Interest Rates—
Acceptance Criteria for *Interim* Rate Levels

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Agenda—Acceptance Criteria for *Interim* Rate Levels

1. Background
2. Interim Rate Criteria
3. Reference Models
4. Summary and Q&A
5. Appendix—Reference Model Results Supporting Interim Rate Criteria

1.

Background

Background

LATF asked the Academy to deliver a series of presentations focused on proposing qualitative **Stylized Facts** and quantitative **Acceptance Criteria** for the three major components of an ESG used for statutory reporting purposes: **Interest Rates**, **Equity Returns**, and **Corporate Bond Fund Returns**.

This presentation presents the Academy's proposal for **Acceptance Criteria for *Interim* Rate Levels**.

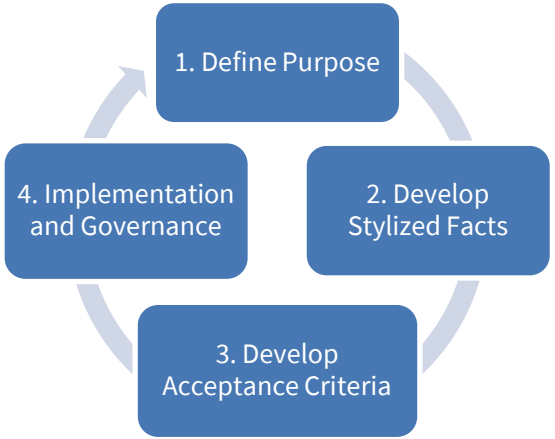
Prior presentations in this series:

- A Framework for Working with ESGs (8/8/22)
- ESG Governance Considerations (8/8/22)
- Equity Returns—Stylized Facts (8/9/22)
- Corporate Credit & Bond Fund Returns—Stylized Facts, Acceptance Criteria, and a Simplified Model (10/27/22)
- Interest Rates—Stylized Facts and Acceptance Criteria (12/11/22)
- Interest Rates—Update on Proposed Acceptance Criteria (8/12/23)

This and future presentations in this series:

- **Acceptance Criteria for *Interim* Rate Levels (9/14/23)**
- Equity Returns—Acceptance criteria, including criteria for the joint distribution of equity returns and interest rates (TBD)

A framework for developing, implementing, and evaluating ESGs and the scenario sets they produce



"Statistical criteria are important in assessing the quality of an ESG. Statistical calibration criteria are usually numerically specified but can also be qualitative in nature. Statistical criteria belong to one of two broad categories: qualitative features and quantitative measures. The issues one must address in both categories are not amenable to a checklist approach, however, and expert judgment plays a role."

(quote from p. 96 of the 2020 CAS/Conning research paper on ESGs)

1. **Define Purpose**: The intended purpose of the ESG informs the economic variables to be simulated and the relative importance of their "stylized facts."

2. **Develop Stylized Facts**: Stylized facts describe properties of the economic variables to be simulated. They are based on historical market data and economic theory and are prioritized relative to the defined purpose at hand. The establishment of stylized facts is critical for selecting candidate ESG models and a key prerequisite for the development of acceptance criteria.

3. **Develop Acceptance Criteria**: A set of quantitative metrics or target values at different time horizons or in different economic conditions that provide a simplified framework for ensuring sets of scenarios produced by the ESG are consistent with key stylized facts.

4. **Implementation and Governance**: ESG models are selected based on their ability to reflect the stylized facts, then calibrated in accordance with acceptance criteria. Validation reports are produced on each candidate scenario set generated by the ESG. These reports compare scenario set statistics to acceptance criteria and contain other charts and tables useful for evaluation and signoff, which is ultimately a matter of judgement (no automatic "pass" or "fail" based only on acceptance criteria). Implementation is an iterative process. It is important to periodically review and recalibrate the ESG as market conditions change over time.

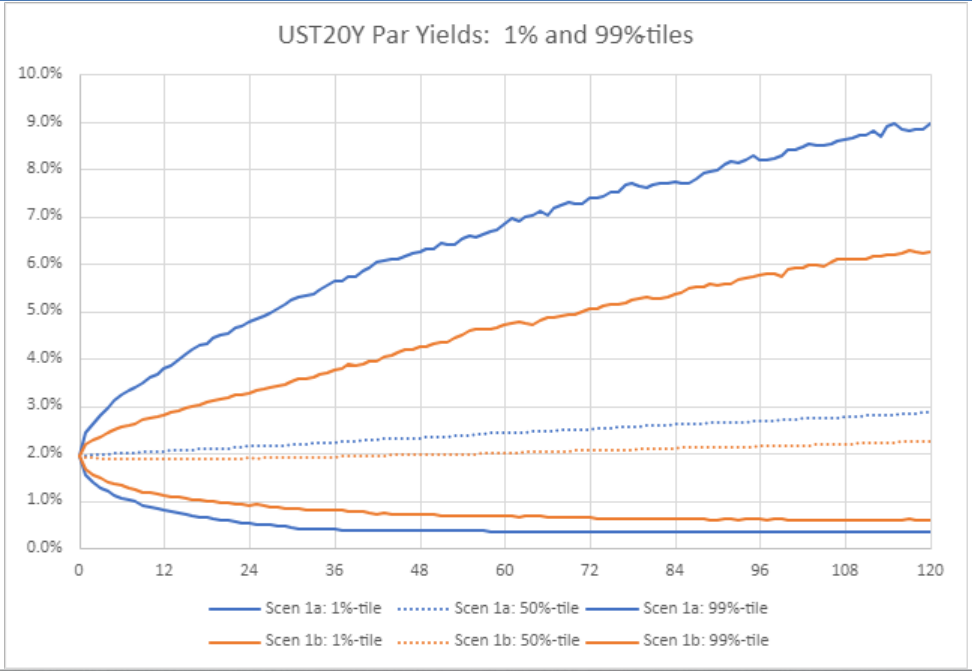
2.

Interim Rate Criteria

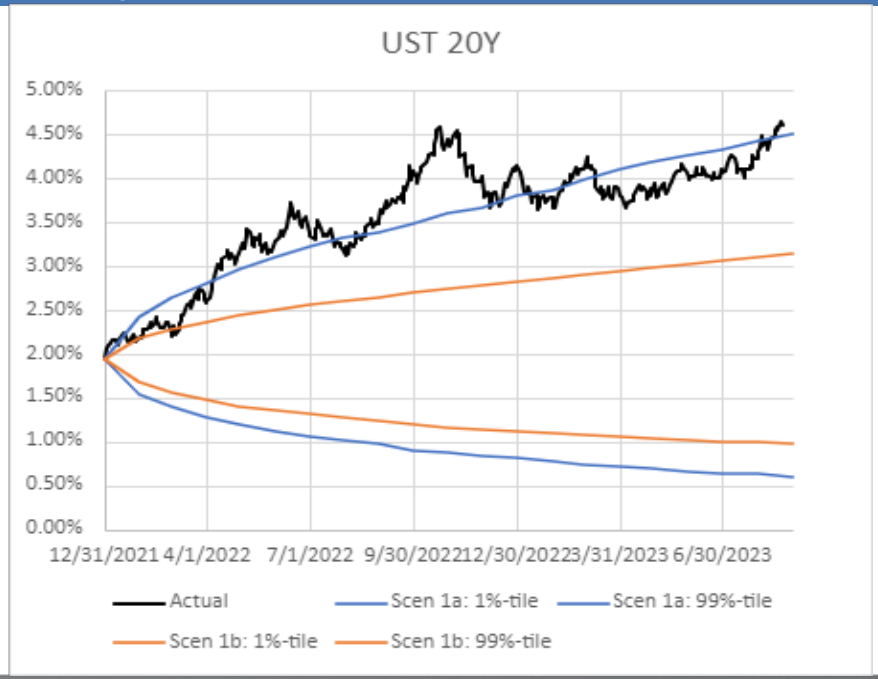
Why is it important to consider ESG behavior during interim periods?

