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Exposure Draft

Valuing Benefits Payable as a Lump Sum

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2018 Pension Committee

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The comment deadline for this exposure draft is November 15, 2018. Please send any comments to pensionanalyst@actuary.org.

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TABLE OF CONTENTS

<u>Valuing Benefits Payable as a Lump Sum</u>

This practice note is not a promulgation of the Actuarial Standards Board (ASB), is not an actuarial standard of practice, is not binding upon any actuary and is not a definitive statement as to what constitutes generally accepted practice in the area under discussion. Events occurring subsequent to the publication of this practice note may make the practices described in the practice note irrelevant or obsolete.

This practice note was prepared by the Pension Committee ("Committee") of the American Academy of Actuaries[1] (Academy) to provide information to actuaries on current and emerging practices in the development of liabilities and cost estimates for pension plans with benefits paid as a lump sum. The intended users of this practice note are the members of actuarial organizations governed by the actuarial standards of practice promulgated by the ASB.

Measurements of defined benefit pension plan obligations include calculations that assign plan costs to time periods, actuarial present value calculations, and estimates of the magnitude of future plan obligations. This practice note does not apply to individual benefit calculations or individual benefit statement estimates. The focus of this practice note is on the application of the concepts discussed herein to accounting for single-employer plans in the U.S. for which the actuary is subject to Actuarial Standard of Practice (ASOP) No. 4, *Measuring Pension Obligations and Determining Pension Plan Costs or Contributions* (ASOP No. 4), ASOP No. 27, *Selection of Economic Assumptions for Measuring Pension Obligations*, and ASOP No. 35, *Selection of Demographic and Other Noneconomic Assumptions for Measuring Pension Obligations*.[2] However, these concepts may be extended to other applications and other types of pension plans. The ASB has approved exposure drafts of revisions of these ASOPs. The proposed revisions, if adopted, would not change the discussion in this practice note.

The Pension Committee welcomes any suggested improvements for future updates of this practice note. Suggestions may be sent to the pension policy analyst of the American Academy of Actuaries at 1850 M Street NW, Suite 300, Washington, DC 20036 or by emailing pensionanalyst@actuary.org.

Overview

Many pension plans offer benefits in the form of a single lump sum payment. In recent years, as sponsors have looked to manage pension risk, this form of payment has become more common. When a lump sum is offered in a traditional pension plan, the amount of the lump sum often varies based on market interest rates. Recognizing the relationship between a lump sum calculated using market bond yields and the value of the underlying annuity calculated based on similar bond yields, the Internal Revenue Service (IRS) requires the use of an "annuity substitution" approach for the

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[1] The American Academy of Actuaries is a 19,500-member professional association whose mission is to serve the public and the U.S. actuarial profession. For more than 50 years, the Academy has assisted public policymakers on all levels by providing leadership, objective expertise, and actuarial advice on risk and financial security issues. The Academy also sets qualification, practice, and professionalism standards for actuaries in the United States.

[2] The ASB has approved exposure drafts for revisions to all of the referenced ASOPs though the revisions are not expected to materially affect the discussion in this practice note.

purpose of valuing certain benefits expected to be paid in lump sum form. This practice note discusses the valuation of these benefits for financial accounting purposes. This practice note utilizes a number of concepts related to interest theory. Appendix B may provide a useful refresher on some of the principles of "interest."

Calculation of a Pension Obligation for Plans Assumed to Pay Future Benefits as a Lump Sum

Methodologies appropriate for valuing lump sums in pension plans have been discussed within the actuarial community for a number of years. These issues have received greater attention as many actuaries have begun to use more granular methods for the development of service and interest cost, which involve the separate application of discount rates to individual years' cash flows.

Before delving into the challenges raised when valuing lump sums with a granular cost method, a review of some of the basic principles of lump sum liability valuation is warranted. Many of these liability-valuation topics were first addressed in an article written by Richard Q. Wendt for the December 2004 edition of *The Pension Forum*[3] published by the Society of Actuaries; this practice note restates and builds on that article.

Non-Interest-Sensitive Lump Sums

Some plans calculate the lump sum equivalent of an annuity benefit on a basis that is not sensitive to underlying interest rates. This basis is usually prescribed in the plan document either directly through the plan formula or implicitly by a fixed conversion factor.[4] Valuation calculations in such circumstances are typically straightforward. A projected lump sum amount can simply be calculated at a future decrement age according to the plan provisions, and this amount can be discounted to the measurement date whenever present values of future benefits are calculated.

