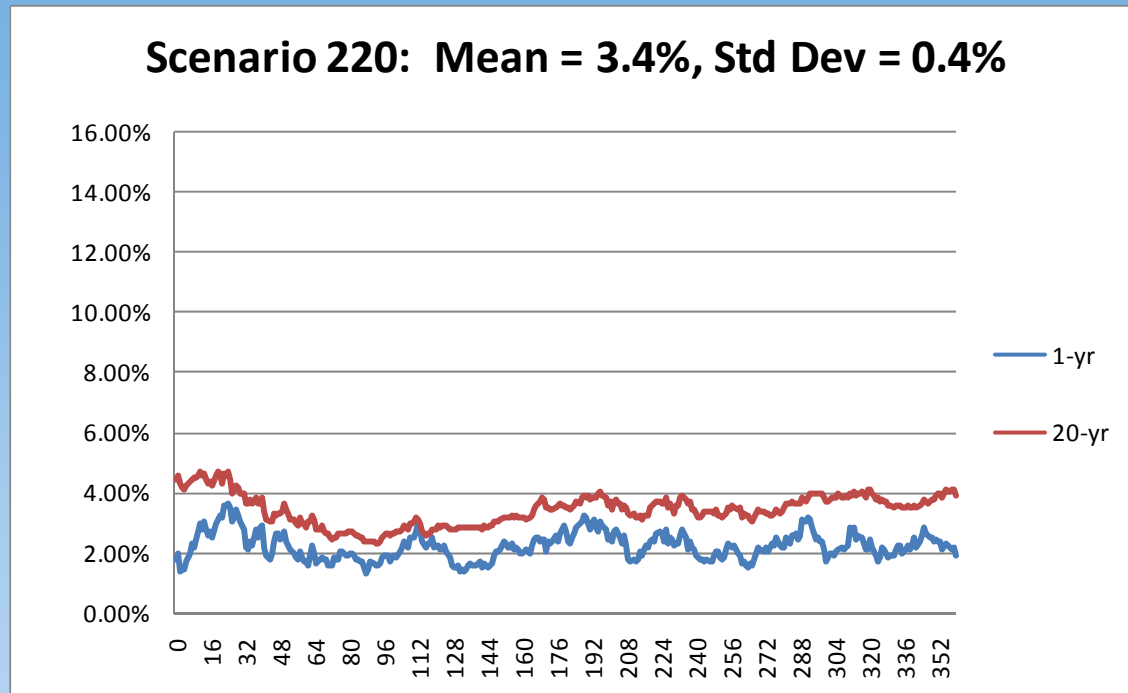
- The entire distribution of future rates is dragged down due to the attraction to a lower rate.
- The effect on the lower end of the distribution is smaller than the effect on the high end because of the lognormal nature of the model whereby the volatility of interest rates is proportional to the level.



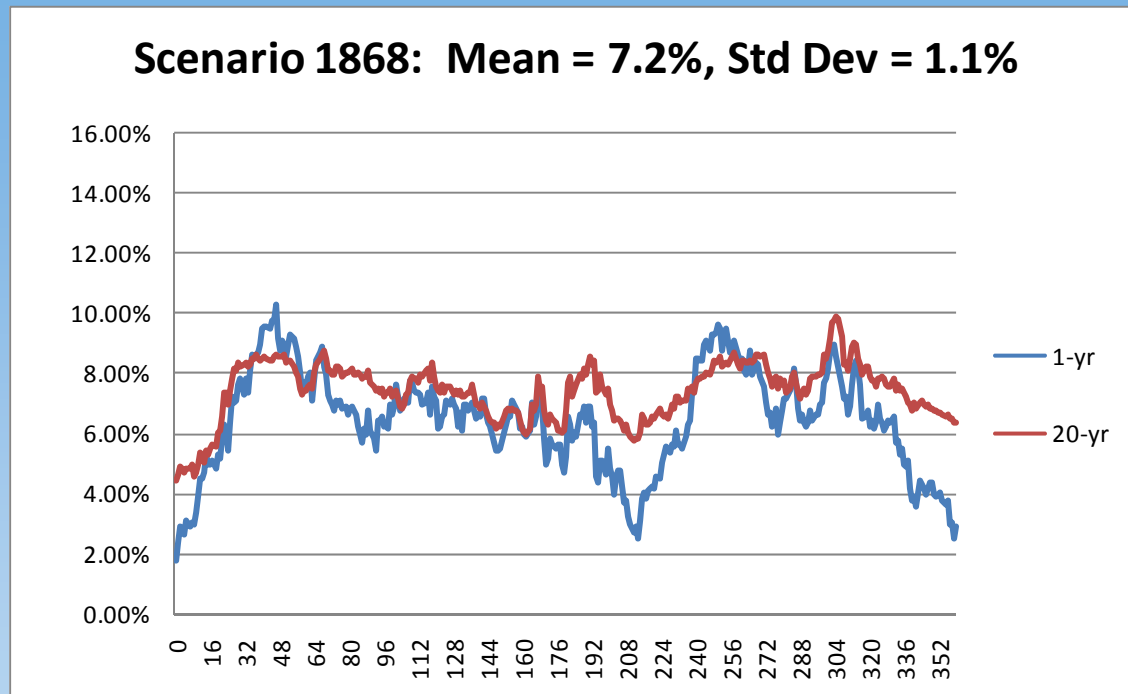
Statistical Illustrations of AAA Generator



