Interest Rates—
Stylized Facts and Acceptance Criteria

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American Academy of Actuaries

Agenda—Interest rates

1. Background
2. Stylized Facts
3. Acceptance Criteria
4. Discussion and Q&A

1.

Background

Background

LATF asked the Academy to deliver a series of presentations focused on proposing qualitative **Stylized Facts** and quantitative **Acceptance Criteria** for the three major components of an ESG used for statutory reporting purposes: **Interest Rates**, **Equity Returns**, and **Corporate Bond Fund Returns**

This presentation proposes **Stylized Facts** and **Acceptance Criteria** for Interest Rates that (a) are independent of any specific ESG model, (b) can be used to identify and evaluate candidate ESG models, and (c) can be used to evaluate a set of stochastic scenarios.

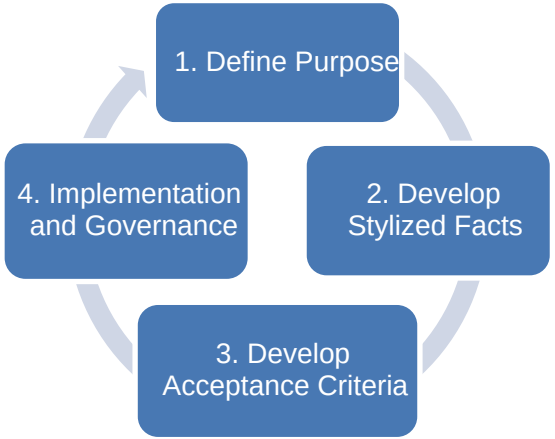
Prior presentations in this series:

- A Framework for Working with ESGs (8/8/22)
- ESG Governance Considerations (8/8/22)
- Equity Returns—Stylized Facts (8/9/22)
- Corporate Credit & Bond Fund Returns—Stylized Facts, Acceptance Criteria, and a Simplified Model (10/27/22)

This and future presentations in this series:

- **Interest Rates—Stylized Facts and Acceptance Criteria**
- Equity Returns—Acceptance Criteria

A framework for developing, implementing, and evaluating ESGs and the scenario sets they produce



"Statistical criteria are important in assessing the quality of an ESG. Statistical calibration criteria are usually numerically specified but can also be qualitative in nature. Statistical criteria belong to one of two broad categories: qualitative features and quantitative measures. The issues one must address in both categories are not amenable to a checklist approach, however, and expert judgment plays a role."

(quote from p. 96 of the 2020 CAS/Conning research paper on ESGs)

1. **Define Purpose**: The intended purpose of the ESG informs the economic variables to be simulated and the relative importance of their "stylized facts."
2. **Develop Stylized Facts** Stylized facts describe properties of the economic variables to be simulated. They are based on historical market data and economic theory and are prioritized relative to the defined purpose at hand. The establishment of stylized facts is critical for selecting candidate ESG models and a key prerequisite for the development of acceptance criteria.
3. **Develop Acceptance Criteria** A set of quantitative metrics or target values at different time horizons or in different economic conditions that provide a simplified framework for ensuring sets of scenarios produced by the ESG are consistent with key stylized facts.
4. **Implementation and Governance**: ESG models are selected based on their ability to reflect the stylized facts, then calibrated in accordance with acceptance criteria. Validation reports are produced on each candidate scenario set generated by the ESG. These reports compare scenario set statistics to acceptance criteria and contain other charts and tables useful for evaluation and signoff, which is ultimately a matter of judgement (no automatic "pass" or "fail" based only on acceptance criteria). Implementation is an iterative process. It is important to periodically review and recalibrate the ESG as market conditions change over time.

Excerpts from the 2020 Casualty Actuarial Society (CAS)/Conning research paper on ESGs

High-level features of a good ESG:

- "It produces simulation results that reflect the economic view of the risk manager.
- Scenarios are consistent with **realistic market dynamics**.
- A large simulation should produce some **extreme but plausible results** (i.e., the simulation covers and moderately exceeds the benchmark stylized facts).
- Component models and architecture must have sufficient flexibility to serve in multiple roles.

If one discusses the essential features of a good ESG with a diverse group of ESG experts, those experts' lists of features and the relative importance of those features will vary. However, they will set forth a common core of ideas that can serve as a checklist of best practices."

A good ESG:

1. "has a solid methodological foundation for the way the models are built and the way the variables are interrelated, and models are parsimonious, practical, and comprehensive.
2. provides a comprehensive suite of macroeconomic and financial variables and a multi-economy capability.
3. can accommodate many types of calibration views across a wide range of benchmarks.
4. produces simulation results that reflect a relevant view.
5. produces some extreme but plausible outcomes.
6. embeds realistic market dynamics.
7. is computationally efficient and numerically stable.
8. has fast and robust recalibration capabilities.
9. meets the requirements of regulators and auditing firms.
10. produces sufficient simulation detail for extensive validation."

The NAIC presented LATF with preliminary goals for interest rates on 12/3/20 and preliminary boundary guidance on 2/17/22

Preliminary goal	Preliminary boundary guidance
1. The model's starting yield curve should fit the actual starting yield curve as closely as possible. 2. The model should produce a variety of yield curve shapes, and they should change over time.	*Yield curve fit and Yield curve shape (priority 4)* a) Review initial actual vs. fitted spot curve differences for a sampling of 5 dates representing different shapes and rate levels for the entire curve and review fitted curves qualitatively to confirm they stylistically mimic the different actual yield curve shapes b) The frequency of different yield curve shapes in early durations should be reasonable considering the shape of the starting yield curve (e.g., a flatter yield curve leads to more inversions). c) The steady state curve has normal shape (not inverted for short maturities, longer vs shorter maturities, or between long maturities)
3. Interest rates can be negative.	*Negative rates (priority 3)* a) All maturities could experience negative interest rates b) Interest rates may remain negative for multi-year time periods c) Rates should generally not be lower than -1.5%

The NAIC presented LATF with preliminary goals for interest rates on 12/3/20 and preliminary boundary guidance on 2/17/22 (continued)

Preliminary goal	Preliminary boundary guidance
4. The model should be capable of producing a reasonable range of results for very long simulations.	*High rates (priority 2)* a) The scenario set should reasonably reflect history, with some allowance for more extreme high and low interest rate environments b) Upper Bound: i. 20% is >= 99th percentile on the 3M yield fan chart, and no more than 5% of scenarios have 3M yields that go above 20% in the first 30 years ii. 20% is >= 99th percentile on the 10Y yield fan chart, and no more than 5% of scenarios have 10Y yields that go above 20% in the first 30 years
5. The ESG should be capable of producing low interest rates for an extended period of time.	*Low for long (priority 1)* a) For scenarios generated as of 12/31/20, at least **10%** of scenarios should have a **10-year** geometric average of the 20-year US Treasury yield that is below its current level (e.g., 1.45% at 12/31/20) b) For scenarios generated as of 12/31/20, at least **5%** of scenarios should have a **30-year** geometric average of the 20-year US Treasury yield that is below its current level (e.g., 1.45% at 12/31/20)

The NAIC presented LATF with preliminary goals for interest rates on 12/3/20 and preliminary boundary guidance on 2/17/22 (continued)

Preliminary goal	Preliminary boundary guidance
6. The model should produce interest rate levels that fluctuate significantly over long periods.	*Volatility (no priority given)* Preliminary boundary guidance not specified
7. The interest rate generator should be arbitrage free.	*Arbitrage free (priority 3)* No longer considered an explicit requirement in the 2/17/22 preliminary boundary guidance since the NAIC's ESG Drafting Group was proposing the use of a generalized fractional floor.
8. The ESG should be calibrated using an appropriate historical period.	*Historical calibration period (no priority given)* Preliminary boundary guidance not specified

2.

Stylized Facts

Groupings for stylized facts about interest rates

Stylized Facts have been grouped into the following three categories:

1. Level of Interest Rates
2. Volatility of Interest Rates
3. Term Structure of Interest Rates (shape of yield curve)

1. Level of Interest Rates

The level of interest rates (the cost of borrowing money) changes due to a variety of complex and interrelated factors (e.g., supply of and demand for financing, business cycle, GDP, inflation, central bank actions to stimulate the economy or control inflation).

a. Short-term rates (which the Fed has more control of) have generally fallen within a range of 0% to 20% and have most often been within the lower part of that range. Long-term rates have generally been within 300 bps of short-term rates.

b. Negative interest rates are possible (have been observed outside the U.S.) but unlikely due to structural and market differences between the U.S. and other economies.

c. Interest rates can exhibit multi-year trends (e.g., up, down, low-for-long). Interest rates can stay at very low levels for several years. Short-term rates can stay very near their lower bound for several years while higher long-term rates continue to fluctuate.

2. Volatility of Interest Rates

The volatility of interest rates varies over time, with periods of both high and low volatility.

a. Monthly changes in interest rates are generally limited in size (less than 80 bps) but changes tend to be greater when the level of interest rates is higher.

b. Monthly changes in short-term rates tend to be larger than monthly changes in long-term rates when short-term rates are not near their lower bound, but the opposite relationship tends to hold when short-term rates are near their lower bound.

c. Volatility tends to increase in stressed markets.