Example 1: Consider a simple example of a pension plan that pays an annuity benefit equal to five annual payments of $10,000 beginning at age 65. Based on the reference yield curve (shown more fully in Appendix A), consider the present value calculated for someone who is currently age 63 with 100% likelihood of retiring two years from now and no other decrements for reasons such as mortality.

If it is assumed the participant elects the annuity, the present value according to the above assumptions is $45,465. The single equivalent discount rate is 2.42% and the Macaulay duration—the weighted average time until payment—is 3.93 years, which is consistent with the payments being evenly spread between years two and six.

[3] The issue of *Pension Forum* in question is available online at https://www.soa.org/library/newsletters/the-pension-forum/2004/december/pfn0412.pdf

[4] Common examples of these types of plans would be non-qualified plans that specify a fixed interest rate for determining the lump sum present value of a set of annuity payments or hybrid defined benefit plans that formulaically define the benefit as a lump sum.

Example 1: Present Value of Annuity

(1) Age at Payment Date	(2) Years from Measurement Date	(3) Applicable Spot Interest Rate	(4) Payment Amount	(5) Present Value Factor	(6) Present Value of Payment: (4) * (5)
63	-	-	-	1.0000	-
64	1.00	1.43%	-	0.9859	-
65	2.00	1.70%	$10,000	0.9668	$ 9,668
66	3.00	2.01%	10,000	0.9420	9,420
67	4.00	2.35%	10,000	0.9113	9,113
68	5.00	2.60%	10,000	0.8796	8,796
69	6.00	2.81%	10,000	0.8468	8,468
Total:					**$45,465**
Macaulay Duration:			**3.93**		
Effective Interest Rate:			**2.42%**		

Example 2: Suppose the participant from Example 1 is instead assumed to elect the plan's lump sum option, which is defined in the plan document as 4.8 times the annual payment for a participant who is age 65. The 4.8 factor does not change based on the interest rate environment. The lump sum payment–$48,000 in this example—is to be paid in two years. The present value of the future lump sum payment is calculated using the two-year spot rate, as demonstrated below in Example 2. The Macaulay duration of the payment to this participant is exactly 2 years.

Example 2: Present Value of Non-Interest-Sensitive Lump Sum

(1) Age at Payment Date	(2) Years from Measurement Date	(3) Applicable Spot Interest Rate	(4) Payment Amount	(5) Present Value Factor	(6) Present Value of Payment: (4) * (5)
63	-	-	-	1.0000	-
64	1.00	1.43%	-	0.9859	-
65	2.00	1.70%	$48,000	0.9668	$46,404
66	3.00	2.01%	-	0.9420	-
67	4.00	2.35%	-	0.9113	-
68	5.00	2.60%	-	0.8796	-
69	6.00	2.81%	-	0.8468	-
Total:					**$46,404**
Macaulay Duration:			**2.00**		
Effective Interest Rate:			**1.70%**		

Examples 1 and 2 support the following observations about non-interest-sensitive lump sums:

- The present value of the two forms of payment for the same benefit is not likely to be the same. This is because the interest rate basis used to convert the annuity to a lump sum is stated in the plan document, while the basis used to discount payments to a present value is based on current market conditions. When these two bases diverge, the present value of the two forms of payment will differ.

- The Macaulay duration is less for the present value of the lump sum than for the present value of the annuity form of payment.
- The single effective interest rate associated with the present value of the lump sum form of payment reflects that shorter time until payment. Interest rates increase with maturity in typical market conditions, so the single effective interest rate for the annuity form of payment will typically exceed that for the lump sum.

The availability of optional forms of payment under the plan with different present values raises the prospect that the participant can select against the plan by choosing the more costly option. The actuary's assumption about the proportion of participants electing the lump sum option—whether it is fixed, or varies with the economic environment—is beyond the scope of this practice note. While this paper does not explore this particular issue, later sections discuss the cost implications for a plan that pays a lump sum based on the greater of two separate conversion bases. In that situation, the participant automatically gets the more costly option, so it is not necessary to make an assumption about the extent to which the participant will select against the plan.