- ESG behavior in earlier years (i.e., how the scenarios evolve from initial conditions to the steady state) can be more significant for the purpose at hand than the theoretical steady state.
  - Different models and calibrations can have similar steady state distributions but be materially different in the earlier years that matter most when using scenarios for purposes of determining statutory reserves and capital for long duration insurance products.
- Interim rate criteria can help ensure a sufficiently wide range of behavior as rates move from initial conditions to the desired steady state distribution.
  - Criteria for rates at specific interim points-in-time can help ensure sufficient rate volatility and dispersion for adequately modeling disintermediation, liquidity, and hedging risks (e.g., the recent run up in interest rates).
  - Criteria for geometric average rates over specific interim horizons can help ensure sufficient low- and high-for-long rate behavior for adequately modeling long-term reinvestment risks.
- Overly narrow rate distributions can imply an overly optimistic degree of certainty and lead to artificial volatility between reporting periods when rates change more than expected.

A fan chart illustrates how two different calibrations (#1a and #1b) of the same model can produce materially different rate dispersion over time



Two scenario sets (#1a and #1b) from the same model (different calibrations) illustrate different levels of rate dispersion and how it compares to history



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Two types of interim rate criteria are proposed for 20-year UST yields

1. Criteria based on percentiles of the distribution of rates at selected points-in-time:
   - Left-Tail Point-in-Time (Low PIT) Criteria (1st percentile)
   - Right-Tail Point-in-Time (High PIT) Criteria (99th percentile)
2. Criteria based on percentiles of the distribution of geometric average rates over selected horizons *(note, a steady state version of this criteria is also proposed):*
   - Low-for-Long (L4L) Criteria (1st percentile)
   - High-for-Long (H4L) Criteria (99th percentile)

Due to a lack of historical data to inform interim rate criteria across multiple starting rate levels, interim rate criteria were developed using multiple reference models calibrated to the Academy's existing *steady state* criteria for interest rates.

- Rather than relying solely on judgment, appropriately calibrated reference models allow insight into interim rate behavior under a variety of conditions where historical data is limited.
- Three reference models were used to model the evolution of the 20-year UST yield: Black-Karasinski (BK), Brennan-Schwartz (BS), and Cox-Ingersoll-Ross (CIR).
- Multiple mean-reversion speeds were tested.
- Proposed criteria are based on the *least binding* statistics from the reference models used.

In addition to low-for-long criteria for the left tail, the Academy believes it is prudent to also establish high-for-long criteria for the right tail. High rates, for example, are a key driver of disintermediation risk for many product types.

Interim rate criteria can help ensure the relationship between mean reversion and volatility produces sufficient dispersion *regardless* of starting rate level.

Unlike steady state criteria, thresholds for interim criteria generally depend on initial conditions and are therefore expressed in tabular form (i.e., many numbers, even for these two rather simple categories of interim criteria).

Proposed interim point-in-time criteria

1. Calculate the [1st] and [99th] percentiles of the distribution of [20-year] UST yields at the end of years [1], [5], and [10].
2. Use the starting level of the 20-year UST yield to look up the corresponding criteria from the table (interpolate if necessary).

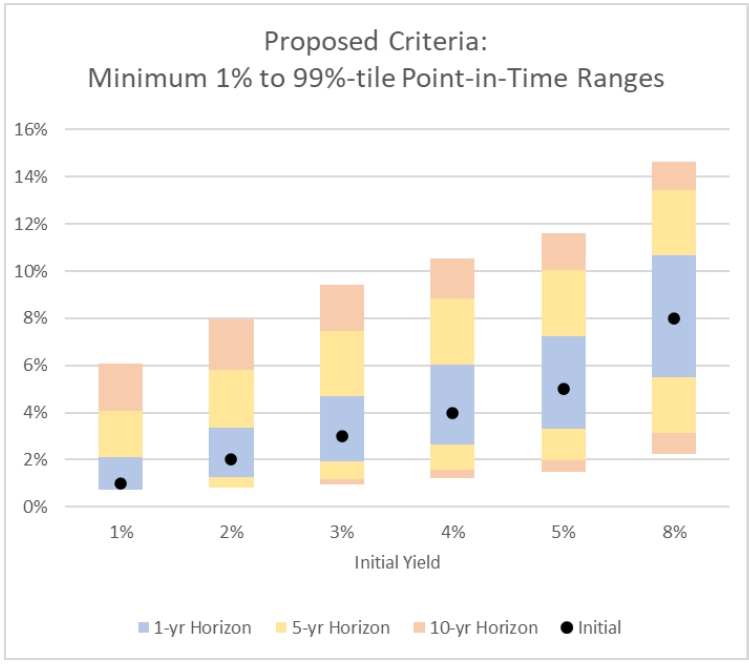
Initial UST20	EOY 1 Point-In-Time		EOY 5 Point-In-Time		EOY 10 Point-In-Time	
	1st percentile should be less than:	99th percentile should be greater than:	1st percentile should be less than:	99th percentile should be greater than:	1st percentile should be less than:	99th percentile should be greater than:
1%	0.54%	1.92%	0.60%	3.89%	0.72%	6.05%
2%	1.22%	3.30%	0.79%	5.75%	0.81%	8.10%
3%	1.92%	4.66%	1.20%	7.48%	0.95%	9.62%
4%	2.62%	6.01%	1.62%	8.83%	1.23%	10.77%
5%	3.31%	7.22%	2.03%	10.03%	1.50%	11.87%
6%	3.99%	8.38%	2.43%	11.21%	1.75%	12.93%
7%	4.68%	9.52%	2.81%	12.35%	2.00%	13.95%
8%	5.46%	10.64%	3.18%	13.46%	2.23%	14.92%
9%	6.26%	11.76%	3.58%	14.56%	2.45%	15.78%
10%	7.06%	12.86%	4.09%	15.62%	2.66%	16.48%

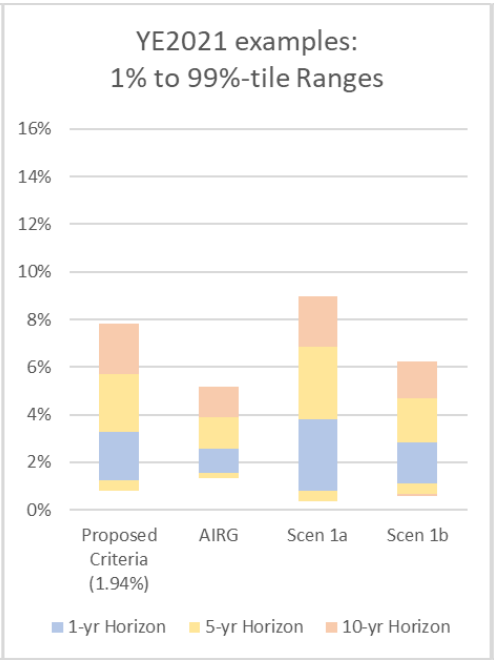
During model implementation and/or calibration, the Academy suggests using this criteria to check interim rate behavior at a variety of starting rate levels, e.g., 2%, 5%, and 8%.