3. Term Structure of Interest Rates (shape of yield curve)

The yield curve embodies the term structure of interest rates and takes a variety of shapes.

a. The normal yield curve shape is upward sloping (long-term rates greater than short-term rates) and concave downward. Normal yield curve shapes can persist for extended periods of time.

b. Non-normal yield curve shapes include inversions (downward sloping), humps, and valleys. Inversions (and other non-normal yield curve shapes) are often associated with key points in the business cycle (e.g., recession indicator) but generally don't persist for extended periods of time.

c. The slope of the yield curve tends to be lower (even negative/inverted) when short-term rates are at relatively high levels.

3.

Acceptance Criteria

Unless otherwise specified, tables and charts on the following slides are based on two primary data sources:

1. *Historical U.S. Treasury yields from the "Historical Curves" tab of the August 2022 Academy Interest Rate Generator (AIRG) located at https://soa.org/resources/tables-calcs-tools/research-scenario/*
2. *Simulated U.S. Treasury yields from "10000_Path_Set_1a_Conning_GFF_Baseline_Equity_123121" located at https://content.conning.com/scenariofiles*

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This section discusses acceptance criteria around four key properties of interest rates identified in the stylized facts

1. Rate level
   - Includes criteria around high, low, and negative rates.
   - Only **steady state** criteria is being proposed at this point. **Interim** rate level criteria, which depend heavily on initial conditions, are being developed and will be proposed later.
2. Rate volatility
   - Criteria varies by rate level (applies to interim and steady state).
3. Yield curve shape
   - Criteria varies by rate level (applies to interim and steady state).
4. Low-for-long
   - Although the ESGWG has not finalized its proposal for this key property of interest rates, we present our qualitative understanding of low-for-long for discussion and feedback.

Acceptance criteria provide a simplified framework for validating key scenario properties but are only part of a larger validation exercise that includes other charts, statistics, and of course, judgment.

Criteria were developed with the following principles in mind:

- The scenario set should include some extreme but plausible scenarios.
- Pathwise behavior is as important as point-in-time distributions.
- Scenarios should be consistent with realistic market dynamics over both short- and long-term horizons.

"The importance of pathwise model behavior is that it is the simulated path that represents the way an insurance company will experience the evolution of the economy. In other words. The pathwise behavior is the only thing of interest when we want to investigate simulation dynamics. If the overall distribution of returns for an asset class is correct but the pathwise behavior does not correspond to the nature of the fluctuations that we see in the historical record, then there is a potential model issue." (p. 107)

"A good ESG will be capable of being calibrated to coherent targets across multiple simulation horizons." (p. 106)

(quotes from the 2020 CAS/Conning research paper on ESGs)

Rate level
Historical PEWs (see appendix for additional information on PEWs)

- Selected 15-year half-life "Percentiles Exponentially Weighted" (PEWs) on historical month-end interest rates are proposed as steady state acceptance criteria for rate level (high, low, and negative).
  - ➢ *Ideally, corresponding percentiles on scenario sets are "plausibly more extreme" than the PEWs.*
- Calculated using data from April 1953, but unlike typical percentiles where data is weighted equally, PEWs give exponentially less weight to older data.
- PEWs are defined by their "half-life." A half-life of 15 years means data that is 15 years older receives half the weight.
- A half-life of 15 years is suggested to give more weight to recent data while not overreacting to short-term fluctuations.

"Stability versus responsiveness: As a common trade-off and concern in general actuarial work, it is important to consider where the happy medium is between a long period of data (enhancing stability) and a recent shorter data period (that promotes responsiveness to more recent conditions)."

(quote from p. 129 of the 2020 CAS/Conning research paper on ESGs)

15-year half-life PEWs at 12/31/21	20Y	1Y
Max	15.52%	16.97%
99th PEW	13.55%	13.86%
95th PEW	9.35%	9.02%
85th PEW	7.54%	6.22%
70th PEW	5.77%	4.88%
60th PEW	4.88%	3.34%
50th PEW	4.33%	2.11%
40th PEW	3.35%	1.31%
30th PEW	2.83%	0.49%
15th PEW	2.31%	0.16%
5th PEW	1.78%	0.10%
1st PEW	1.15%	0.07%
Min	0.98%	0.05%

Criteria for the distribution of steady state interest rates

	20Y Criteria	1Y Criteria	"Buffers" could provide guidance on "too extreme"
Max	> 15.52%	> 16.97%	[300 bps]
99th Percentile	> 13.55%	> 13.86%	[275 bps]
95th Percentile	> 9.35%	> 9.02%	[250 bps]
85th Percentile	> 7.54%	> 6.22%	[225 bps]
70th Percentile	> 5.77%	> 4.88%	[200 bps]
50th Percentile	> 3.35% and < 4.88%	> 1.31% and < 3.34%	n/a
30th Percentile	< 2.83%	< 0.49%	[60 bps]
15th Percentile	< 2.31%	< 0.16%	[70 bps]
5th Percentile	< 1.78%	< 0.10%	[80 bps]
1st Percentile	< 1.15%	< 0.07%	[90 bps]
Min	< 0.98%	< 0.05%	[100 bps]

- Criteria is based on 15-year half-life PEWs.
  - Scenarios should be "plausibly more extreme" than the PEWs.
  - But scenarios that exceed the PEWs by more than a "buffer" may be "too extreme".
- Test statistics:
  - Percentiles of the [20Y] and [1Y] rate distributions at month [600] (year [50]).
  - Max and Min of the [20Y] and [1Y] rate distributions are from projection months [480] through [720] (years [40] through [60]).
- Note, the range for the 50th percentile (Median) is based on the 40th and 60th PEW.

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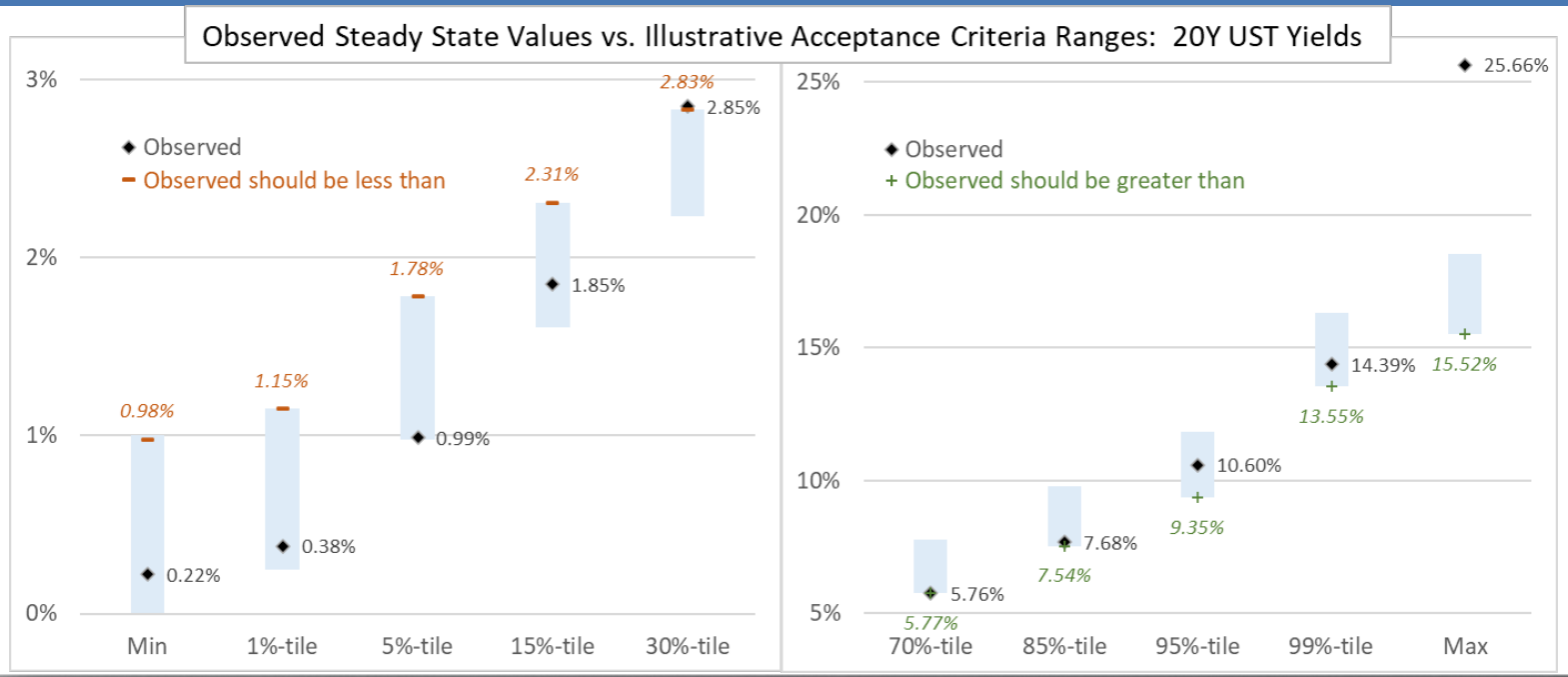
Rate level