Interest-Sensitive Lump Sums

In other situations, plans define lump sums with reference to interest rates in effect at the time of the payment. These designs are common because Internal Revenue Code (IRC) Section 417(e) requires that for a traditional plan that defines the benefit in terms of an annuity, any lump sum paid must be no less than the present value of the annuity payable at Normal Retirement Age based on a mandated interest rate based on high-quality corporate bonds. This introduces a new issue for consideration in the actuarial valuation: whether to value a projected lump sum (and what assumptions to use in calculating the lump sum amount to be valued) or the underlying annuity.

As with many other methodological choices, actuaries' professional judgment is an important component of the decision-making process. An understanding of any applicable guidance and regulations is also, of course, necessary.

In performing ERISA funding valuations for plans covered by IRC Section 430, the IRS requires the use of the annuity substitution technique in valuing certain lump sum payments.[5] This technique recognizes the underlying interest sensitivity of the lump sum by valuing the original annuity payments (for a traditional plan that defines the benefit in terms of an annuity) instead of the lump sum cash flow. The result is that the pension obligation calculation and the effective interest rate are largely unaffected by the election of the lump sum.[6]

With respect to other pension obligation calculations where the IRS' rules are not specifically applicable, two primary categories of approaches to measuring pension obligations have developed:

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[5] 1.430(d)-1(f)(4)(iii).

[6] The portion of participants assumed to elect a lump sum payment may have some effect on both of these quantities if a different mortality table is used for calculating lump sums according to plan terms than for determining the present value of annuities according to actuarial valuation assumptions.

- “Best-estimate” approaches—An actuarial assumption is set for the conversion factors expected to be in place at the time of the projected lump sum payment.
- “Settlement” approaches—The pension obligation is determined as the amount that would be required to eliminate the interest rate risk associated with the conversion between payment forms.[7]

These approaches are discussed in more detail below.

The appropriateness of a given approach can vary based on the purpose of the measurement. For example, Accounting Standards Codification (ASC) 960 (used for plan financial statements) specifically permits the use of either a best-estimate approach (using the expected rate of return on plan assets as a discount rate) or a settlement approach.[8]

Under ASC 715 (which covers accounting for retirement benefits in the plan sponsor’s financial statements), some actuaries and auditors look to the definition of a discount rate which “shall reflect the rates at which the pension benefits could be effectively settled.”[9] The statement goes on to say, “the objective of selecting assumed discount rates using [rates of return on high-quality corporate bonds] is to measure a single amount that, if invested at the measurement date … would provide the necessary future cash flows to pay the pension benefits when due.”[10] The use of corporate bond yields to convert an annuity into a lump sum payment is substantially similar to the discounting process for calculating the pension obligation for the underlying annuity. Some actuaries therefore consider this passage to suggest a settlement approach to measuring the pension obligation associated with different benefit forms when the conversion factors vary with changes in market interest rates.

Others take a more narrow reading of the words “discount rate” in the above passages and consider the discount rate concept to apply only to the determination of the present value of expected payments. The interest rates used to convert between benefit forms are then analogous to all other assumptions used in the projection of benefit amounts, and would be governed by the requirement that “each significant assumption used shall reflect the best estimate solely with respect to that individual assumption.”[11]

Both approaches are commonly used for ASC 715 valuations. The ultimate decision generally rests with plan sponsors subject to their auditors’ approval.

International Accounting Standard (IAS) 19 (applicable to most non-U.S. plan sponsors’ financial statements) requires the discount rate to be based on high-quality corporate bonds aligned with the timing of the benefit payments.[12] There is no specific reference in IAS 19 to the settlement concept. Assumptions other than the discount rate are required to be “best estimates of the variables that will determine the ultimate cost of providing [the] benefits.”[13] Many actuaries read the explicit guidance toward corporate bonds and the best-estimate language as suggesting a best-estimate approach.

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[7] Calculations still generally rely on a number of non-market, best-estimate assumptions for other factors (e.g., mortality, form of payment, timing of commencement, etc.).
[8] ASC 960-20-35-1 and ASC 960-20-31-1A.
[9] ASC 715-30-35-43.
[10] ASC 715-30-35-44.
[11] ASC 715-30-35-42.
[12] IAS 19 Revised 2011 paragraphs 83 and 85.
[13] IAS 19 Revised 2011, paragraph 76.

However, other actuaries distinguish between the projected amount of a given benefit payment and the variables that will determine the ultimate cost of providing that benefit. The relationship between lump sum conversion factors and the discount rates used to measure pension obligations (and the fact that the two might be expected to move in tandem) could support an approach similar to the settlement approaches discussed in this paper.