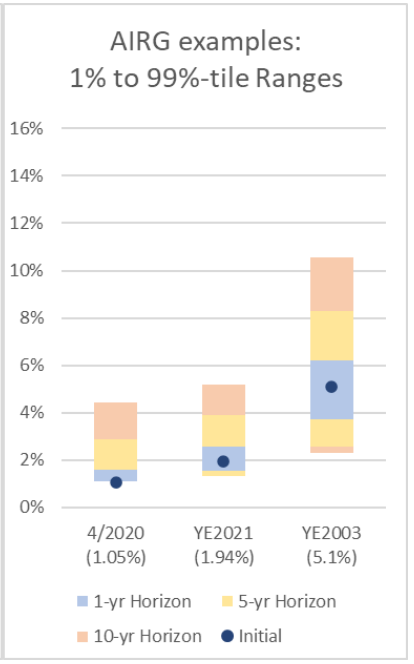
During monthly production, this criteria can be applied by using the starting rate level to interpolate between neighboring values.

Note that only a single row (interpolated to the starting level of the UST20 yield) is needed to apply this criteria to a set of scenarios.

Stacked column charts can illustrate how the interim rate dispersion criteria widen as the starting rate level increases







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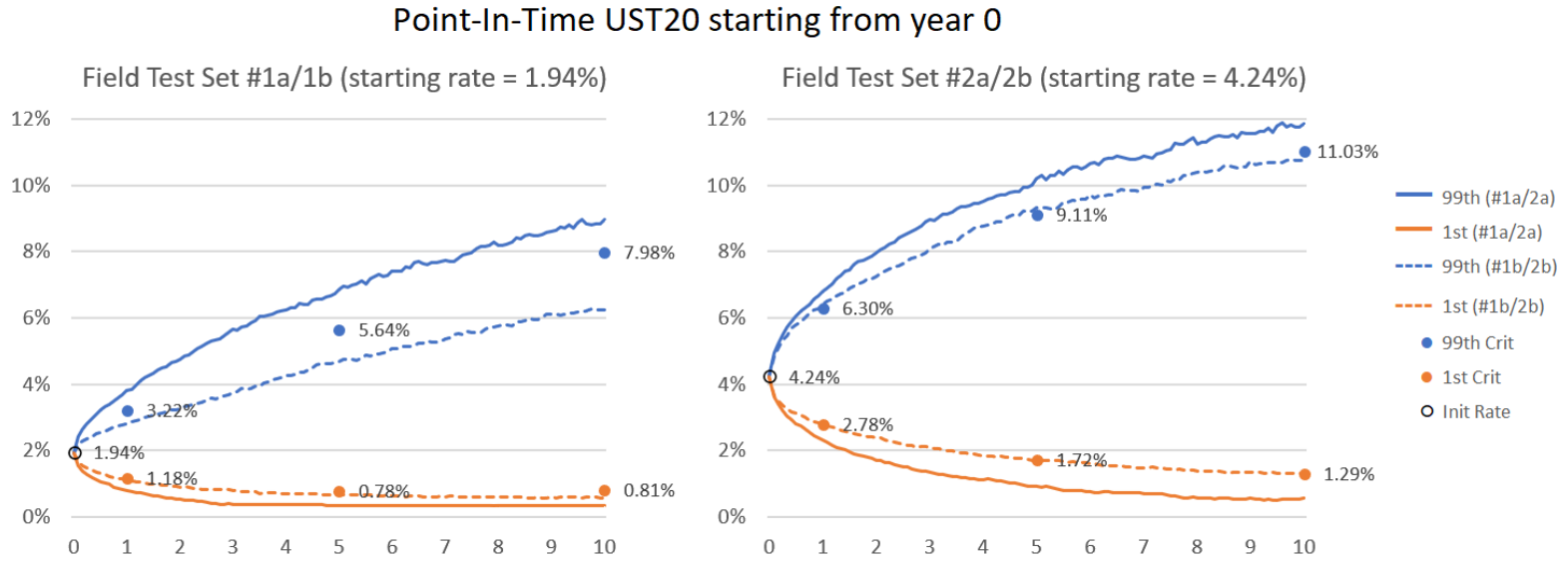
Illustrative application of interim point-in-time criteria for the 20-year UST yield



Sets satisfy this interim point-in-time criteria if the percentiles (blue/orange lines) lie outside the criteria (blue/orange dots)

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Proposed Low- and High-for-Long (L4L and H4L) geometric average criteria for UST20

1. For each scenario, calculate the geometric average of the [20-year] UST yield over the first [10] and [30] years of the projection.
2. Calculate the [1st] and [99th] percentiles of the distribution of geometric average rates (for both the 10 and 30-year horizons).
3. Look up criteria based on the starting level of the 20-year UST yield (interpolate if necessary).