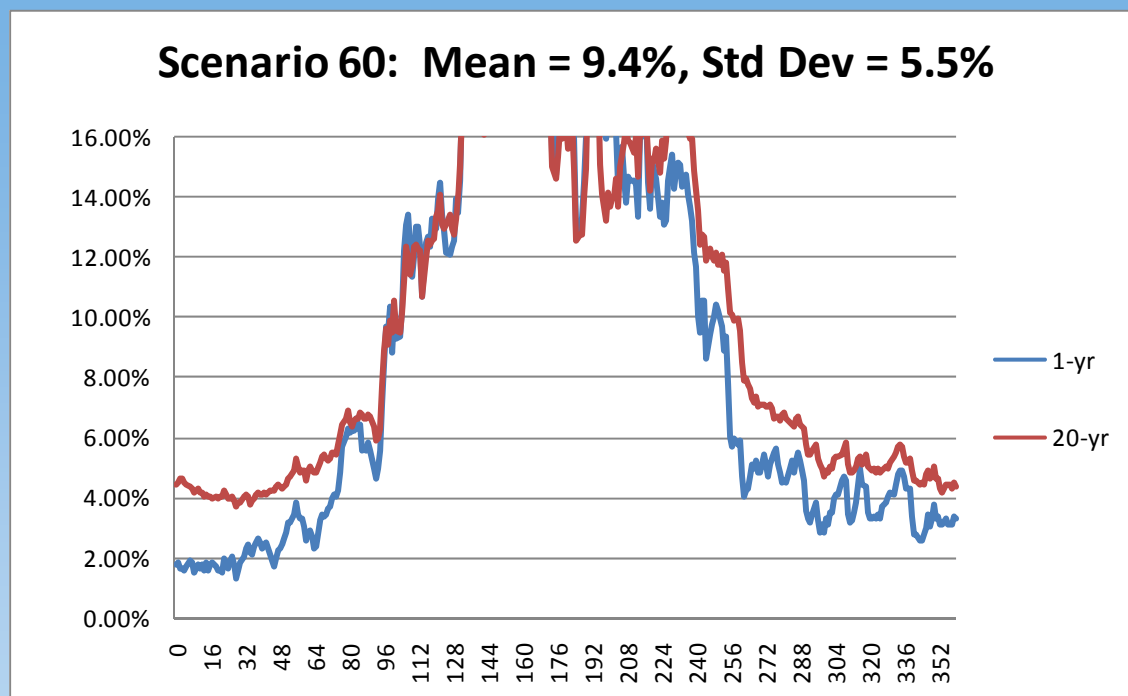
Persistent Low Interest Rates



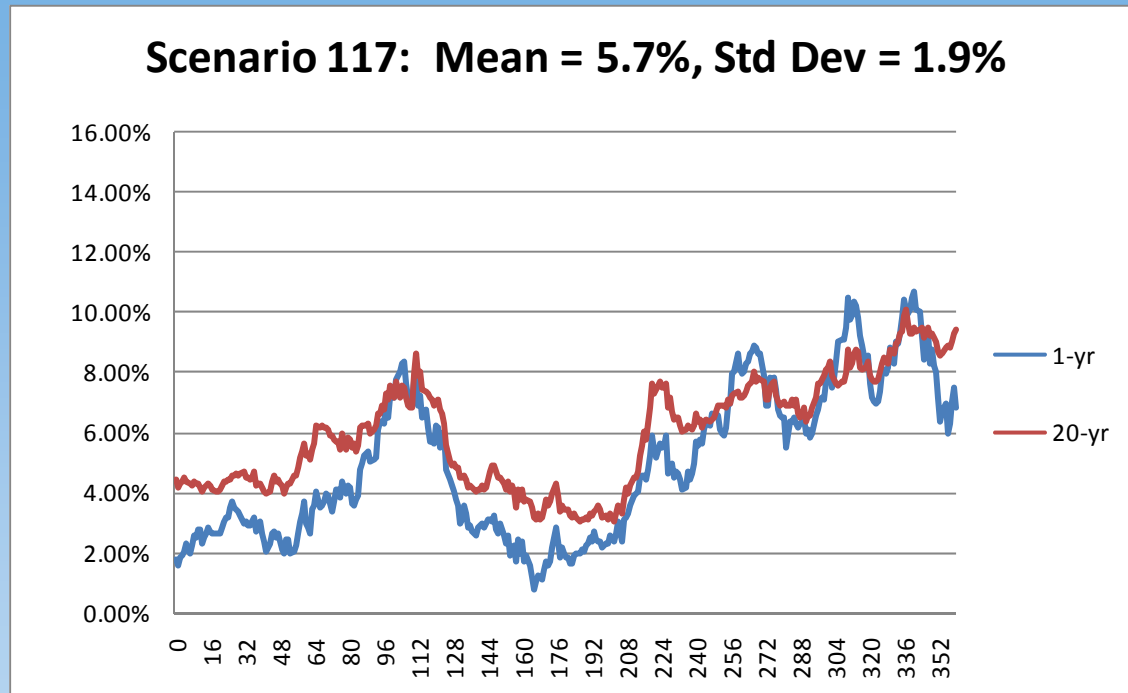
Persistent High Interest Rates