Other regulatory frameworks may use similar concepts but are beyond the scope of this practice note.

Best-Estimate Approach

The best-estimate approach requires setting assumptions for future economic (and demographic) variables, including interest rates implicit in the construction of the annuity-to-lump-sum conversion factors. In accordance with ASOP No. 27 section 3.12, those assumptions should be consistent with other best-estimate assumptions used in the valuation—notably the assumptions used for inflation and real returns used in the development of the expected rate of return on assets assumption, the salary scale, and other economic assumptions.

In a manner similar to the non-interest-sensitive lump sums, under the best-estimate approach, the lump sum is considered to be a fixed amount based on the best estimate of the lump sum conversion at the assumed payment date and therefore shortens the duration of the plan's liabilities.

The plan design in Example 3 is the same as that described in Example 2, except that the lump sum conversion is based on the long-term corporate bond rates measured on the lump sum payment date. Example 3 further supposes that the best estimate of that yield curve in two years' time is equivalent to an effective interest rate of 2.50%.

Example 3: Present Value of Interest-Sensitive Lump Sum—Best-Estimate Approach

(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Annuity Payment	(5) Discount Factor at Best Estimate Lump Sum Rate (2.50%)	(6) Annuity Payments Discounted to Lump Sum Date: (4) * (5)	(7) Lump Sum Payment: ∑ Column (6)	(8) Present Value Factor	(9) Present Value of Payment: (7) * (8)
63	-	-				-	1.0000	-
64	1.00	1.43%				-	0.9859	-
65	2.00	1.70%	$10,000	1.0000	$10,000	$47,620	0.9668	$46,037
66	3.00	2.01%	10,000	0.9756	9,756	-	0.9420	-
67	4.00	2.35%	10,000	0.9518	9,518	-	0.9113	-
68	5.00	2.60%	10,000	0.9286	9,286	-	0.8796	-
69	6.00	2.81%	10,000	0.9060	9,060	-	0.8468	-
Total:								**$46,037**
Macaulay Duration:			**2.00**					
Effective Interest Rate:			**1.70%**					

Under the best-estimate approach, the pension obligation represents a measure of the present value of the expected lump sum cash flow. If the plan sponsor invested in bonds that actually matched this cash flow, the investment would only be sufficient to precisely settle the pension obligation if the assumption of 2.50% were consistent with the market rates in effect when the participant is age 65. If the actual effective lump sum rate on the payment date is lower than the 2.5% assumed rate, the accumulated value of the investment would be insufficient. Similarly, if the actual effective rate on the lump sum payment date is more than 2.5%, the accumulated value of the investment would exceed the lump sum payable. The duration of the pension obligation in this example matches that of Example 2.

In Example 3, the pension obligation can be thought of as a zero-coupon bond maturing in two years in an amount equal to the expected lump sum payment of $47,620. The current price of that bond, which is determined using the two-year spot rate of 1.70%, is $46,037. That bond will pay exactly $47,620 in two years, regardless of whether the effective lump sum interest rate is actually 2.50% on the lump sum payment date.

A subsequent change in the interest rate environment may influence the best-estimate assumption. The sensitivity of the best-estimate assumption to changes in market interest rates (if any) is not reflected in the duration statistic, which treats the lump sum as being fixed, rather than interest-sensitive.

Settlement Approaches

Another common alternative for valuations not subject to IRS funding rules is use of a settlement-like approach to setting the discount rate. Market factors are used to estimate the price at which the risk of the pension obligation could be eliminated, which reflects the interest rate risk within the payment amounts (i.e., the lump sums) in addition to the discount on those amounts. Although this approach does not reflect the anticipated timing of the actual payments to participants, it is nevertheless commonly accepted under ASC 715-30 as well as ASC 960. For interest-sensitive lump sums, the effective duration of pension obligations reflects the interest sensitivity embedded in the calculation of the lump sum under a settlement approach.

To understand why this happens, consider the underlying principle of a settlement basis: estimating the market price of satisfying a pension obligation with (near) certainty. Achieving this risk defeasance (at least with respect to the discount rate) generally can be accomplished by investing in a bond portfolio whose cash flows match the cash flows of the underlying pension obligation.[14] Because the amount of any lump sum cash flow depends on the underlying annuity cash flows, to eliminate the interest rate risk the bond portfolio must align to those underlying annuity cash flows. Settlement techniques for valuing interest-sensitive lump sums emphasize the considerable similarity between discount rate application and lump sum conversion, although converting an annuity to a lump sum discounts annuity payments to the lump sum payment date, rather than to the measurement date. (Appropriate adjustment would be made where the lump sum basis is variable, but differs from the pension obligation discounting basis, as discussed later in this practice note.) Adherents of settlement approaches argue that since discount rates are set on a settlement basis, lump sum conversion should be measured using a compatible approach, such as those discussed below.