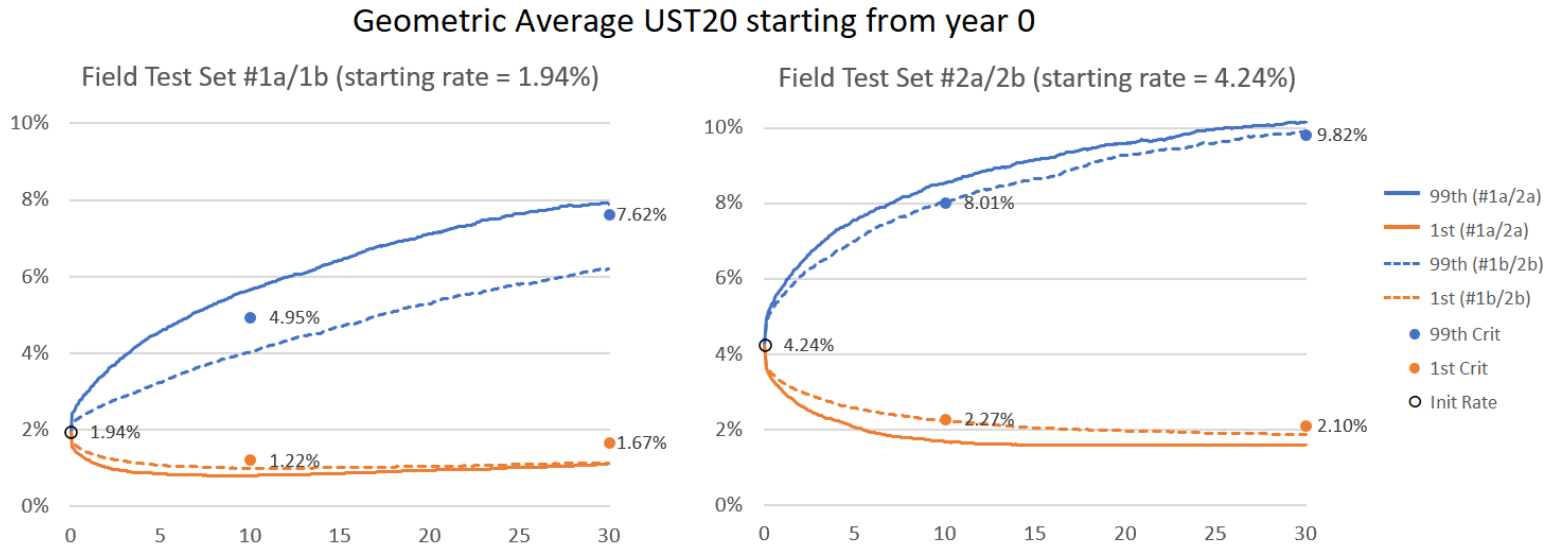
Period	Initial UST20	10-year Geom Avg		30-year Geom Avg	
		1st percentile should be less than:	99th percentile should be greater than:	1st percentile should be less than:	99th percentile should be greater than:
Interim (years 0-10 or 0-30)	1%	0.94%	3.43%	1.50%	6.25%
	2%	1.23%	5.05%	1.68%	7.71%
	3%	1.62%	6.55%	1.86%	8.72%
	4%	2.15%	7.74%	2.06%	9.62%
	5%	2.66%	8.87%	2.26%	10.46%
	6%	3.15%	9.96%	2.50%	11.16%
	7%	3.63%	11.03%	2.78%	11.61%
	8%	4.10%	12.07%	3.06%	11.99%
	9%	4.64%	13.08%	3.34%	12.33%
	10%	5.21%	14.01%	3.65%	12.63%
Steady State (years 70-80 or 70-100)	**Any**	1.34%	13.57%	1.94%	11.45%

During model implementation and/or calibration, the Academy suggests using this criteria to check interim rate behavior at a variety of starting rate levels, e.g., 2%, 5%, and 8%.

During monthly production, this criteria can be applied by using the starting rate level to interpolate between neighboring values.

As with the Point-In-Time criteria, note that only a single row (interpolated to the starting level of the UST20 yield) is needed to apply this interim criteria to a set of scenarios.

Illustrative application of *interim* Low-for-Long (L4L) and High-for-Long (H4L) criteria for geometric averages of the 20-year UST yield



Sets satisfy this Interim Geom Avg criteria if the percentiles (blue/orange lines) lie outside the criteria (blue/orange dots)

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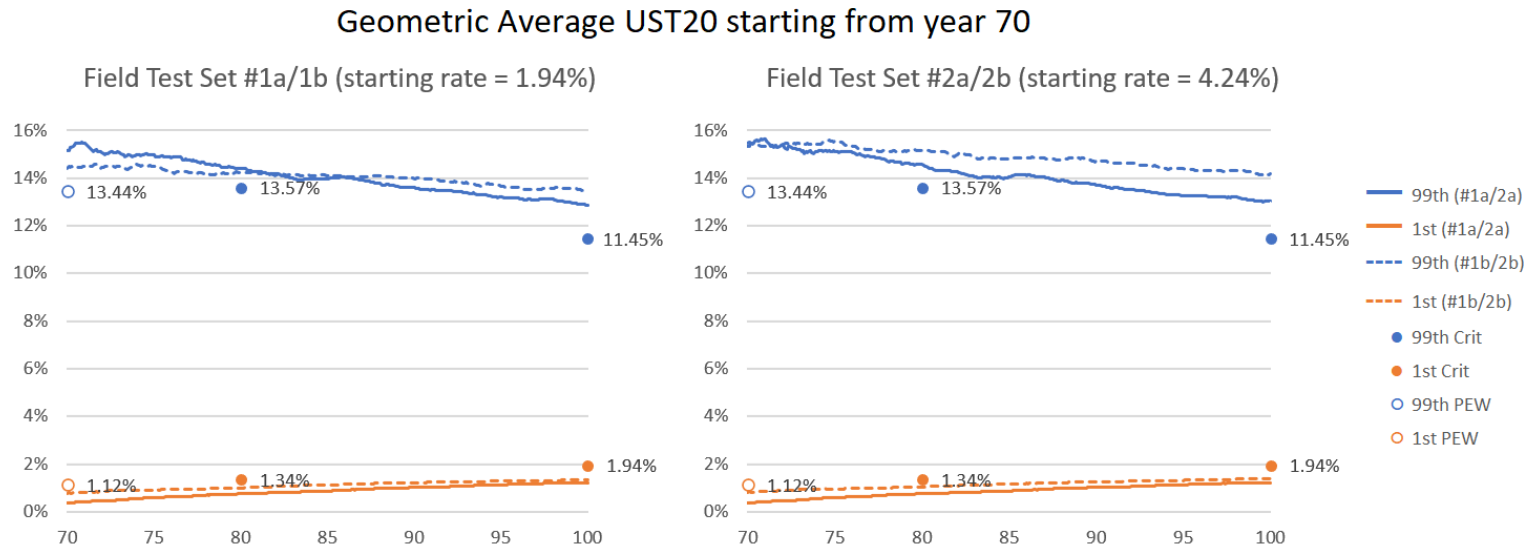
Illustrative application of *steady state* Low-for-Long (L4L) and High-for-Long (H4L) criteria for geometric averages



Sets satisfy this Steady State Geom Avg (and PEW) criteria if the percentiles (blue/orange lines) lie outside the criteria (blue/orange dots)

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Comparison to NAIC's low-for-long criteria used for the field test

1. NAIC preliminary boundary guidance for goal #5 (*"The ESG should be capable of producing low interest rates for an extended period of time"*) — *For scenarios of the 20-year UST yield generated as of 12/31/20 (i.e., starting at a level of 1.45%):*
   a. *After 10 years, at least 10% of scenarios should have a geometric average below (the starting level of) 1.45%*
   b. *After 30 years, at least 5% of scenarios should have a geometric average below (the starting level of) 1.45%*

2. For comparison, the Academy's 20-year UST reference models (used to develop the criteria in this presentation) were started at 1.45% to determine rate levels corresponding to the NAIC's preliminary boundary guidance:

		Percentiles of geometric average UST20 rates *when starting at 1.45%*									Least Binding (max)
		Black Karasinski (BK)			Brennan-Schwartz (BS)			Cox Ingersoll Ross (CIR)			
Horizon	**Percentile**	HL=10yr	HL=12yr	HL=15yr	HL=10yr	HL=12yr	HL=15yr	HL=10yr	HL=12yr	HL*=15yr	
1st 10 yrs	**10th**	1.30%	1.25%	1.21%	**1.48%**	1.41%	1.35%	1.48%	1.39%	1.33%	**1.48%**
1st 30 yrs	**5th**	1.67%	1.54%	1.40%	1.89%	1.77%	1.64%	**1.96%**	1.80%	1.66%	**1.96%**

** HL = Half-life of time required for median UST20 to reach steady state*

   a. 10-year geometric average (starting at 1.45%) – Seven of the nine reference model calibrations produced a 10th percentile below 1.45%, with a maximum (i.e., least-binding) overall percentile of 1.48% (slightly above 1.45%).
   b. 30-year geometric average (starting at 1.45%) – Only one of the nine reference model calibrations produced a 5th percentile below 1.45%, with a maximum (i.e., least-binding) overall percentile of at 1.96% (moderately above 1.45%).