Illustrative application of criteria to field test scenario set #1a

	20Y Criteria	1Y Criteria	Buffers	20Y Stat	20Y Result	1Y Stat	1Y Result
Max	> 15.52%	> 16.97%	[300 bps]	25.66%	> Buffer (714 bps)	29.60%	> Buffer (963 bps)
99th Percentile	> 13.55%	> 13.86%	[275 bps]	14.39%	In range	15.40%	In range
95th Percentile	> 9.35%	> 9.02%	[250 bps]	10.60%	In range	11.09%	In range
85th Percentile	> 7.54%	> 6.22%	[225 bps]	7.68%	In range	7.41%	In range
70th Percentile	> 5.77%	> 4.88%	[200 bps]	5.76%	< PEW (1 bp)	4.71%	< PEW (17 bps)
50th Percentile	> 3.35% and < 4.88%	> 1.31% and < 3.34%	n/a	4.20%	In range	2.35%	In range
30th Percentile	< 2.83%	< 0.49%	[60 bps]	2.85%	> PEW (2 bps)	0.40%	In range
15th Percentile	< 2.31%	< 0.16%	[70 bps]	1.85%	In range	0.07%	In range
5th Percentile	< 1.78%	< 0.10%	[80 bps]	0.99%	In range	-0.26%	In range
1st Percentile	< 1.15%	< 0.07%	[90 bps]	0.38%	In range	-0.53%	In range
Min	< 0.98%	< 0.05%	[100 bps]	0.22%	In range	-0.79%	In range

Illustrative application of criteria to field test scenario set #1a (continued)



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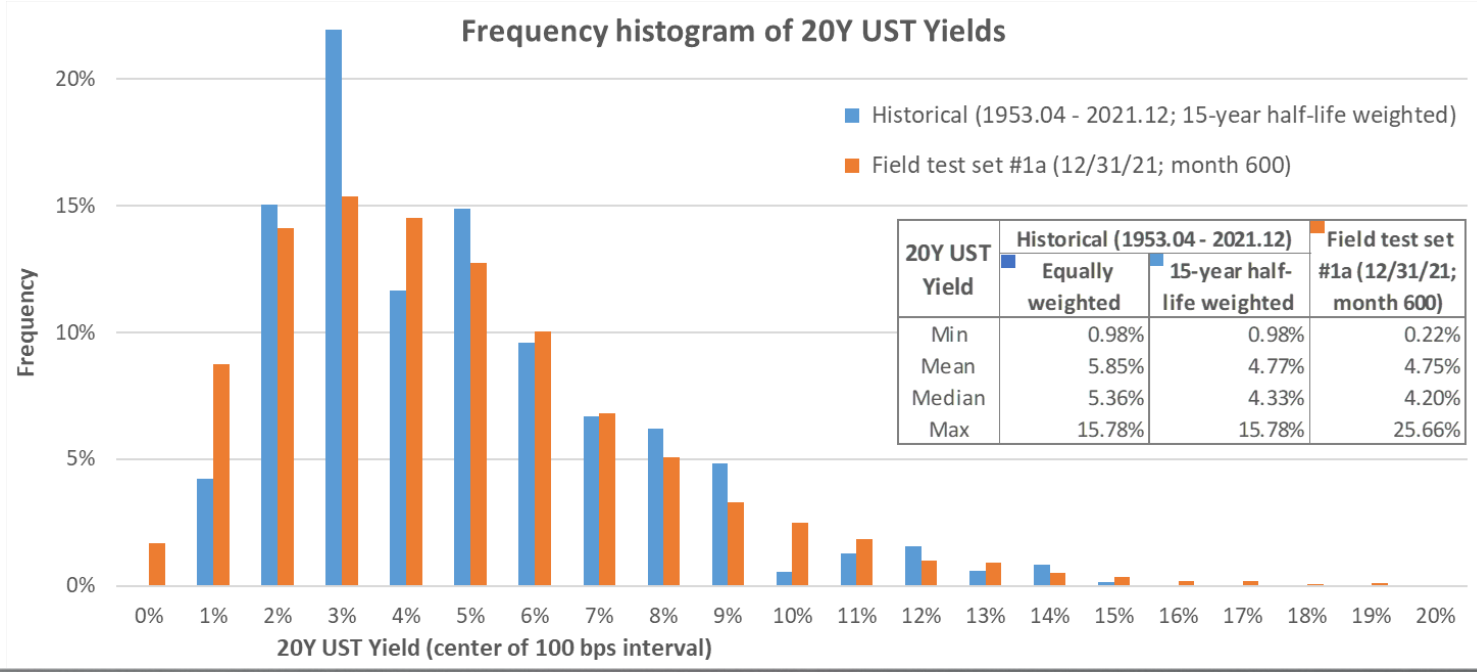
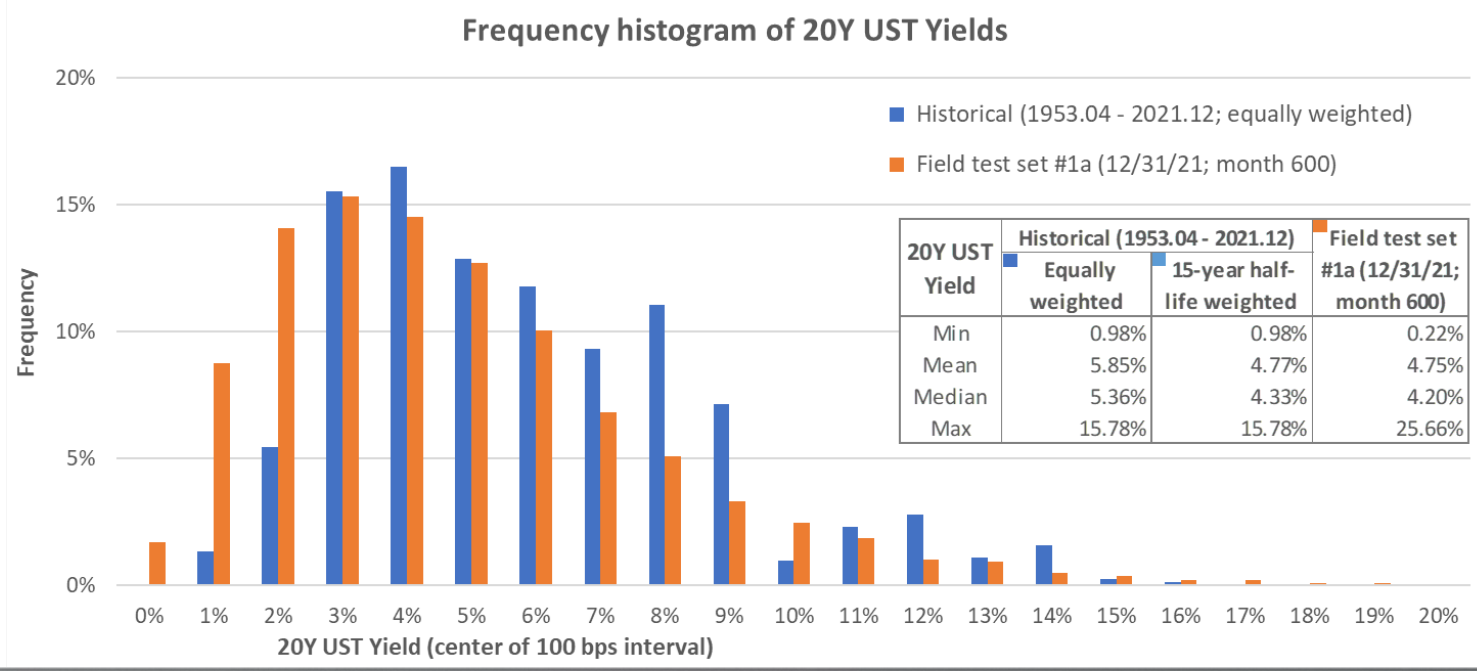
Supplemental chart for evaluating rate levels on consistent basis with PEWs



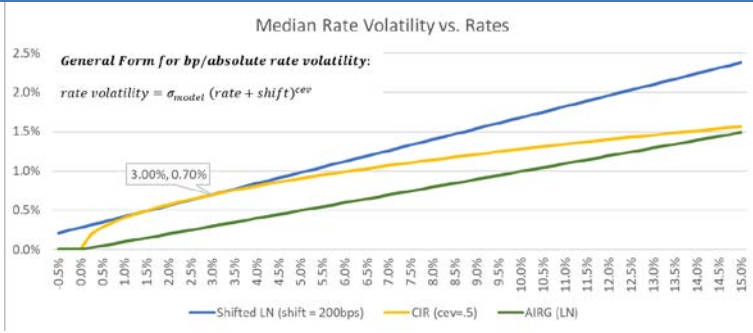
20Y UST Yield	Historical (1953.04 - 2021.12) Equally weighted	Historical (1953.04 - 2021.12) 15-year half-life weighted	Field test set #1a (12/31/21; month 600)
Min	0.98%	0.98%	0.22%
Mean	5.85%	4.77%	4.75%
Median	5.36%	4.33%	4.20%
Max	15.78%	15.78%	25.66%