[14] The possibility of default or downgrade is generally disregarded for this purpose.

There are three commonly used techniques to accomplish this calculation: Annuity Substitution, Individual Implied Lump Sum Rates, and Aggregate Implied Lump Sum Rates.

Annuity Substitution

The annuity substitution method is the method described previously and required by the IRS for funding purposes for plans subject to IRC Section 430, and it can also be considered for other measurement purposes. Under this approach, the underlying annuity is valued in place of the lump sum option. Adjustments may be incorporated to reflect differences between the basis used to convert annuities to lump sums and the basis used to value annuities (e.g., use of unisex versus gender-specific mortality).

Example 4 below assumes no such differences, since as previously noted all examples in the practice note ignore the effect of mortality for simplicity.

Example 4: Present Value of Interest-Sensitive Lump Sum—Annuity Substitution

(1) Age at Payment Date	(2) Years from Measurement Date	(3) Applicable Spot Interest Rate	(4) Payment Amount	(5) Present Value Factor	(6) Present Value of Payment: (4) * (5)
63	-	-	-	1.0000	-
64	1.00	1.43%	-	0.9859	-
65	2.00	1.70%	$10,000	0.9668	$9,668
66	3.00	2.01%	10,000	0.9420	9,420
67	4.00	2.35%	10,000	0.9113	9,113
68	5.00	2.60%	10,000	0.8796	8,796
69	6.00	2.81%	10,000	0.8468	8,468
Total:					**$45,465**
Macaulay Duration:			**3.93**		
Effective Interest Rate:			**2.42%**		

As all of the inputs to this example are identical to those in Example 1, this calculation produces the same present value, duration, and effective interest rate as Example 1.

To understand why discounting the underlying annuity payments provides a reasonable settlement value, consider what the settlement present value actually represents: a portfolio of assets that could be held to guarantee payment of the underlying pension obligation. The yield curve represents assumed market pricing of zero coupon bonds. In this example, the plan could pay $45,465 today to purchase five zero-coupon bonds maturing two, three, four, five, and six years from now. Those bonds would appreciate or depreciate in value over the coming two years; however, they would have a cash flow that aligns with the annuity cash flow used to determine the lump sum. Consequently, if rates rise the value of the assets would fall but the amount of the lump sum would fall in tandem. This would be true regardless of the future shape or level of the yield curve.

The preceding paragraph explains why the annuity substitution method approximates the settlement of interest-sensitive lump sums when pension obligation discount and lump sum conversion rates reflect the same corporate bond yields. The method does, however, have an important limitation: Although it produces an accurate measure of the cost of settling the pension obligation, it does not reflect the plan's expected cash flow. By ignoring the timing of lump sum payments (which are paid earlier than the associated annuity payments even if they are interest-sensitive), the cash flow stream understates the near-term liquidity needs of the plan. This may not have any significant implications for measurement of pension obligations, but it may make annuity substitution unsuitable for other purposes.

<u>Individual Implied Lump Sum Rates</u>

Another approach to recognizing the interest sensitivity of lump sum payments is to reflect the expected timing of lump sum payments, but to calculate the lump sum payments in a way that is consistent with their settlement value as of the measurement date. To do this, forward rates are developed from interest rates observed on the measurement date. If an implied lump sum payment is calculated using these forward rates as the future value of annuity payments, when that lump sum is subsequently discounted to the measurement date, the result is equal to the present value of the underlying annuity cash flows. This approach is referred to as "individual" because implied lump sum rates are calculated for each projected lump sum payment in the valuation. The rates are "implied" because they are inferred from current market conditions. They represent the rates that relate currently observed values of payments at different time periods. They do not necessarily represent a prediction of what interest rates will be in the future. This approach is also sometimes referred to as the "implied forward rates" approach.