   This suggests that 1.b. above may have been one of the NAIC's more constraining pieces of preliminary boundary guidance.

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3.

Reference Models

Rationale for reference models used to inform interim rate criteria

- Goals for selecting reference models to that resulting interim rate criteria accommodates reasonably calibrated models:
  - Sample a variety of potential distributions and dynamic relationships between volatility and rate level, calibrated to multiple mean-reversion speeds, resulting in more inclusive criteria with sufficient tolerance for different model forms.
  - Long-standing, well-understood models that are used elsewhere in actuarial practice and finance.
  - Relatively simple models with parameters that allow for direct setting of various mean reversion speeds, and effective targeting of existing steady state criteria for mean reversion level and volatility (manageable calibration exercise).

Model Class	Distribution of Monthly Rate Changes	Volatility is proportional to	Example of other actuarial use
Black Karasinski (BK)	Lognormal	Level of shifted rate	AIRG (*non*-shifted version of BK)
Brennan Schwartz (BS)	Normal	Level of shifted rate	CIA's ESG criteria
Cox Ingersoll Ross (CIR)	Normal	Square root of level of shifted rate	CIA's ESG criteria

Three reference models for the 20-year UST yield were used to support the development of interim criteria

Black Karasinski (BK)

- Based on an Extended Lognormal Model, like the AIRG, but with a constant volatility scalar, σ:
  - $lr_t = log(rate_t + shift)$ $\quad lr_{t+1} = lr_t + \beta(\log(\tau + shift) - lr_t) + \sigma\left(\frac{(rate_t+shift)^{cev}}{(rate_t+shift)}\right) Z_t$
  - With *cev=1* the extended lognormal model simplifies to a shifted Black Karasinski form, which is like the AIRG model but with a constant diffusion term.

Brennan Schwartz (BS)

- Based on a Generalized Shifted CEV model with *shift=0* and *cev=1.*

Cox Ingersoll Ross (CIR)

- Based on a Generalized Shifted CEV model with *shift=0* and *cev=0.5.*

Generalized Shifted CEV model:

$$rate_{t+1} = rate_t + \beta(\tau - rate_t) + \sigma\,(rate_t + shift)^{cev}\,Z_t$$

All reference models were calibrated to steady state criteria previously proposed by the Academy.

Each model was calibrated to 4 different half-lives within the previously proposed half-life criteria range of 10–20 years (i.e., 10, 12, 15, and 20 years).

Reference model parameters, mean reversion speeds, and associated residuals

Model Parameters	Black Karasinski (BK)				Brennan Schwartz (BS)				Cox Ingersoll Ross (CIR)			
Half-Life (HL)	**10yr**	**12yr**	**15yr**	**20yr**	**10yr**	**12yr**	**15yr**	**20yr**	**10yr**	**12yr**	**15yr**	**20yr**
shift	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	-0.0025	-0.0025	-0.0025	-0.0025
cev	1	1	1	1	1	1	1	1	0.5	0.5	0.5	0.5
sigma	0.05124	0.04691	0.04186	0.03572	0.05774	0.05268	0.04691	0.04330	0.01445	0.01323	0.01184	0.01027
beta	0.00576	0.00480	0.00384	0.00288	0.00576	0.00480	0.00384	0.00288	0.00576	0.00480	0.00384	0.00288
tau	0.048	0.048	0.048	0.048	0.051	0.051	0.051	0.055	0.055	0.0545	0.055	0.0545
implied vol target	0.0071	0.0065	0.0058	0.00495	0.008	0.0073	0.0065	0.0060	0.0083	0.0076	0.0068	0.0059

Mean Reversion Speed and Weight of Initial Rates at year 50 and 80				
Half-Life (HL)	**10yr**	**12yr**	**15yr**	**20yr**
Annual Mean Reversion Speed (MRS)	6.7%	5.6%	4.5%	3.4%*
Weight of initial rates at year 50 (Wt_{50})	3.12%	5.57%	9.92%	17.68%†
Weight of initial rates at year 80 (Wt_{80})	0.39%	0.98%	2.48%	6.25%

** MRS = 1 - $0.5^{(1/HL)}$, e.g., 3.4% = 1 - $0.5^{(1/20)}$* *† Wt_{yr} = $(1 - MRS)^{yr}$, e.g., 17.68% = $(1 - 3.4\%)^{50}$*

Reference "volatility target" measures annualized model rate volatility at specified reference point. Assuming monthly model parameters and a reference rate = 3%, i.e., $volatility\ target = \sqrt{12}\ \sigma\ (.03 + shift)^{cev}$

All modeled rates are subject to hard floor/cap of 0.25%/20%, and soft floor/cap of 0.5%/18%

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Reference models (starting with 20Y UST at 1.45%) compared against previously proposed *steady state* criteria (red if outside desired range)