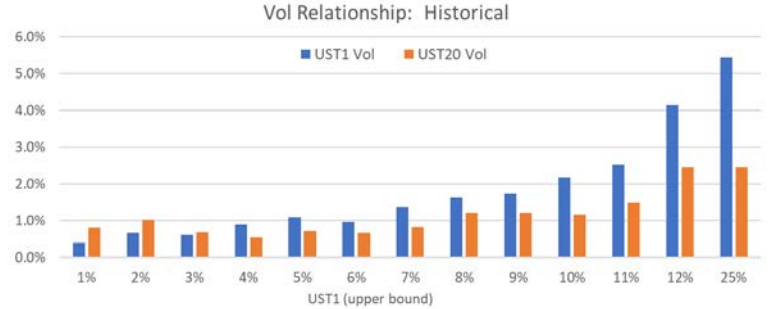
Supplemental chart for evaluating rate levels on consistent basis with PEWs

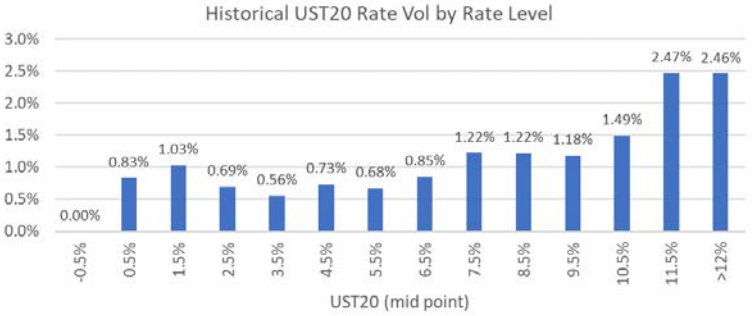


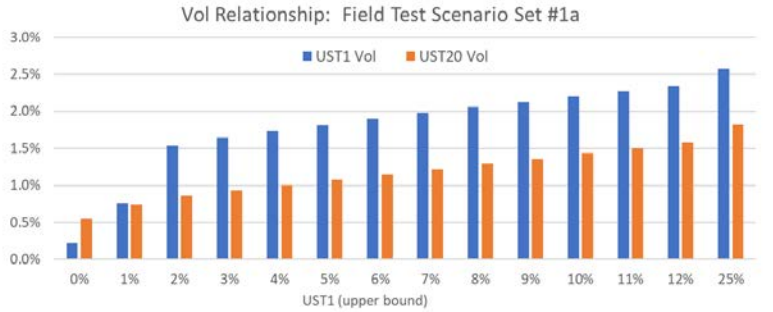
20Y UST Yield	Historical (1953.04 - 2021.12) Equally weighted	Historical (1953.04 - 2021.12) 15-year half-life weighted	Field test set #1a (12/31/21; month 600)
Min	0.98%	0.98%	0.22%
Mean	5.85%	4.77%	4.75%
Median	5.36%	4.33%	4.20%
Max	15.78%	15.78%	25.66%

Rate volatility
Background









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Historical volatility statistics

Annualized standard deviation of monthly yield changes from 1953.04 to 2021.12, bucketed by yield level at beginning of month (BOM):

Bucket	Yield Level (BOM	1Y	20Y
Low	[⩽ 3%]	0.59%	0.61%
Medium	[> 3%, ≤ 8%]	1.16%	0.74%
High	[> 8%]	3.32%	1.54%

Note that short (1Y) rate volatility tends to exceed long (20Y) rate volatility, except when rates are low.

Volatility criteria

- For the relevant test statistics on the candidate scenario set, calculate the annualized standard deviation of monthly yield changes across all scenarios, bucketed by the rate level at the BOM.
  - Calculate the above test statistics for both the first [10] years and steady state, e.g., years [40] to [60].
- The above test statistics should be "reasonably close" to the historical volatility statistics in the table to the left.
  - For example, the above test statistics should be within [X]% of historical volatility statistics.

Illustrative application of rate volatility criteria to field test scenario set #1a

Tabular comparison of annualized standard deviation of 1Y and 20Y UST rates to history

				First [10] years		Steady state*	
	Bucket	Yield Level (BOM)	History	Simulated	Difference	Simulated	Difference
1Y UST	Low	[≤ 3%]	0.59%	1.06%	47 bps above	1.05%	46 bps above
	Medium	[> 3%, ≤ 8%]	1.16%	1.88%	72 bps above	1.85%	69 bps above
	High	[> 8%]	3.32%	2.31%	101 bps below	2.31%	101 bps below
	Bucket	Yield Level (BOM)	History	Simulated	Difference	Simulated	Difference
20Y UST	Low	[≤ 3%]	0.61%	0.66%	5 bps above	0.68%	7 bps above
	Medium	[> 3%, ≤ 8%]	0.74%	1.00%	26 bps above	1.11%	37 bps above
	High	[> 8%]	1.54%	1.61%	7 bps above	1.69%	15 bps above

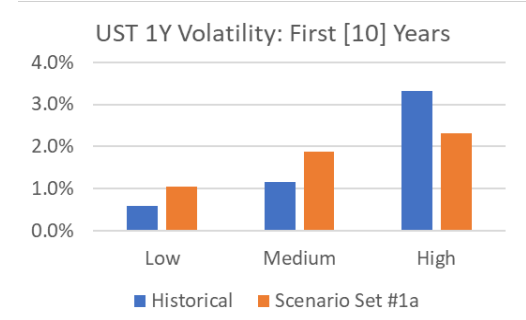
** Years [40] to [60]*

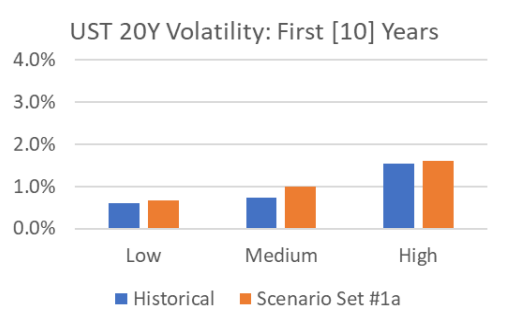
Illustrative application of rate volatility criteria to field test scenario set #1a

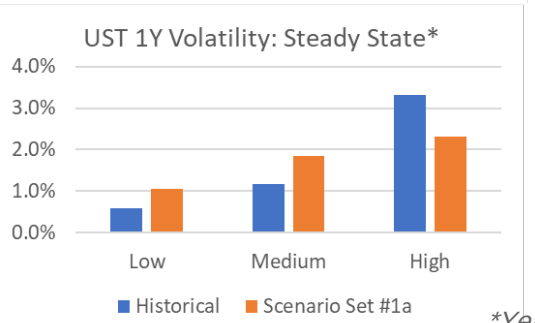
Graphical comparison of annualized standard deviation of 1Y and 20Y UST rates to history

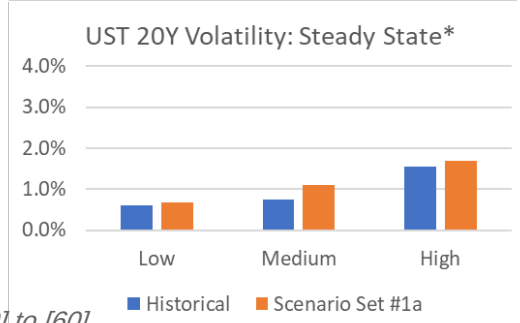
Observations on Set #1a:

- Initial and steady state volatility are similar
- Volatility is generally higher than history
- In the Low bucket:
  - 1Y volatility roughly double history
  - 20Y volatility roughly equal to history









**Years [40] to [60]*

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Historical yield curve slope statistics

Selected percentiles on the distribution of slope (month-end [20Y] less month-end [1Y] yields) from 1953.04 to 2021.12, bucketed by [20Y] rate:

Bucket	Yield Level (BOM)	% Inverted	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	[≤3%]	0%	0.0%	0.3%	**0.5%**	1.1%	1.6%	2.0%	**2.3%**	2.6%	2.9%
Medium	[>3%, ≤8%]	17%	-1.4%	-0.5%	**-0.1%**	0.4%	0.9%	1.8%	**3.3%**	3.8%	4.3%
High	[>8%]	25%	-3.4%	-1.5%	**-0.8%**	0.3%	1.2%	1.8%	**2.1%**	2.7%	3.9%

Criteria

- For the test statistics on the candidate scenario set, calculate selected percentiles on the distribution of slope ([20Y] less [1Y] yield) across all scenarios, bucketed by the level of the [20Y] yield level.
  - Calculate above for both the first [10] years and steady state, e.g., years [40] to [60].
- The [15th] and [85th] percentiles should be "plausibly more extreme" than history.

Historical data indicates the distribution of curve shapes (particularly inversions) varies by rate level.

