Proposed “quadrant” criteria for the joint distribution of interest rates and equity returns

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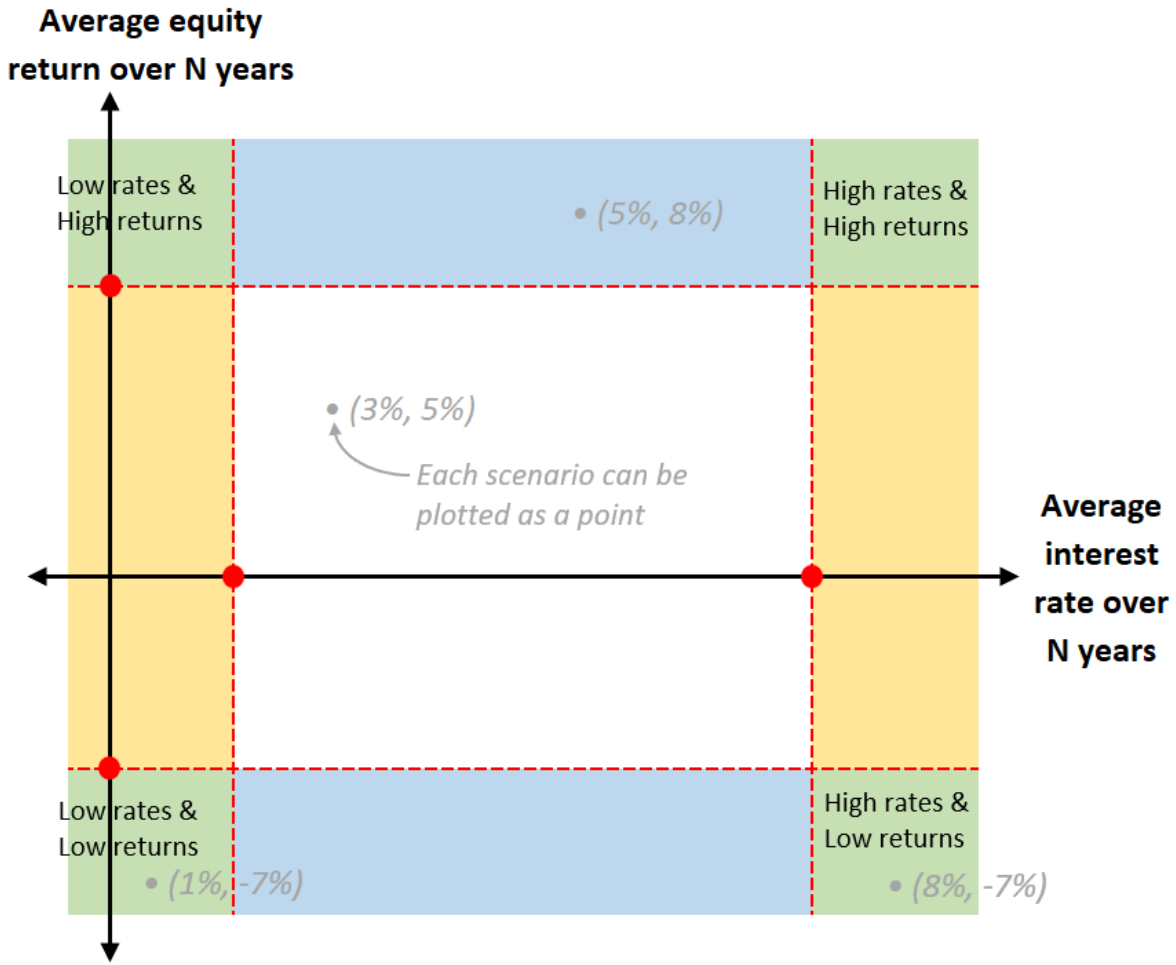
“Quadrant” criteria can be used to evaluate the frequency and severity of scenarios in the “four corners” of the joint distribution of interest rates and equity returns



- Proposed quadrant criteria are focused on deciles of the joint distribution of geometric average rates and returns over a horizon.
- Two forms of this criteria are proposed
  - **“Frequency” quadrant criteria:**
    - Quadrants are defined by explicit bounds (red lines) based on deciles from reference model(s).
    - Relevant statistic is the number of scenarios in a quadrant.
  - **“Severity” quadrant criteria:**
    - Quadrants are defined using deciles from the scenario set being evaluated, first interest rates then equity returns (number of scenarios in a quadrant is always 10,000 * 10% * 10% = 100).
    - Relevant statistic is the average geometric average across scenarios in a quadrant (for both rates and returns).
    - Implied interest rate / equity return linkage can also be estimated.