Model	Black Karasinski (BK)				Brennan Schwartz (BS)				Cox Ingersoll Ross (CIR)			
Half-Life (HL)	10yr	12yr	15yr	20yr	10yr	12yr	15yr	20yr	10yr	12yr	15yr	20yr
Worse-than-History Frequency												
Pr(>16.97%) = 0.5% to 1.5%	0.76%	0.77%	0.76%	0.74%	0.90%	0.88%	0.88%	1.19%	0.51%	0.53%	0.56%	0.55%
Pr(<0.05%) = 0.5% to 1.5%	1.01%	1.05%	1.06%	1.02%	1.03%	1.02%	0.98%	0.99%	0.99%	1.04%	0.99%	1.07%
Rate Volatility												
Vol(BOM rate <= 3%) = 0.31% to 0.92%	0.57%	0.52%	0.47%	0.40%	0.64%	0.58%	0.52%	0.48%	0.69%	0.63%	0.56%	0.49%
Vol(3% < BOM rate <= 8%) = 0.37% to 1.12%	1.11%	1.01%	0.90%	0.77%	1.21%	1.11%	0.99%	0.92%	1.12%	1.02%	0.91%	0.79%
Vol(BOM rate > 8%) = 0.78% to 2.33%	2.11%	1.93%	1.72%	1.45%	2.46%	2.24%	1.99%	1.86%	1.59%	1.46%	1.30%	1.13%
Rate Level (Bounds/PEWs)												
Min = 0.00% to 0.50%	0.32%	0.33%	0.34%	0.36%	0.41%	0.40%	0.40%	0.38%	0.40%	0.39%	0.39%	0.39%
1st = 0.22% to 1.12%	1.03%	1.03%	0.98%	0.99%	0.99%	0.99%	1.02%	1.03%	0.97%	1.00%	1.03%	1.01%
5th = 0.98% to 1.78%	1.64%	1.64%	1.63%	1.64%	1.52%	1.51%	1.52%	1.52%	1.63%	1.60%	1.61%	1.57%
15th = 1.61% to 2.31%	2.53%	2.53%	2.54%	2.55%	2.18%	2.19%	2.20%	2.27%	2.51%	2.49%	2.53%	2.50%
30th = 2.23% to 2.83%	3.48%	3.47%	3.48%	3.48%	2.94%	2.95%	2.97%	3.07%	3.53%	3.49%	3.52%	3.48%
50th = 3.35% to 4.89%	4.72%	4.73%	4.74%	4.76%	4.02%	4.03%	4.03%	4.24%	4.87%	4.78%	4.86%	4.82%
70th = 5.77% to 7.77%	6.33%	6.33%	6.36%	6.38%	5.54%	5.51%	5.55%	5.88%	6.53%	6.45%	6.53%	6.49%
85th = 7.56% to 9.81%	8.37%	8.35%	8.38%	8.32%	7.62%	7.60%	7.65%	8.13%	8.51%	8.44%	8.49%	8.43%
95th = 9.50% to 12.00%	11.45%	11.49%	11.44%	11.34%	11.14%	11.12%	11.09%	11.79%	11.24%	11.25%	11.25%	11.21%
99th = 13.44% to 16.19%	15.06%	15.04%	14.99%	15.01%	15.20%	15.25%	15.22%	15.80%	14.67%	14.54%	14.69%	14.64%
Max = 17.00% to 20.00%	20.00%	19.34%	19.26%	18.65%	20.00%	19.61%	19.51%	19.49%	18.79%	18.50%	18.46%	18.37%

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Some additional observations from testing multiple reference models

1. There is moderate variation across model forms, with BS better suited to PEWs/boundary conditions, and producing more stabilized low-for-long (L4L) statistics. Other model forms had difficulty hitting 15% PEW criterion but produced lower (i.e., more conservative) L4L statistics vs. BS. No single model is overly the least binding.
2. Extending half-life (lowering mean reversion speed) results in more pro-cyclical/volatile geometric average levels relative to initial rates.
3. BK (i.e., lognormal) models tend to produce lower (i.e., more conservative) L4L statistics when starting at lower rates relative to relative to BS and CIR.
4. BS models produce lower (i.e., more conservative) L4L statistics when starting at higher rates, while other models generally converge when starting rates are close to the median.
5. Due to the high weight of initial rates associated with the 20-year half-life mean reversion speed (i.e., 17.68% weighting after 50 years, 6.25% weighting after 80 years), only 10, 12, and 15-year half-life mean reversion speeds were used to support the proposed criteria.
6. Overall, 30-year-horizon L4L statistics are within 50-70 basis points (bps) across all reference models.
7. Criteria were set based on the *least binding* result from the nine selected reference model calibrations (calibrated to the Academy's previously proposed steady state criteria) in order to accommodate a range of reasonably calibrated models. An additional tolerance (cushion) could be considered to accommodate an even wider range of models.

4.

Summary and Q&A

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Summary

Categories of interest rate criteria proposed by the Academy:

** criteria proposed in this presentation (i.e., 1.a and 3.)*

1. Rate Level Criteria
   - (a) Steady state "PEW" Criteria; (b) Interim "Point-In-Time" Criteria* (varies by starting rate level)
2. Min/Max & Worse-Than-History (WTH) Frequency Criteria
   - Developed for both rate level and slope
   - For slope, varies by level of 20-year rate (can be used for both interim and steady state)
3. Rate Low- and High-for-Long (Geometric Average) Criteria*
   - Both interim and steady state (interim criteria varies by starting rate level)
4. Rate Volatility Criteria
   - Varies by beginning-of-month rate level (can be used for both interim and steady state)
5. Slope Criteria
   - Varies by level of 20-year rate (can be used for both interim and steady state)
6. Mean Reversion Speed Criteria
   - Developed for both rate level and slope
   - For application across the entire projection (interim and steady state)

Comments regarding newly proposed interim rate criteria (1.a and 3.):

Insurance company projected cashflows can vary significantly depending on how interest rates transition from known starting values to the targeted steady state distribution.

Compared to steady state, interim criteria are more complex since it must cover a broad range of initial conditions where historical data is often limited.

Although more complex, interim criteria are an important component of this limited but practical set of acceptance criteria proposed for use in this regulatory setting.

Without robust, interim rate behavior under a variety of initial conditions, models may introduce artificial volatility in reserve and capital calculations when rates move substantially from one period to the next.

Thank You

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5.

Appendix—Reference Model Results Supporting Interim Rate Criteria

Reference model results supporting 10-year Low-for-Long criteria (left tail)

Period	Starting level of UST20	10-year horizon Geometric Average at 1st percentile (UST20)									Criteria = least binding (i.e., max)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (years 0-10)	1%	0.8%	0.7%	0.7%	**0.9%**	0.9%	0.8%	0.9%	0.9%	0.8%	0.9%
	2%	1.1%	1.1%	1.1%	**1.2%**	1.2%	1.2%	1.2%	1.2%	1.1%	1.2%
	3%	1.5%	1.6%	**1.6%**	1.6%	1.6%	1.6%	1.5%	1.5%	1.5%	1.6%
	4%	2.0%	2.1%	**2.1%**	1.9%	1.9%	2.0%	1.9%	1.9%	2.0%	2.1%
	5%	2.4%	2.5%	**2.7%**	2.2%	2.3%	2.4%	2.3%	2.3%	2.5%	2.7%
	6%	2.9%	3.0%	**3.1%**	2.6%	2.7%	2.9%	2.7%	2.8%	3.0%	3.1%
	7%	3.3%	3.4%	**3.6%**	2.9%	3.1%	3.3%	3.1%	3.3%	3.5%	3.6%
	8%	3.7%	3.9%	**4.1%**	3.2%	3.4%	3.7%	3.6%	3.8%	4.1%	4.1%
	9%	4.1%	4.3%	4.6%	3.6%	3.8%	4.1%	4.1%	4.3%	**4.6%**	4.6%
	10%	4.4%	4.7%	5.0%	3.9%	4.2%	4.5%	4.6%	4.8%	**5.2%**	5.2%
Steady State (years 70-80)	Any	1.2%	1.2%	1.1%	1.3%	1.2%	1.2%	**1.3%**	1.3%	1.2%	1.3%

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Reference model results supporting 10-year High-for-Long criteria (right tail)