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Yield curve slope
Illustrative application of criteria to field test scenario set #1a

Historical

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	0%	0.0%	0.3%	**0.5%**	1.1%	1.6%	2.0%	**2.3%**	2.6%	2.9%
Medium	17%	-1.4%	-0.5%	**-0.1%**	0.4%	0.9%	1.8%	**3.3%**	3.8%	4.3%
High	25%	-3.4%	-1.5%	**-0.8%**	0.3%	1.2%	1.8%	**2.1%**	2.7%	3.9%

Field test #1a *(first [10] years)*

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	6%	-4.5%	-0.2%	**0.6%**	1.0%	1.3%	1.6%	**1.9%**	2.2%	3.1%
Medium	35%	-9.2%	-2.6%	**-1.3%**	-0.3%	0.7%	1.5%	**2.3%**	3.0%	4.5%
High	62%	-10.0%	-5.2%	**-3.4%**	-2.0%	-0.7%	0.5%	**1.3%**	2.2%	3.7%

Difference *(field test #1a less historical)*

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	6%	-4.6%	-0.5%	**0.1%**	-0.2%	-0.3%	-0.5%	**-0.4%**	-0.4%	0.2%
Medium	18%	-7.9%	-2.1%	**-1.2%**	-0.6%	-0.3%	-0.3%	**-1.0%**	-0.8%	0.3%
High	37%	-6.7%	-3.7%	**-2.5%**	-2.3%	-1.9%	-1.3%	**-0.8%**	-0.5%	-0.2%

Notes:

- Slope = [20Y] less [1Y] yield
- Bucketed by [20Y] yield
- Buckets:
  - Low [$\leq 3\%$]
  - Medium [$> 3\%, \leq 8\%$]
  - High [$> 8\%$]
- The [15th] percentile is more extreme than history if the difference is negative.
- The [85th] percentile is more extreme than history if the difference is positive.

Illustrative application of criteria to field test scenario set #1a

Historical

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	0%	0.0%	0.3%	**0.5%**	1.1%	1.6%	2.0%	**2.3%**	2.6%	2.9%
Medium	17%	-1.4%	-0.5%	**-0.1%**	0.4%	0.9%	1.8%	**3.3%**	3.8%	4.3%
High	25%	-3.4%	-1.5%	**-0.8%**	0.3%	1.2%	1.8%	**2.1%**	2.7%	3.9%

Field test #1a *(steady state, e.g., years [40] to [60])*

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	4%	-4.5%	0.3%	**0.9%**	1.2%	1.6%	2.0%	**2.3%**	2.6%	3.1%
Medium	19%	-10.5%	-2.0%	**-0.4%**	0.7%	1.7%	2.5%	**3.1%**	3.5%	4.6%
High	39%	-11.3%	-3.6%	**-1.8%**	-0.5%	0.6%	1.5%	**2.2%**	2.8%	4.2%

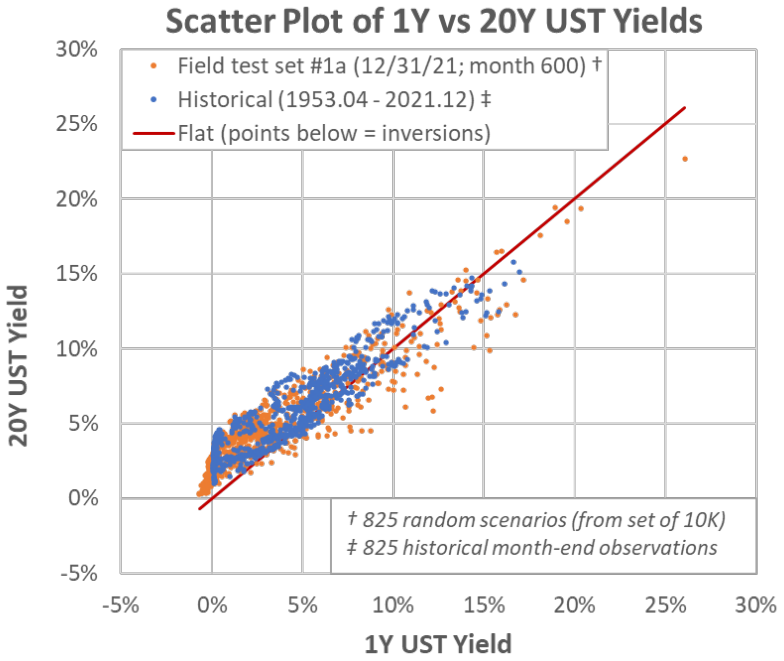
Difference *(field test #1a less historical)*

Bucket	Inv %	Min	5%	**15%**	30%	Median	70%	**85%**	95%	Max
Low	4%	-4.5%	0.0%	**0.4%**	0.1%	0.0%	0.0%	**0.0%**	0.0%	0.2%
Medium	2%	-9.2%	-1.5%	**-0.3%**	0.4%	0.8%	0.8%	**-0.2%**	-0.3%	0.4%
High	14%	-8.0%	-2.0%	**-1.0%**	-0.8%	-0.6%	-0.3%	**0.1%**	0.1%	0.3%

Notes:

- Slope = [20Y] less [1Y] yield
- Bucketed by [20Y] yield
- Buckets:
  - Low [$\leq 3\%$]
  - Medium [$> 3\%, \leq 8\%$]
  - High [$> 8\%$]
- The [15th] percentile is more extreme than history if the difference is negative.
- The [85th] percentile is more extreme than history if the difference is positive.

Supplemental chart for evaluating rate yield curve slope



Statistics

Historical (1953.04 - 2021.12)

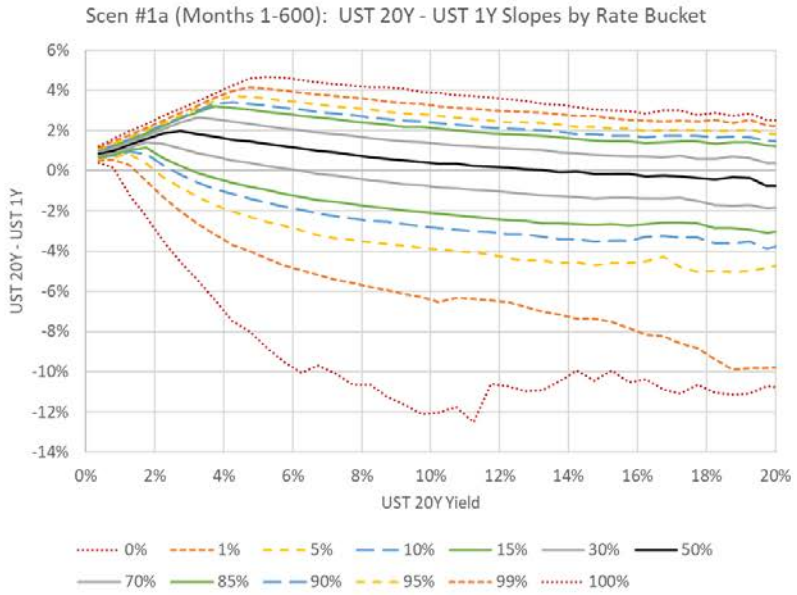
- Min spread = -336 bps
- Mean spread = 122 bps
- Max spread = 425 bps
- StDev spread = 132 bps
- Inversion freq. = 16%
- Mean inversion = -63 bps

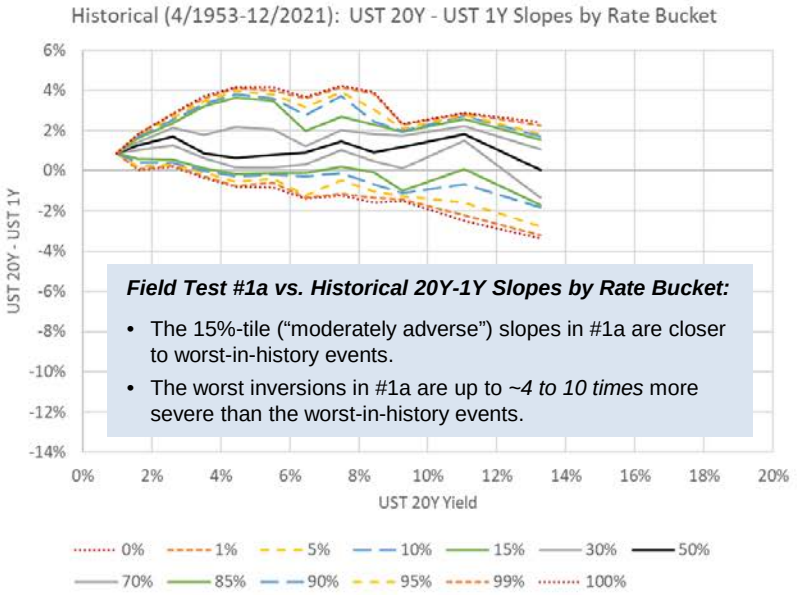
Field test set #1a (12/31/21; month 600) *

- Min spread = -926 bps
- Mean spread = 129 bps
- Max spread = 448 bps
- StDev spread = 159 bps
- Inversion freq. = 17%
- Mean inversion = -146 bps

** Stats based on all 10K scenarios*

Supplemental chart for evaluating rate yield curve slope





Field Test #1a vs. Historical 20Y-1Y Slopes by Rate Bucket:

- The 15%-tile ("moderately adverse") slopes in #1a are closer to worst-in-history events.
- The worst inversions in #1a are up to *~4 to 10 times* more severe than the worst-in-history events.