Frequency quadrant criteria — Interest rate bounds

- LATF's exposed "T5" criteria for interest rates (i.e., the low-for-long and high-for-long criteria) were proposed by the ESGS in our 9/14/23 presentation to LATF.
- "T5" uses 1st and 99th percentiles as criteria for low-for-long and high-for-long interest rates, but such percentiles are rather severe as quadrant criteria for the *joint* distribution of interest rates and equity returns.
  - 10,000 scenarios * 1% * 1% → about 1 scenario per quadrant.
- The "T5" table was expanded by adding10th and 90th percentiles using the same methodology as before, i.e., the least-binding scenario set percentile from a range of identified reference models (see our 9/14/23 presentation for additional detail).
  - 10,000 scenarios * 10% * 10% → about 100 scenarios per quadrant.
- For any given scenario set, interest rate bounds for frequency quadrant criteria are determined by noting the starting level of the 20-year Treasury rate (UST20) and then interpolating 10th and 90th percentiles from the expanded "T5" table.

Expanded "T5" Table — Percentile criteria for the distribution of geometric average UST20 rates (use 10th and 90th percentiles as the low and high interest rate bounds for the frequency quadrant criteria).

Starting UST20	First 10 years of projection				First 30 years of projection			
	1st	10th	90th	99th	1st	10th	90th	99th
1%	0.9%	1.3%	2.4%	3.4%	1.5%	2.1%	4.2%	6.2%
2%	1.2%	1.7%	3.7%	5.0%	1.7%	2.4%	5.1%	7.7%
3%	1.6%	2.3%	4.8%	6.6%	1.9%	2.6%	6.0%	8.7%
4%	2.1%	2.9%	5.9%	7.7%	2.1%	2.9%	6.8%	9.6%
5%	2.7%	3.5%	6.9%	8.9%	2.3%	3.2%	7.6%	10.5%
6%	3.1%	4.2%	7.9%	10.0%	2.5%	3.6%	8.2%	11.2%
7%	3.6%	4.7%	8.9%	11.0%	2.8%	4.0%	8.7%	11.6%
8%	4.1%	5.4%	9.8%	12.1%	3.1%	4.3%	9.2%	12.0%
9%	4.6%	6.0%	10.7%	13.1%	3.3%	4.7%	9.7%	12.3%
10%	5.2%	6.7%	11.6%	14.0%	3.6%	5.1%	10.0%	12.6%

Interpolated values for starting UST20 at 1.94% (12/31/21) and 4.24% (12/31/19 + 200bps)

Starting UST20	First 10 years of projection				First 30 years of projection			
	1st	10th	90th	99th	1st	10th	90th	99th
1.94%	1.22%	1.70%	3.61%	4.95%	1.67%	2.34%	5.08%	7.62%
4.24%	2.27%	3.07%	6.18%	8.01%	2.11%	2.98%	6.98%	9.82%

Note: The highlighted values are the interest rate bounds used to apply the frequency quadrant criteria to field test scenario sets 1a, 5a, and 6 (starting UST20 of 1.94%) and 2a, 6a, and 5b (starting UST of 4.24%).

Frequency quadrant criteria — Equity return bounds

- The ESGS is currently only proposing quadrant criteria for low equity returns (quadrant criteria for high equity returns could be developed if regulators are interested).
- As with interest rates, the bound for low equity returns is based on the 10th percentile of the distribution of geometric average equity returns over the first 10 and 30 years of the projection.
- Unlike interest rates, equity return bounds do not depend on the starting level (no interpolation required).
- The proposed equity bounds are simply the 10th percentile from the 10,000 S&P 500 scenarios produced by the NAIC's currently prescribed ESG (AIRG):

Low equity return bounds for the frequency quadrant criteria (based on the 10,000 AIRG S&P 500 scenarios)

	10th Percentile	
Horizon	**Geometric average return**	**Gross wealth factor (GWF)**
First 10 years of projection	1.14%	1.12
First 30 years of projection	3.83%	3.09

Note: GWFs are simply an alternative way to express geometric average returns, e.g., $(1 + 1.14\%)^{10} = 1.12$.