Period	Starting level of UST20	10-year horizon Geometric Average at 99th percentile (UST20)									Criteria = least binding (i.e., min)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (years 0-10)	1%	4.3%	3.9%	**3.4%**	4.7%	4.2%	3.8%	4.9%	4.5%	4.0%	3.4%
	2%	5.9%	5.5%	**5.0%**	6.2%	5.8%	5.3%	6.2%	5.8%	5.4%	5.0%
	3%	7.4%	7.0%	**6.6%**	7.7%	7.3%	6.8%	7.3%	7.0%	6.6%	6.6%
	4%	8.8%	8.5%	8.0%	9.2%	8.8%	8.3%	8.4%	8.1%	**7.7%**	7.7%
	5%	10.1%	9.8%	9.3%	10.6%	10.2%	9.8%	9.5%	9.2%	**8.9%**	8.9%
	6%	11.3%	11.0%	10.6%	11.8%	11.5%	11.2%	10.5%	10.3%	**10.0%**	10.0%
	7%	12.2%	12.1%	11.8%	12.7%	12.6%	12.4%	11.5%	11.3%	**11.0%**	11.0%
	8%	13.1%	13.0%	12.9%	13.5%	13.5%	13.4%	12.5%	12.3%	**12.1%**	12.1%
	9%	13.8%	13.8%	13.8%	14.2%	14.2%	14.2%	13.4%	13.3%	**13.1%**	13.1%
	10%	14.3%	14.4%	14.5%	14.7%	14.8%	14.8%	14.1%	14.1%	**14.0%**	14.0%
Steady State (years 70-80)	Any	**13.6%**	13.7%	14.1%	13.6%	13.8%	14.1%	13.7%	13.9%	14.2%	13.6%

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Reference model results supporting 30-year Low-for-Long criteria (left tail)

Period	Starting level of UST20	30-year horizon Geometric Average at 1st percentile (UST20)									Criteria = least binding (i.e., max)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (years 0-30)	1%	1.2%	1.1%	1.0%	1.5%	1.4%	1.2%	**1.5%**	1.4%	1.2%	1.5%
	2%	1.4%	1.3%	1.3%	1.7%	1.6%	1.5%	**1.7%**	1.6%	1.4%	1.7%
	3%	1.7%	1.6%	1.6%	1.8%	1.8%	1.7%	**1.9%**	1.8%	1.7%	1.9%
	4%	2.0%	1.9%	1.9%	2.0%	2.0%	1.9%	**2.1%**	2.0%	1.9%	2.1%
	5%	2.2%	2.2%	2.2%	2.2%	2.1%	2.1%	**2.3%**	2.2%	2.2%	2.3%
	6%	2.4%	2.4%	**2.5%**	2.3%	2.3%	2.3%	2.5%	2.4%	2.5%	2.5%
	7%	2.6%	2.7%	**2.8%**	2.5%	2.5%	2.6%	2.7%	2.7%	2.7%	2.8%
	8%	2.9%	2.9%	**3.1%**	2.7%	2.7%	2.8%	2.9%	2.9%	3.0%	3.1%
	9%	3.1%	3.2%	3.3%	2.8%	2.9%	3.0%	3.1%	3.2%	**3.3%**	3.3%
	10%	3.3%	3.4%	3.6%	3.0%	3.1%	3.2%	3.4%	3.4%	**3.6%**	3.6%
Steady State (years 70-100)	Any	1.7%	1.6%	1.5%	1.8%	1.7%	1.6%	**1.9%**	1.8%	1.7%	1.9%

Reference model results supporting 30-year High-for-Long criteria (right tail)

Period	Starting level of UST20	30-year horizon Geometric Average at 99th percentile (UST20)									Criteria = least binding (i.e., min)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (years 0-30)	1%	7.4%	6.9%	**6.2%**	7.8%	7.4%	6.7%	7.7%	7.3%	6.7%	6.2%
	2%	8.5%	8.2%	**7.7%**	8.6%	8.4%	8.0%	8.5%	8.2%	7.8%	7.7%
	3%	9.3%	9.2%	8.9%	9.4%	9.3%	9.1%	9.3%	9.0%	**8.7%**	8.7%
	4%	10.0%	10.0%	9.9%	10.1%	10.2%	10.1%	9.9%	9.8%	**9.6%**	9.6%
	5%	10.6%	10.7%	10.8%	10.8%	10.9%	11.0%	10.6%	10.6%	**10.5%**	10.5%
	6%	**11.2%**	11.3%	11.5%	11.3%	11.5%	11.7%	11.2%	11.2%	11.2%	11.2%
	7%	**11.6%**	11.9%	12.1%	11.8%	12.1%	12.4%	11.7%	11.9%	12.0%	11.6%
	8%	**12.0%**	12.3%	12.6%	12.2%	12.5%	12.9%	12.2%	12.4%	12.6%	12.0%
	9%	**12.3%**	12.7%	13.0%	12.6%	12.9%	13.3%	12.7%	12.9%	13.2%	12.3%
	10%	**12.6%**	13.0%	13.4%	12.9%	13.3%	13.7%	13.1%	13.4%	13.7%	12.6%
Steady State (years 70-100)	Any	11.6%	12.0%	12.4%	**11.5%**	11.9%	12.3%	12.1%	12.4%	12.8%	11.5%

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Reference model results supporting 1-year Point-in-Time criteria at 1st percentile (left tail)

Period	Starting level of UST20	1-year Point-in-Time at 1st percentile (UST20)									Criteria = least binding (i.e., max)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 1)	1%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%
	2%	1.1%	1.2%	1.2%	1.0%	1.1%	1.2%	0.9%	1.0%	1.1%	1.2%
	3%	1.7%	1.8%	1.9%	1.6%	1.7%	1.8%	1.5%	1.6%	1.7%	1.9%
	4%	2.4%	2.5%	2.6%	2.2%	2.3%	2.4%	2.2%	2.3%	2.4%	2.6%
	5%	3.0%	3.1%	3.3%	2.7%	2.9%	3.1%	2.8%	3.0%	3.2%	3.3%
	6%	3.6%	3.8%	4.0%	3.3%	3.5%	3.7%	3.5%	3.7%	3.9%	4.0%
	7%	4.2%	4.4%	4.7%	3.9%	4.1%	4.4%	4.2%	4.4%	4.7%	4.7%
	8%	4.9%	5.1%	5.3%	4.4%	4.7%	5.0%	5.0%	5.2%	5.5%	5.5%
	9%	5.5%	5.7%	6.0%	5.0%	5.3%	5.6%	5.7%	6.0%	6.3%	6.3%
	10%	6.1%	6.3%	6.7%	5.6%	5.9%	6.3%	6.5%	6.7%	7.1%	7.1%

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Reference model results supporting 1-year Point-in-Time criteria at 99th percentile (right tail)