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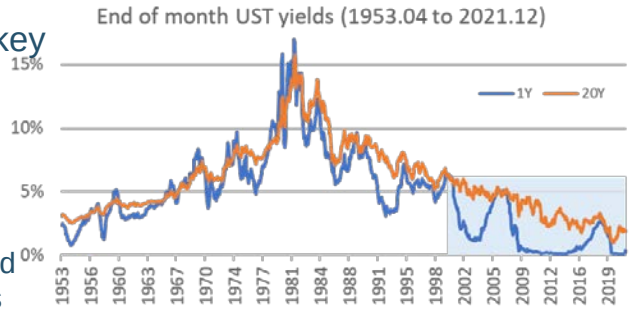
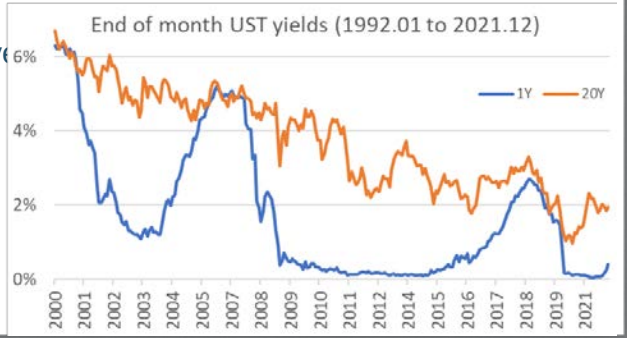
4. Low-for-long
Qualitative understanding

Although the ESGWG has not finalized its proposal for this key property of interest rates, we present our qualitative understanding of low-for-long for discussion and feedback.

Historical observations on low-for-long interest rate behavior:

1. (a) The long rate [20Y] stays below a threshold [3%] for an extended period of time [5+ years]. (b) During this time, the long rate continues to fluctuate as usual.
2. (a) The short rate [1Y] is "stuck" in a very narrow range [50bps] above zero. (b) During this time, short rate volatility (which normally exceeds long rate volatility) drops to near zero.
3. Low-for-long is a relatively recent phenomenon (post-2000 in the US; limited historical data).





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4.

Discussion and Q&A

Thank You

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- Amanda Barry Moilanen, Life Policy Analyst, barrymoilanen@actuary.org

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Appendix

Additional information on Percentiles Exponentially Weighted (PEWs)

The development of historical statistics for economic variables such as interest rates and equity rates involves subjective decisions such as how much history to include. One way to make use of all available data, but to focus more heavily on more recent data, is to develop exponentially weighted averages and percentiles.

An AWE is an Average Weighted Exponentially, with parameter Alpha. The most recent historical period, typically a month, gets an initial weight of 100%. Each prior historical period gets (1-α) times the weight of the next most recent period. Based on the number of historical periods of available data, the weights are then normalized so that their sum is 100%. The AWE is simply the weighted average of all the available or selected data. The “half-life” is then the period of time for which the cumulative weight reaches 50%.

PEWs apply the same concept to develop exponentially weighted percentiles. The historical values are unchanged, but their relative weight is dependent on when they occurred. Values are rank-ordered, with percentiles based on the sum of the relative weights up to the particular value. It may be desirable to assign percentiles at the center of each value’s weight range, especially if extreme values are important or statistical distributions will be fitted to the percentiles.

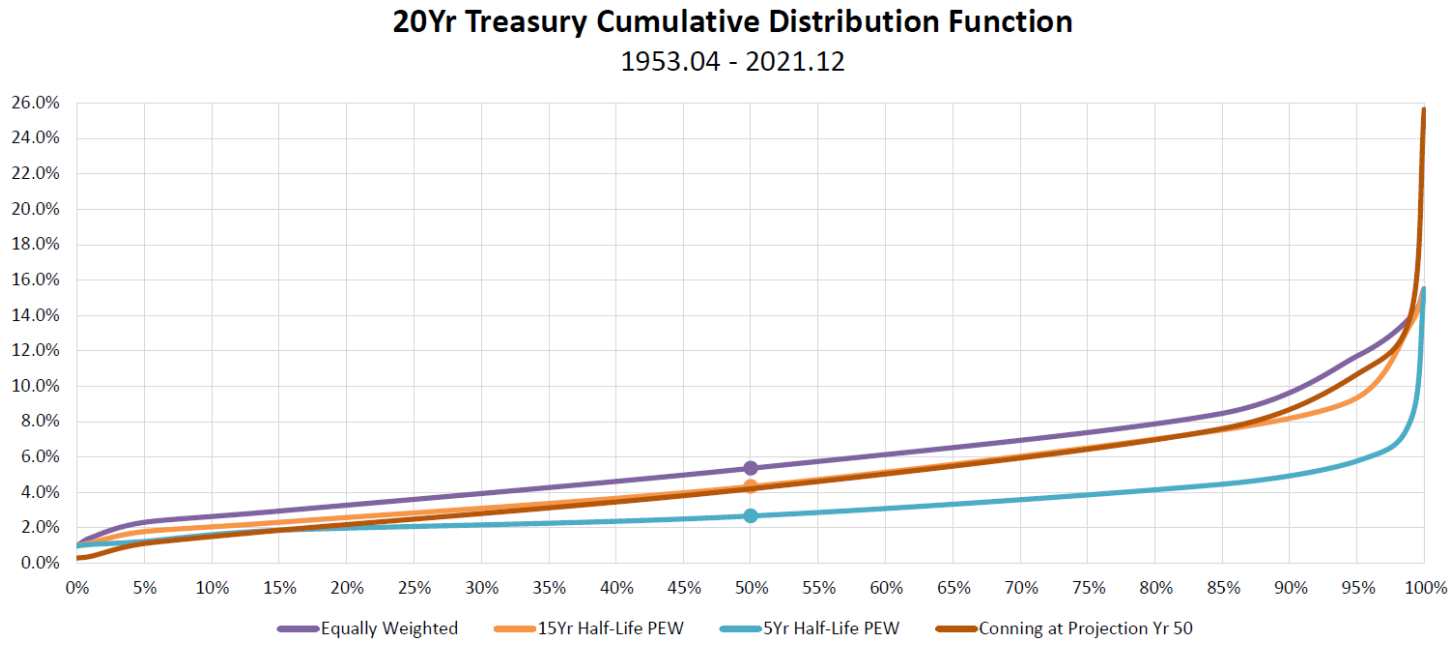
PEWs

Historical UST 20Y PEWs at different half-lives (12/31/2021)

	Equally Weighted	20Yr Half-Life	15Yr Half-Life	10Yr Half-Life	5Yr Half-Life
Maximum	15.52 %	15.52 %	15.52 %	15.52 %	15.52 %
99th PEW	13.92 %	13.63 %	13.55 %	12.49 %	8.11 %
95th PEW	11.70 %	10.44 %	9.35 %	8.59 %	5.78 %
85th PEW	8.48 %	7.94 %	7.54 %	6.47 %	4.47 %
70th PEW	7.09 %	6.20 %	5.77 %	4.87 %	3.08 %
50th PEW	5.36 %	4.64 %	4.33 %	3.31 %	2.66 %
30th PEW	4.06 %	3.05 %	2.83 %	2.63 %	2.20 %
15th PEW	2.95 %	2.47 %	2.31 %	2.08 %	1.85 %
5th PEW	2.31 %	1.85 %	1.78 %	1.45 %	1.23 %
1st PEW	1.43 %	1.18 %	1.15 %	1.05 %	1.05 %
Minimum	0.98 %	0.98 %	0.98 %	0.98 %	0.98 %
Max minus Min	14.54 %	14.54 %	14.54 %	14.54 %	14.54 %
99th minus 1st	12.48 %	12.45 %	12.40 %	11.44 %	7.06 %
95th minus 5th	9.39 %	8.59 %	7.57 %	7.14 %	4.55 %
85th minus 15th	5.53 %	5.47 %	5.23 %	4.39 %	2.62 %
70th minus 30th	3.03 %	3.15 %	2.94 %	2.24 %	0.88 %

* Historical 20 year Treasury rates are from: 1) 1953.04 - 1977.01 - monthly average rates from Fed H15 monthly history report;
2) 1977.02 - 1993.09 - estimated month-end 20 year rates by averaging 10 year and 30 year; 3) 1993.10 - current: actual month end rates.