Note: There are other reasonable bases for this criteria besides the AIRG's 10,000 equity scenarios, such as the least-binding (slightly less extreme) reference model basis used to develop C3 Phase II equity GWF criteria. For example, compared to the 10-year 10th percentile of 1.14% (1.12 GWF) above:

- *LATF's exposed "E1" criteria (the former C3 Phase II equity GWF criteria, based on data through 2005) would correspond to a 10-year 10th percentile of 1.50% (1.16 GWF).*
- *The updated C3 Phase II equity GWF criteria in the Academy's 11/22/23 letter to LATF (based on data through 2022) would correspond to a 10-year 10th percentile of 2.01% (1.22 GWF).*

Illustrative application of quadrant criteria

The following slides use these scenario sets to illustrate the proposed quadrant criteria:

Scenario Set	Equity — Model / Calibration / Linkage Approach	Starting Date	Starting UST20
1a	GEMS / NAIC / Constant mean equity risk premium with recentering	12/31/21	1.94%
2a		12/31/19 + 200 bps	4.24%
1a-AIRG	AIRG / AIRG / Constant mean equity return	12/31/21	1.94%
2a-AIRG		12/31/19 + 200 bps	4.24%
5a	GEMS / Conning / Constant mean equity risk premium	12/31/21	1.94%
5b		12/31/19 + 200 bps	4.24%
6	GEMS / ACLI / Constant mean equity return	12/31/21	1.94%
6a		12/31/19 + 200 bps	4.24%

Notes:

1. *All scenario sets listed in this table use the GEMS Interest Model with the Generalized Fractional Floor.*
2. *Scenario sets 1a, 2a, 5a, 5b, and 6 were part of the NAIC's ESG field test (1a-AIRG, 2a-AIRG, and 6a were not).*
3. *All scenario sets are comprised of 10,000 scenarios.*

Frequency quadrant criteria — Illustrative application

	Horizon	Scenario set	1a	2a	1a-AIRG		5a	5b	6	6a
		Starting rate level	*1.94%*	*4.24%*	*1.94%*		*1.94%*	*4.24%*	*1.94%*	*4.24%*
		Linkage approach	*Const mean ERP (with recentering)*		*Const mean return*		*Const mean ERP*		*Const mean return*	
IR & EQ Quadrant Frequency	10 years	Low IR / Low EQ	307	283	194	179	539	354	184	164
		High IR / Low EQ	91	22	164	119	211	52	171	126
	30 years	Low IR / Low EQ	528	562	232	189	972	629	225	170
		High IR / Low EQ	12	4	142	110	87	19	120	104
IR Tail Frequency	10 years	Low IR	1,951	1,834	1,951	1,834	1,951	1,834	1,951	1,834
		High IR	1,723	1,252	1,723	1,252	1,723	1,252	1,723	1,252
	30 years	Low IR	2,389	1,900	2,389	1,900	2,389	1,900	2,389	1,900
		High IR	1,312	1,047	1,312	1,047	1,312	1,047	1,312	1,047
EQ Tail Frequency	10 years	Low EQ	1,227	928	1,000	1,000	2,136	1,226	933	933
	30 years	Low EQ	1,175	1,324	1,000	1,000	2,648	1,754	940	940
IR & EQ Bounds (geometric average rate/return)	*10 years*	*Low IR bound*	*1.70%*	*3.07%*	*1.70%*	*3.07%*	*1.70%*	*3.07%*	*1.70%*	*3.07%*
		High IR bound	*3.61%*	*6.18%*	*3.61%*	*6.18%*	*3.61%*	*6.18%*	*3.61%*	*6.18%*
	30 years	*Low IR bound*	*2.34%*	*2.98%*	*2.34%*	*2.98%*	*2.34%*	*2.98%*	*2.34%*	*2.98%*
		High IR bound	*5.08%*	*6.98%*	*5.08%*	*6.98%*	*5.08%*	*6.98%*	*5.08%*	*6.98%*
	10 years	*Low EQ bound*	*1.14%*	*1.14%*	*1.14%*	*1.14%*	*1.14%*	*1.14%*	*1.14%*	*1.14%*
	30 years	*Low EQ bound*	*3.83%*	*3.83%*	*3.83%*	*3.83%*	*3.83%*	*3.83%*	*3.83%*	*3.83%*