Period	Starting level of UST20	1-year Point-in-Time at 99th percentile (UST20)									Criteria = least binding (i.e., min)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 1)	1%	2.2%	2.1%	1.9%	2.4%	2.2%	2.1%	2.6%	2.4%	2.2%	1.9%
	2%	3.7%	3.5%	3.3%	3.9%	3.7%	3.4%	4.0%	3.8%	3.6%	3.3%
	3%	5.1%	4.9%	4.7%	5.3%	5.1%	4.8%	5.3%	5.1%	4.8%	4.7%
	4%	6.5%	6.3%	6.0%	6.8%	6.5%	6.2%	6.5%	6.3%	6.0%	6.0%
	5%	7.9%	7.7%	7.3%	8.2%	7.9%	7.6%	7.7%	7.5%	7.2%	7.2%
	6%	9.3%	9.0%	8.7%	9.7%	9.3%	8.9%	8.9%	8.7%	8.4%	8.4%
	7%	10.7%	10.4%	10.0%	11.1%	10.7%	10.3%	10.1%	9.8%	9.5%	9.5%
	8%	12.0%	11.7%	11.3%	12.6%	12.2%	11.7%	11.2%	10.9%	10.6%	10.6%
	9%	13.4%	13.0%	12.6%	14.0%	13.6%	13.0%	12.3%	12.1%	11.8%	11.8%
	10%	14.7%	14.3%	13.9%	15.5%	15.0%	14.4%	13.4%	13.2%	12.9%	12.9%

Reference model results supporting 5-year Point-in-Time criteria at 1st percentile (left tail)

Period	Starting level of UST20	5-year Point-in-Time at 1st percentile (UST20)									Criteria = least binding (i.e., max)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 5)	1%	0.5%	0.5%	0.5%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%
	2%	0.7%	0.7%	0.8%	0.8%	0.8%	0.8%	0.7%	0.7%	0.7%	0.8%
	3%	1.0%	1.1%	1.2%	1.0%	1.0%	1.1%	0.9%	0.9%	1.0%	1.2%
	4%	1.4%	1.5%	1.6%	1.2%	1.3%	1.4%	1.1%	1.2%	1.3%	1.6%
	5%	1.7%	1.8%	2.0%	1.4%	1.6%	1.7%	1.4%	1.5%	1.7%	2.0%
	6%	2.0%	2.2%	2.4%	1.6%	1.8%	2.1%	1.7%	1.9%	2.2%	2.4%
	7%	2.3%	2.5%	2.8%	1.9%	2.1%	2.4%	2.1%	2.3%	2.6%	2.8%
	8%	2.6%	2.9%	3.2%	2.1%	2.3%	2.7%	2.4%	2.7%	3.1%	3.2%
	9%	2.9%	3.2%	3.5%	2.3%	2.6%	3.0%	2.8%	3.1%	3.6%	3.6%
	10%	3.2%	3.5%	3.9%	2.5%	2.9%	3.3%	3.2%	3.6%	4.1%	4.1%

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Reference model results supporting 5-year Point-in-Time criteria at 99th percentile (right tail)

Period	Starting level of UST20	5-year Point-in-Time at 99th percentile (UST20)									Criteria = least binding (i.e., min)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 5)	1%	5.0%	4.5%	3.9%	5.6%	5.0%	4.4%	5.9%	5.3%	4.7%	3.9%
	2%	7.0%	6.4%	5.7%	7.4%	6.8%	6.1%	7.4%	6.8%	6.2%	5.7%
	3%	8.8%	8.2%	7.5%	9.2%	8.5%	7.8%	8.7%	8.1%	7.6%	7.5%
	4%	10.4%	9.8%	9.1%	11.0%	10.3%	9.5%	9.9%	9.4%	8.8%	8.8%
	5%	12.0%	11.4%	10.7%	12.8%	12.1%	11.3%	11.0%	10.6%	10.0%	10.0%
	6%	13.5%	12.9%	12.2%	14.5%	13.8%	13.0%	12.2%	11.7%	11.2%	11.2%
	7%	14.8%	14.4%	13.7%	15.9%	15.4%	14.6%	13.3%	12.8%	12.3%	12.3%
	8%	15.9%	15.6%	15.1%	16.8%	16.6%	16.1%	14.3%	13.9%	13.5%	13.5%
	9%	16.7%	16.6%	16.2%	17.4%	17.3%	17.0%	15.3%	15.0%	14.6%	14.6%
	10%	17.3%	17.2%	17.0%	17.8%	17.8%	17.6%	16.2%	16.0%	15.6%	15.6%

Reference model results supporting 10-year Point-in-Time criteria at 1st percentile (left tail)

Period	Starting level of UST20	10-year Point-in-Time at 1st percentile (UST20)									Criteria = least binding (i.e., max)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 10)	1%	0.6%	0.6%	0.5%	0.7%	0.7%	0.7%	0.7%	0.7%	0.6%	0.7%
	2%	0.7%	0.7%	0.7%	0.8%	0.8%	0.8%	0.8%	0.7%	0.7%	0.8%
	3%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.8%	0.9%	0.9%
	4%	1.1%	1.1%	1.2%	1.0%	1.0%	1.1%	1.0%	1.0%	1.0%	1.2%
	5%	1.3%	1.4%	1.5%	1.1%	1.2%	1.3%	1.1%	1.1%	1.2%	1.5%
	6%	1.4%	1.6%	1.8%	1.2%	1.3%	1.4%	1.2%	1.3%	1.5%	1.8%
	7%	1.6%	1.8%	2.0%	1.3%	1.4%	1.6%	1.4%	1.5%	1.7%	2.0%
	8%	1.8%	2.0%	2.2%	1.4%	1.5%	1.8%	1.5%	1.7%	2.0%	2.2%
	9%	1.9%	2.1%	2.4%	1.5%	1.7%	2.0%	1.7%	1.9%	2.3%	2.4%
	10%	2.1%	2.3%	2.7%	1.6%	1.8%	2.1%	1.9%	2.2%	2.6%	2.7%

Reference model results supporting 10-year Point-in-Time criteria at 99th percentile (right tail)

Period	Starting level of UST20	10-year Point-in-Time at 99th percentile (UST20)									Criteria = least binding (i.e., min)
		Black-Karasinki (BK)			Brennan-Schwartz (BS)			Cox-Ingersoll-Ross (CIR)			
		10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	10yr HL	12yr HL	15yr HL	
Interim (end of year 10)	1%	8.0%	7.1%	6.1%	8.8%	7.8%	6.7%	8.7%	7.9%	7.0%	6.1%
	2%	10.0%	9.1%	8.1%	10.6%	9.6%	8.5%	9.9%	9.2%	8.4%	8.1%
	3%	11.6%	10.9%	9.9%	12.2%	11.4%	10.4%	11.0%	10.3%	9.6%	9.6%
	4%	13.1%	12.4%	11.6%	13.8%	13.1%	12.2%	12.0%	11.4%	10.8%	10.8%
	5%	14.3%	13.8%	13.1%	15.0%	14.6%	13.9%	12.9%	12.5%	11.9%	11.9%
	6%	15.2%	14.9%	14.4%	15.9%	15.7%	15.2%	13.9%	13.4%	12.9%	12.9%
	7%	15.9%	15.8%	15.4%	16.6%	16.5%	16.3%	14.7%	14.4%	14.0%	14.0%
	8%	16.4%	16.4%	16.2%	17.0%	17.0%	16.9%	15.4%	15.2%	14.9%	14.9%
	9%	16.8%	16.8%	16.8%	17.3%	17.4%	17.4%	16.0%	15.9%	15.8%	15.8%
	10%	17.1%	17.1%	17.2%	17.6%	17.6%	17.6%	16.5%	16.5%	16.5%	16.5%