Chart of UST 20Y PEWs at different half-lives (12/31/2021)



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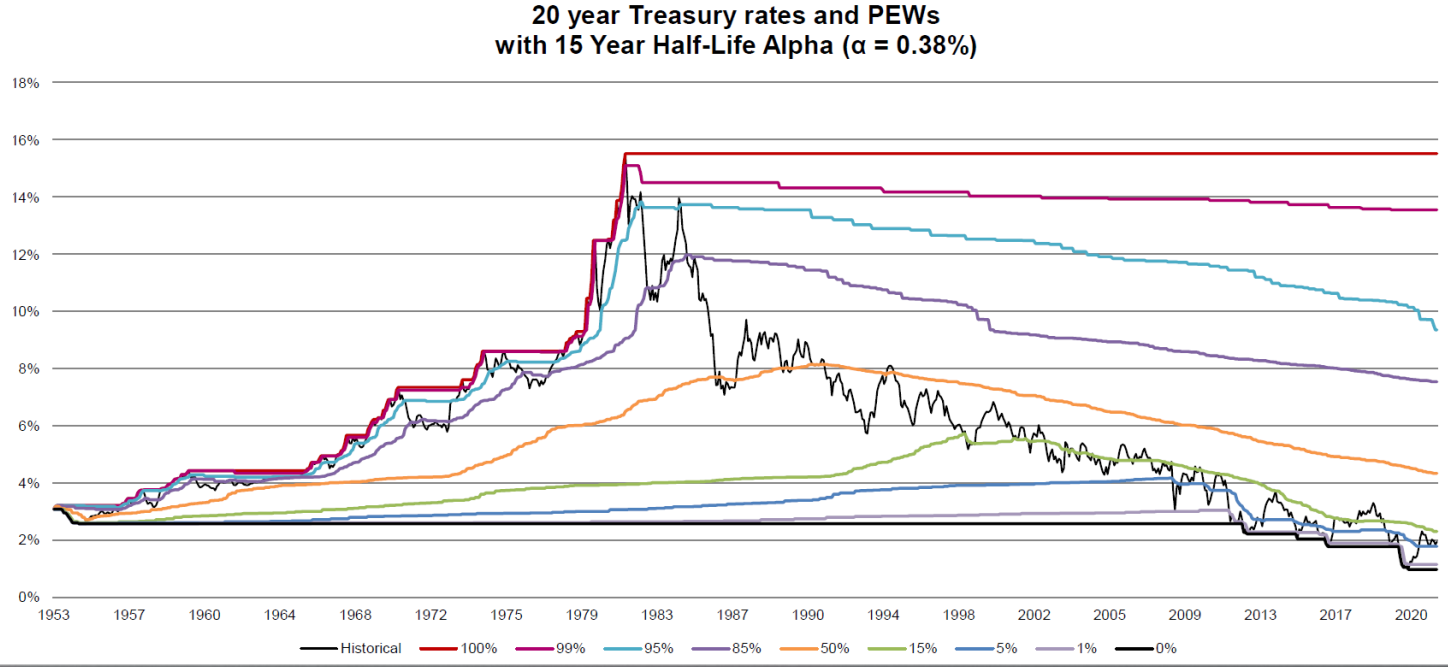
Historical movement in 15-year half-life PEWs

	PEWs from April 1953 through:						
	1971.12 (50 years ago)	1991.12 (30 years ago)	2001.12 (20 years ago)	2011.12 (10 years ago)	2016.12 (5 years ago)	2020.12 (1 year ago)	2021.12 Current
Maximum	7.34 %	15.52 %	15.52 %	15.52 %	15.52 %	15.52 %	15.52 %
99th PEW	7.24 %	14.32 %	14.03 %	13.88 %	13.63 %	13.55 %	13.55 %
95th PEW	6.85 %	13.28 %	12.48 %	11.45 %	10.64 %	10.04 %	9.35 %
85th PEW	6.18 %	11.19 %	9.18 %	8.34 %	8.01 %	7.60 %	7.54 %
Mean (AWE)	4.60 %	8.03 %	7.38 %	6.24 %	5.46 %	4.91 %	4.77 %
50th PEW	4.20 %	8.11 %	7.05 %	5.68 %	4.91 %	4.47 %	4.33 %
15th PEW	3.30 %	4.25 %	5.46 %	4.19 %	2.77 %	2.49 %	2.31 %
5th PEW	2.86 %	3.49 %	3.96 %	3.61 %	2.31 %	1.78 %	1.78 %
1st PEW	2.60 %	2.80 %	2.93 %	2.66 %	1.90 %	1.15 %	1.15 %
Minimum	2.57 %	2.57 %	2.57 %	2.57 %	1.78 %	0.98 %	0.98 %
99th minus 1st	4.64 %	11.52 %	11.10 %	11.22 %	11.73 %	12.40 %	12.40 %
95th minus 5th	3.99 %	9.79 %	8.52 %	7.84 %	8.33 %	8.26 %	7.57 %
85th minus 15th	2.88 %	6.94 %	3.72 %	4.15 %	5.24 %	5.11 %	5.23 %

* Percentiles Exponentially Weighted (PEW) are determined by the specified alpha
* Historical 20 year Treasury rates are from: 1) 1953.04 - 1977.01 - monthly average rates from Fed H15 monthly history report; 2) 1977.02 - 1993.09 - estimated month-end 20 year rates by averaging 10 year and 30 year; 3) 1993.10 - current: actual month end rates.

Chart of historical movement in 15-year half-life PEWs



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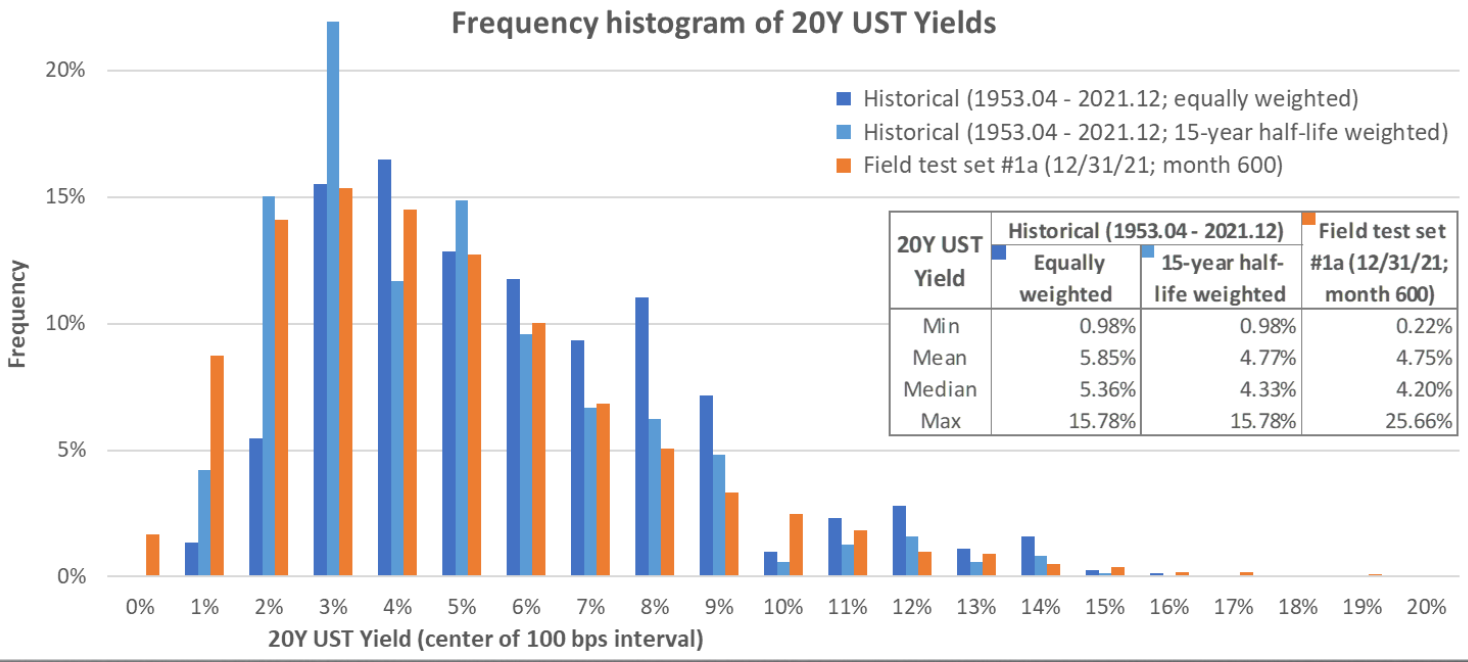
Supplemental chart for evaluating rate levels on consistent basis with PEWs



20Y UST Yield	Historical (1953.04 - 2021.12) Equally weighted	Historical (1953.04 - 2021.12) 15-year half-life weighted	Field test set #1a (12/31/21; month 600)
Min	0.98%	0.98%	0.22%
Mean	5.85%	4.77%	4.75%
Median	5.36%	4.33%	4.20%
Max	15.78%	15.78%	25.66%

Yield curve slope (bucketed by 20Y rate)
Historical Slope Data (4/1953 - 12/2020)

Observations:

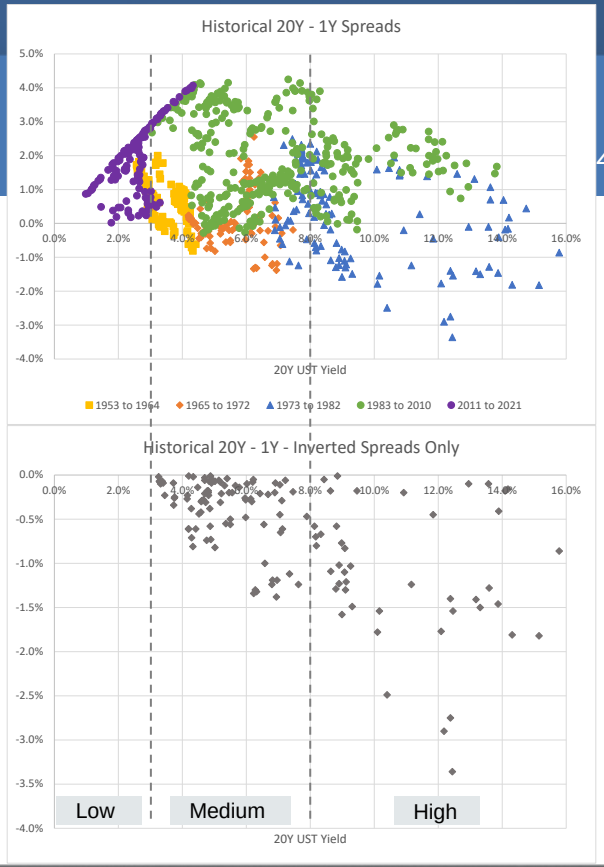
- No inversions for UST 20-year yields below 3%
- Severity of inversions generally increases with rate levels
- Other variations in curve steepness by rate level
- Recommend slope criteria based on simplified Low / Medium / High 20Y yield buckets to capture historical dynamics while not being overly constraining
- Also considers alignment with volatility buckets