Example 5 illustrates this method for a lump sum payable in two years, and it requires a calculation of the implied forward rates two years from now.[15]

Example 5: Present Value of Interest-Sensitive Lump Sum—Individual Implied Lump Sum Rates

(1) Age at Pmt. Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Two-Year Forward Rate	(5) Annuity Payment	(6) Discount Factor at Two-Year Forward Rates	(7) Annuity Payments Discounted to Lump Sum Date: (5) * (6)	(8) Lump Sum Payment: ∑ Col. (7)	(9) Present Value Factor	(10) Present Value of Payment: (8) * (9)
63	-	-		-				1.0000	-
64	1.00	1.43%		-				0.9859	-
65	2.00	1.70%		$10,000	1.0000	$10,000	$47,028	0.9668	$45,465
66	3.00	2.01%	2.63%	10,000	0.9744	9,744		0.9420	-
67	4.00	2.35%	3.00%	10,000	0.9426	9,426		0.9113	-
68	5.00	2.60%	3.20%	10,000	0.9099	9,099		0.8796	-
69	6.00	2.81%	3.37%	10,000	0.8759	8,759		0.8468	-

[15] For example, the two-year forward rate shown in year 4 of 3.00% is the rate that connects the year 4 spot rate of 2.35% to the year 2 spot rate of 1.70%. It is derived as $[1.0235^4/1.0170^2]^{(1/(4-2))}$ -1. See Appendix B for more details.

Total:		4.7028	$47,028		**$45,465**
Macaulay Duration:	**2.00**				
Effective Interest Rate:	**1.70%**				

The implied lump sum payment of $47,028 can be derived by taking each year's annuity payment and discounting it back to the lump sum payment date using each of the individual two-year forward rates applied between the annuity payment date and the lump sum payment date. In this example, the final payment of $10,000 shown in the last row of the table is then discounted back four years at the 3.37% rate to arrive at a present value of $8,759, while the prior year's annuity payment is discounted for three years at 3.20%, and so on. If the implied lump sum payment is discounted (at the 2-year spot rate of 1.70%), a present value of $45,465 is derived.

As can be seen in Example 5, this approach produces the same pension obligation as annuity substitution in Example 4, but it produces a lower effective interest rate due to the shorter maturity of the payment valued. It should also be noted that the projected lump sum amounts produced under the individual implied lump sum rates methodology are not intended to represent an estimate of the lump sum amounts that will actually be paid, but are instead amounts that are consistent with the pricing reflected in the current yield curve.

For example, assume that a payment is due in two years equal to the present value of a $10,000 payment due six years from now (i.e., the final annuity payment in the example). Two ways to settle this pension obligation are:

- Purchase a zero-coupon bond today that will mature to $10,000 in six years, or
- Commit to the purchase of a zero-coupon bond in two years that will mature to $10,000 four years after that.

Because both options provide the same cash flow in all scenarios, they should have the same present value.[16] The six-year zero-coupon bond costs $8,468 today based on the six-year spot rate of 2.81%. The value today of a payment due in two years would be the face value of that payment discounted for two years at 1.70%. Thus, the price at which the four-year bond would be purchased in the second option (assuming a current commitment to making the purchase) must be $8,468 x 1.017^2, or $8,759 (otherwise the two settlement options, which both settle the pension obligation today, will have a different present value today). This is equivalent to the price of a four-year bond using the implied 4-year spot rate two years from now of 3.37% (i.e., $10,000 discounted for four years at 3.37%).

This calculation is not a prediction of what the spot rate will be in two years. Rather it is simply a mathematical convenience for splitting the current price of a six-year zero-coupon rate into a two-year and four-year piece. Although the two-year zero-coupon bond is expected to yield 1.70% over the next two years, this does not necessarily mean that the six-year bond will also yield 1.70% if sold after two years. Suppose that it is expected, for example, to instead earn 2.50% over the next two years on the six-year bond. Then the consistent expectation for the future pricing on the bond would be that the four-year spot yield will be $(1.0281^6/1.025^2)^{(1/4)}-1 = 2.97\%$. Just because 2.97% represents an expectation for the four-year spot yield two years from now, using that rate in our

[16] Ignoring the effect of downgrade or default, which would affect the value two years from now of the six-year bond.

calculation does not give a "better" measure of the pension obligation if the objective is a settlement measurement. However, it does suggest that if the pension obligation is backed by buying a bond that matures in two years (instead of in six) based on a best estimate, then a 2.97% rate would be used to determine the expected payment, rather than the 3.37% implied forward rate. As discussed below, however, buying that bond would only guarantee that the pension obligation would be met if the four-year spot rate really is 2.97% two years from now and not some other rate. Under this approach a two-year bond maturing at $8,895 ($10,000 x 1.0297^{-4}) would be bought, which is the lump sum value of the $10,000 annuity payment. The cost of that bond today would be $8,600, calculated as $8,895 x 1.017^{-2}.