Observations:

1. Constant mean ERP approaches tend to oversample the Low IR / Low EQ quadrant and undersample the High IR / Low EQ quadrant (often an important source of risk, e.g., disintermediation).

Notes:

1. *Quadrants are determined as the scenarios with geometric average rates / returns falling within the specified bounds based on deciles.*
2. *Interest Rate (IR) is the 20-year Treasury (UST20). Equity Return (EQ) is the S&P 500 index.*

Severity quadrant criteria — Illustrative application

	Horizon	Scenario set	1a	2a	1a-AIRG	5a	5b	6	6a
		Starting rate level	*1.94%*	*4.24%*	*1.94%*	*1.94%*	*4.24%*	*1.94%*	*4.24%*
		Linkage approach	*Const mean ERP (with recentering)*		*Const mean return*	*Const mean ERP*		*Const mean return*	
IR & EQ Quadrant Severity (geometric average equity return in quadrant)	10 years	Low IR / Low EQ	-2.2%	-2.3%	-0.8%	-6.2%	-4.7%	-1.1%	-1.3%
		High IR / Low EQ	0.9%	3.1%	-1.0%	-2.0%	1.4%	-1.5%	-1.4%
	30 years	Low IR / Low EQ	1.3%	0.7%	2.9%	-1.7%	-1.1%	3.0%	2.9%
		High IR / Low EQ	6.0%	7.0%	2.6%	3.2%	5.6%	2.5%	2.6%
IR Tail Severity (geometric average interest rate in tail)	10 years	Low IR	1.1%	2.2%	1.1%	1.1%	2.2%	1.1%	2.2%
		High IR	4.8%	7.4%	4.8%	4.8%	7.4%	4.8%	7.4%
	30 years	Low IR	1.5%	2.1%	1.5%	1.5%	2.1%	1.5%	2.1%
		High IR	6.5%	8.4%	6.5%	6.5%	8.4%	6.5%	8.4%
Implied IR & EQ Linkage (in low EQ tail)	10 years	Low EQ	88%	104%	-7%	122%	121%	-9%	-1%
	30 years	Low EQ	90%	97%	-6%	95%	103%	-9%	-5%

Observations:

1. Under the constant mean ERP approaches, average equity returns in the High IR / Low EQ 30-year quadrant are rather high and may not sufficiently capture the risk of lower returns in that quadrant (e.g., 6.0 %, 7.0%).
2. In the 1a and 2a (constant mean ERP with recentering) Low IR / Low EQ quadrant, it is unintuitive that the average equity return decreases (e.g., 1.3% → 0.7%) when the starting rate level increases (i.e., exhibits negative linkage when the goal for 1a and 2a was positive linkage).

Notes:

1. *Quadrants are determined by selecting the 1,000 scenarios (out of 10,000) with the lowest or highest geometric average interest rate, and then selecting the 100 scenarios (out of those 1,000) with the lowest equity return (i.e., quadrants are always comprised of 100 scenarios).*
2. *Interest Rate (IR) is the 20-year Treasury (UST20). Equity Return (EQ) is the S&P 500 index.*
3. *Implied IR & EQ Linkage = Horizon years * Ln((1+High IR & Low EQ geom avg return)/(1+Low IR & Low EQ geom avg return)) * 10% / ((High IR geom avg rate) – (Low IR geom avg rate)) = Ln(High IR & Low EQ GWF / Low IR & Low EQ GWF) * 10% / ((High IR geom avg rate) – (Low IR geom avg rate)).*

Questions?

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