ALL Spreads

Rate Bucket (20Y)	Inverted Months	Total Months	% Inverted	Min Spread	15%	50%	85%	Max Spread	Avg Spread
[0%, 1%)	0	1	0%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%
[1%, 2%)	0	23	0%	0.0%	0.6%	1.3%	1.6%	1.8%	1.2%
[2%, 3%)	0	107	0%	0.2%	0.5%	1.7%	2.4%	2.9%	1.6%
[3%, 4%)	11	102	11%	-0.3%	0.1%	0.9%	3.2%	3.7%	1.4%
[4%, 5%)	32	146	22%	-0.8%	-0.2%	0.6%	3.6%	4.1%	1.2%
[5%, 6%)	18	89	20%	-0.8%	-0.1%	0.8%	3.5%	4.2%	1.3%
[6%, 7%)	18	93	19%	-1.4%	-0.1%	0.9%	2.0%	3.7%	0.9%
[7%, 8%)	10	82	12%	-1.2%	0.2%	1.5%	2.7%	4.3%	1.5%
[8%, 9%)	14	79	18%	-1.6%	-0.1%	0.9%	2.3%	3.9%	1.1%
[9%, 10%)	7	29	24%	-1.5%	-1.0%	1.2%	1.9%	2.3%	0.8%
> 10%	24	74	32%	-3.4%	-1.5%	1.4%	2.2%	2.9%	0.7%
All	**134**	**825**	**16%**	**-3.4%**	**-0.1%**	**1.1%**	**2.7%**	**4.3%**	1.2%

INVERTED Spreads Only

Rate Bucket (20Y)	Inverted Months	Total Months	% Inverted	Most Negative Spread	15%	50%	85%	Least Negative Spread	Avg Spread
[0%, 1%)	0	1	0%	n/a	n/a	n/a	n/a	n/a	n/a
[1%, 2%)	0	23	0%	n/a	n/a	n/a	n/a	n/a	n/a
[2%, 3%)	0	107	0%	n/a	n/a	n/a	n/a	n/a	n/a
[3%, 4%)	11	102	11%	-0.3%	-0.3%	-0.1%	-0.1%	0.0%	-0.1%
[4%, 5%)	32	146	22%	-0.8%	-0.6%	-0.3%	-0.1%	0.0%	-0.3%
[5%, 6%)	18	89	20%	-0.8%	-0.5%	-0.2%	-0.1%	0.0%	-0.3%
[6%, 7%)	18	93	19%	-1.4%	-1.3%	-0.4%	-0.1%	0.0%	-0.7%
[7%, 8%)	10	82	12%	-1.2%	-1.0%	-0.5%	-0.1%	-0.1%	-0.5%
[8%, 9%)	14	79	18%	-1.6%	-1.2%	-0.7%	-0.2%	0.0%	-0.8%
[9%, 10%)	7	29	24%	-1.5%	-1.3%	-1.1%	-0.8%	-0.2%	-1.0%
> 10%	24	74	32%	-3.4%	-2.2%	-1.4%	-0.2%	-0.1%	-1.4%
All	**134**	**825**	**16%**	**-3.4%**	**-1.3%**	**-0.4%**	**-0.1%**	**0.0%**	**-0.6%**



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Yield curve slope (bucketed by 1Y rate)
Historical Slope Data (4/1953 - 12/2020)

Observations:

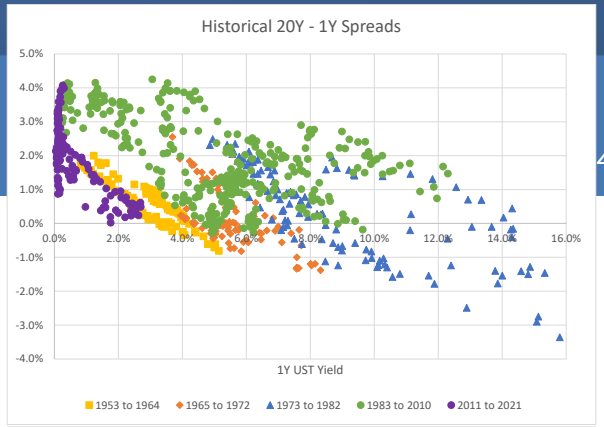
- No inversions for UST 1-year yields below 3%
- Severity of inversions generally increases with rate levels
- Other variations in curve steepness by rate level
- Recommend slope criteria based on simplified Low / Medium / High yield buckets to capture historical dynamics while not being overly constraining
- May bucket by 20Y instead of 1Y yields based on preference

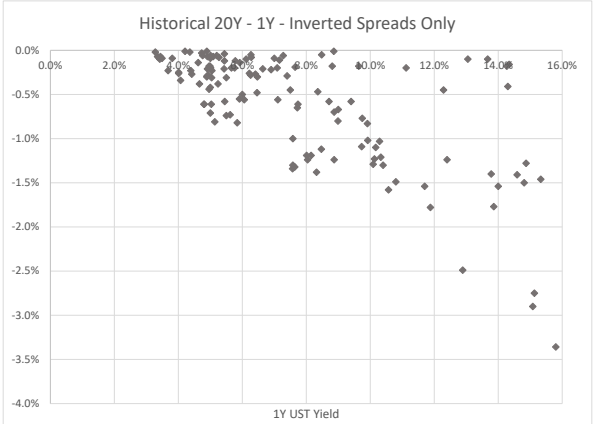
ALL Spreads

Rate Bucket (1Y)	Inverted Months	Total Months	% Inverted	Min Spread	15%	50%	85%	Max Spread	Avg Spread
[0%, 1%)	0	128	0%	0.5%	1.6%	2.5%	3.6%	4.1%	2.5%
[1%, 2%)	0	69	0%	0.0%	1.0%	1.6%	3.6%	4.2%	2.1%
[2%, 3%)	0	71	0%	0.1%	0.4%	0.9%	3.0%	3.6%	1.3%
[3%, 4%)	8	103	8%	-0.2%	0.2%	0.8%	2.8%	4.3%	1.2%
[4%, 5%)	22	89	25%	-0.6%	-0.2%	0.5%	1.7%	3.9%	0.8%
[5%, 6%)	26	116	22%	-0.8%	-0.1%	0.9%	1.8%	3.0%	0.8%
[6%, 7%)	12	76	16%	-0.6%	0.0%	1.2%	1.9%	2.5%	1.1%
[7%, 8%)	14	56	25%	-1.3%	-0.3%	0.7%	1.9%	2.9%	0.8%
[8%, 9%)	13	38	34%	-1.4%	-0.9%	0.5%	1.8%	2.8%	0.5%
[9%, 10%)	7	26	27%	-1.1%	-0.7%	1.5%	2.1%	2.5%	0.9%
> 10%	32	53	60%	-3.4%	-1.6%	-0.4%	1.5%	2.0%	-0.3%
All	**134**	**825**	**16%**	**-3.4%**	**-0.1%**	**1.1%**	**2.7%**	**4.3%**	1.2%

INVERTED Spreads Only

Rate Bucket (1Y)	Inverted Months	Total Months	% Inverted	Most Negative Spread	15%	50%	85%	Least Negative Spread	Avg Spread
[0%, 1%)	0	128	0%	n/a	n/a	n/a	n/a	n/a	n/a
[1%, 2%)	0	69	0%	n/a	n/a	n/a	n/a	n/a	n/a
[2%, 3%)	0	71	0%	n/a	n/a	n/a	n/a	n/a	n/a
[3%, 4%)	8	103	8%	-0.2%	-0.1%	-0.1%	-0.1%	0.0%	-0.1%
[4%, 5%)	22	89	25%	-0.6%	-0.4%	-0.2%	0.0%	0.0%	-0.2%
[5%, 6%)	26	116	22%	-0.8%	-0.7%	-0.2%	-0.1%	0.0%	-0.3%
[6%, 7%)	12	76	16%	-0.6%	-0.5%	-0.3%	-0.1%	0.0%	-0.3%
[7%, 8%)	14	56	25%	-1.3%	-1.3%	-0.5%	-0.1%	-0.1%	-0.6%
[8%, 9%)	13	38	34%	-1.4%	-1.2%	-0.8%	-0.2%	0.0%	-0.8%
[9%, 10%)	7	26	27%	-1.1%	-1.0%	-0.8%	-0.5%	-0.2%	-0.7%
> 10%	32	53	60%	-3.4%	-1.8%	-1.4%	-0.3%	-0.1%	-1.3%
All	**134**	**825**	**16%**	**-3.4%**	**-1.3%**	**-0.4%**	**-0.1%**	**0.0%**	**-0.6%**





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