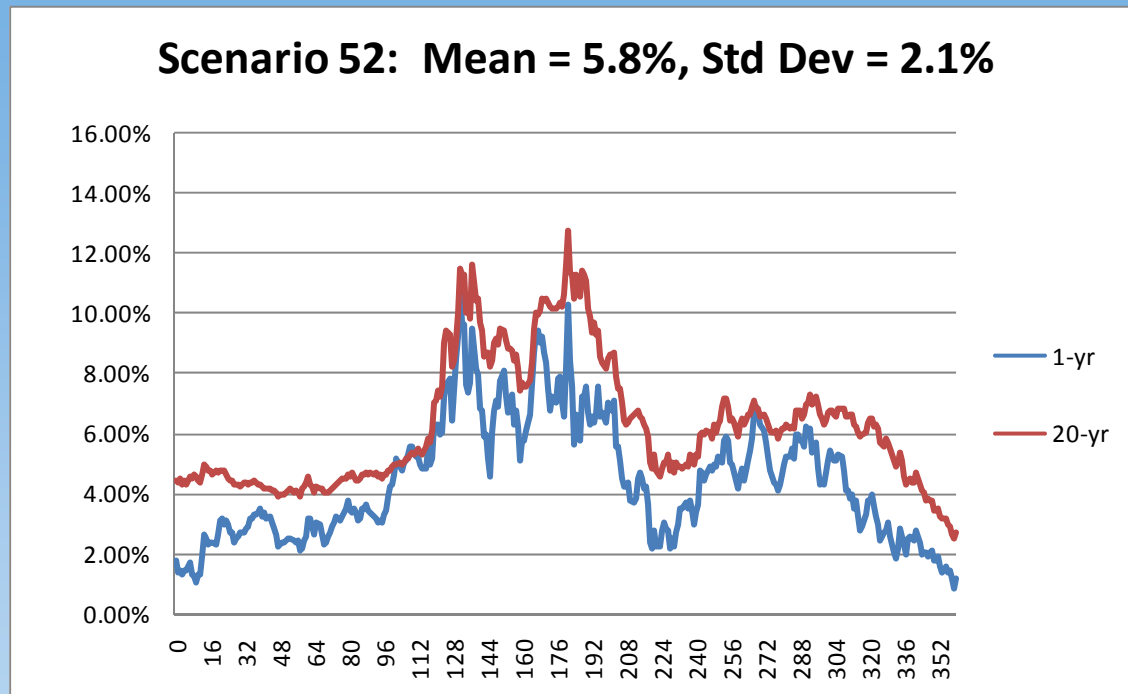
Wide Range of Interest Rates



More Typical Scenario



Another More Typical Scenario



QUESTIONS?