Assuming, however, that one can invest or borrow equally based on the current yield curve, the 3.37% forward rate, while not a prediction of future rates, is the four-year yield available two years from now that can be locked in today by simultaneously borrowing for two years at 1.70% and spending the proceeds today on a bond that matures in six years and yields 2.81%. If the $8,468 needed today is borrowed to purchase a bond maturing in six years at $10,000, then the net cash position does not change today; while a commitment has been made, no investment of money has been made yet. In two years, the repayment of the loan will be $8,759, which represents the initial cash outlay. Four years after that, the bond will make a $10,000 payment. So from the point at which the $8,759 repayment of the loan was required to the date that the proceeds of the bond investment are received, the annual return on the investment is 3.37% = ($10,000 / $8,759)$^{(1/4)}$-1. That will be true regardless of what the actual spot rate is in four years. It will be true even if the spot rate two years from now matches the best-estimate assumption of 2.97%, rather than the implied rate of 3.37%. The 3.37% return can be locked in by committing to the investment today at rates determined by today's market conditions.

When the participant elects a lump sum in two years, the bond can be sold to cover the lump sum payment. If the four-year spot yield two years from now is 2.97% (consistent with a best estimate) then the bond will be worth $8,895, which will precisely cover the portion of the lump sum payment associated with that final $10,000 annuity payment. This is true even though the cash outlay required to repay the loan was only $8,759. Because a transaction has been entered into that settles the pension obligation and requires only a cash outlay of $8,759 in two years, regardless of how the interest rate changes, the pension obligation is the $8,468 derived above.

<u>Aggregate Implied Lump Sum Rates</u>

Another way to value lump sums under a settlement approach is to determine an effective interest rate based on the underlying annuities reflected in the lump sums, and to apply that rate to determine a lump sum payment. Not surprisingly, this methodology also produces the same pension obligation as the annuity substitution approach in Example 4. This approach is typically applied for an entire population—the lump sum conversion rate used for all participants corresponds to the annuity-based discount rate for the entire population. Since a single aggregate rate is derived, this approach is referred to as "Aggregate Implied Lump Sum Rates." As with the individual implied lump sum rates approach, this conversion rate is not necessarily a best estimate of the anticipated lump sum conversion rate. Rather, it is a mathematical device used to calibrate the pension obligation to its targeted value, the pension obligation measured for the annuity payment form, as shown here in Example 6.

Example 6: Present Value of Interest-Sensitive Lump Sum—Aggregate Implied Lump Sum Rates

(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Annuity Payment	(5) Discount Factor at Effective Interest Rate	(6) Annuity Payments Discounted to Lump Sum Date: (4) * (5)	(7) Lump Sum Payment: ∑ Column (6)	(8) Present Value Factor	(9) Present Value of Payment: (7) * (8)
63	-	-	-	-			1.0000	-
64	1.00	1.43%	-	-			1.0000	-
65	2.00	1.70%	$10,000	1.0000	$10,000	$47,692	0.9533	$45,465
66	3.00	2.01%	10,000	0.9764	9,764		0.9308	-
67	4.00	2.35%	10,000	0.9533	9,533		0.9088	-
68	5.00	2.60%	10,000	0.9308	9,308		0.8873	-
69	6.00	2.81%	10,000	0.9088	9,088		0.8663	-
Total:								**$45,465**
Macaulay Duration:			**2.00**					
Effective Interest Rate:			**2.42%**					

Using the 2.42% effective interest rate derived from annuity cash flows both to determine the amount of the lump sum and then to value that lump sum produces the same $45,465 pension obligation as the annuity in Example 1.

Constructing a Theoretical Matching Portfolio Consistent With the Pension Obligation

The pension obligation represents the value of assets that, if set aside today, would be sufficient to make the payments for benefits accrued to date (as defined for purposes of measuring the pension obligation) if all assumptions are met in the future. Inherent in any settlement approach is the idea that a portfolio of assets consisting of long-term, zero-coupon bonds aligned to the payment date(s) of the benefit will provide for the benefit payments regardless of subsequent changes in bond yields.