Appendix



AAA's Economic Scenario
Implementation Work Group
(ESIWG)
Update to LHATF

Nancy Bennett, Chair, ESIWG

June, 2009 NAIC Meeting



Discussion Topics

- Use of Economic Generators: Current State
- Use of Economic Generators: Future State
- Recent ESIWG Activity
- ESWG/ESIWG Plans
- ESWG/ESIWG Position on Generators
- NAIC/LHATF Role



Use of Economic Generators: Current State

- The C3P1 and C3P2 calculations are based on multiple economic scenarios.
- C3P1 is based on a pre-packaged set of 12 or 50 interest rate scenarios generated by the company, based on the AAA ESWG interest rate generator. Recall that the chosen scenario sets are based on interest rate mismatch for representative annuities and investment strategies.
- For some companies, C3P2 calculations are based on a set of prepackaged scenarios published by the ESWG. These scenarios satisfy calibration criteria recommended and approved by the NAIC. For other companies, C3P2 calculations are based on scenarios generated from a proprietary generator that also satisfy calibration criteria.
- VACARVM calculations, effective for year end 2009, will use the scenarios provided for the C3P2 calculation.



Use of Economic Generators: Future State

- Stochastic reserve calculations requiring a scenario generator are specified in VM-20 (Life Products), VM-21 (Variable Annuities), and an anticipated VM-22(Annuities).
- With the development of PBA for life insurance (reserves and C3P3), the ESWG developed a more robust interest rate generator and calibration criteria to support stochastic calculations for all products.
 - The ESWG has recommended that this interest rate generator be used in the C3P1 calculation, replacing the existing generator that produces the set of 12 and 50 scenarios.
 - The generator could be used to generate updated prepackaged scenarios and calibration criteria for bond funds and/or interest rate scenarios for the C3P2 and VACARVM calculations for YE2009; updated bond returns from this new generator would likely require approval by the NAIC and could affect company preparation for the new VACARVM requirements.



Recent ESIWG Activity

- Released updated interest rate scenario generator:
 - IR generator is a stochastic log volatility model and generates realistic scenarios. Generator includes a mean reversion parameter updated for recent experience and an automatic process for updating the parameters based on updated historical yield curves.
 - 10,000 scenarios updated for September 30, 2008 have been released
 - Scenario picking tool and 1000 interest scenarios calibrated to September 30, 2008 environment have been released
 - Statistics generator has been released
- Responding to LHATF sub-group's questions
 - Sensitivity of scenario statistics to changes in parameters
 - Additional discussion of certain development choices



ESWG/ESIWG Plans

- Expand documentation with FAQ Document and Getting Started Guide
- Continue to enhance generators
 - Additional user flexibility
 - Develop ability to generate bond fund returns in enhanced IR generator
 - Enhance equity generator to include process for automatically updating parameters based on recent historical experience
- Continue work with LHATF and LRBCWG
 - Discuss process for approving generators
 - Define process for generating economic scenarios on an ongoing basis



ESWG/ESIWG Position on Generators

- Use of one interest rate generator and one equity generator for all principle-based reserve and capital calculations
- Permit the use of a company generator with prescribed calibration criteria in addition to prescribed prepackaged scenarios
- ESWG generator and calibration criteria have been developed with practical considerations in mind
 - Will not require frequent development.
 - Generator includes process to automatically update parameters
 - ESWG generator considered to be a “safe harbor” generator sufficient for regulatory minimums. However, more sophisticated generators will capture additional risks in the scenarios and the use of more sophisticated generators should be allowed in PBA.



NAIC/LHATF Role

- Approve the recently released interest rate generator and calibration criteria
- Discuss the maintenance of the generator process and output on a routine basis (e.g. prepackaged scenarios vs. generators/calibration criteria, updated parameters, resources)
- Update Valuation Manual and RBC Instructions to reflect consistent, clear alternatives