As previously developed in Example 1, the present value of the assumed annual payments of $10,000 at ages 65 through 69 is $45,465 under the yield curve described above, with a single effective interest rate of 2.42%. The theoretical basis of that measurement of pension obligations requires that a portfolio of zero-coupon bonds (with maturities two, three, four, five, and six years from now) could be purchased for that same sum. While these specific securities may not actually be available in the market, the yield curve is a representation of market pricing of payments of various maturities. It describes what these hypothetical bonds would be expected to cost if the market were complete.

To illustrate the settlement of interest rate risk, consider Example 7, in which a plan purchases those maturities of bonds when the participant is age 63 and holds them until the associated payments are made. Assuming that all assumptions are met (apart from the single effective interest rate), the bonds do not default, and disregarding future benefit accruals, the bond payments would precisely meet the cash flow needs of the plan regardless of the interest rate environment. Accordingly, it would be expected that the market value of this bond portfolio will continue to equal the pension obligation at any future date. Assume, for example, that after that initial year, the yield curve moves as indicated. While the interest rates have declined (the single effective interest rate is now 1.55%), because the asset portfolio matches the benefit payments with incoming cash flows, the value of the portfolio continues to align with the pension obligation such that both the market value of the asset portfolio and the pension obligation are $47,754 when the participant is age 64.

Example 7: Present Value of Annuity—Sensitivity to Interest Rate Change

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
64	-	-	1.0000	-	-	-	-
65	1.00	0.41%	0.9959	$10,000	$9,959	$10,000	$9,959
66	2.00	0.62%	0.9877	10,000	9,877	10,000	9,877
67	3.00	1.14%	0.9665	10,000	9,665	10,000	9,665
68	4.00	1.76%	0.9325	10,000	9,325	10,000	9,325
69	5.00	2.29%	0.8929	10,000	8,929	10,000	8,929
Totals:					**$47,754**		**$47,754**

The pension obligation for each payment and the zero-coupon bond assumed to be purchased to match that payment have the same present value. The values of each are calculated as the present value using the then-current yield curve, regardless of how yields have changed (or more accurately, bond pricing defines the yield curve, which in turn is used to measure the pension obligation).

Once benefit payments commence, each bond payment would be used to pay the corresponding benefit payment. By the time the individual is age 69, only a single payment of $10,000 would remain, matched by the payment from the final bond that is about to mature.

Non-Interest-Sensitive Lump Sums

For non-interest-sensitive lump sums, the plan has made an underlying promise for payment on the lump sum date that is independent of the interest rate environment at the time of the payment. Thus, in Example 8, the matching portfolio for a single sum payment of $48,000 two years from now at age 65 is a zero-coupon bond with a single payment in that amount on that date. According to the yield curve used in the baseline examples, that single-payment bond would cost $46,404 (see Example 2) when the participant is age 63.

Assuming the same one-year movement in interest rates as the preceding annuity-based Example 7, the single-payment bond would continue to align with the pension obligation associated with the lump sum, which would be a present value of $47,804.

Example 8: Present Value of Fixed Lump Sum—Sensitivity to Interest Rate Change

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
64	-	-	1.0000	-	-	-	-
65	1.00	0.41%	0.9959	$48,000	$47,804	$48,000	$47,804
66	2.00	0.62%	0.9877	-	-	-	-
67	3.00	1.14%	0.9665	-	-	-	-
68	4.00	1.76%	0.9325	-	-	-	-
69	5.00	2.29%	0.8929	-	-	-	-
Totals:					**$47,804**		**$47,804**

One year later (immediately before the assumed payment) the bond would be worth $48,000 (the payment about to be made), which would match the $48,000 lump sum. Because the lump sum payment is not interest-sensitive, this result would not vary with changes in rates.

Interest-Sensitive Lump Sums

As described earlier, due to regulatory requirements, traditional U.S. qualified retirement plans (those that define the benefit as an annuity) commonly offer lump sums that are dependent on the interest rate environment at or near the time of payment. The selection of a best-estimate or settlement approach lead to different matching portfolios for these interest-sensitive lump sums.

Best-Estimate Approach

Under the best-estimate approach, an assumption is made as to the interest rates that will apply to determine lump sums paid at each point in the future. In Example 3, when the participant was age 63, the assumed single effective interest rate in two years was 2.50%. That produced an assumed lump sum payable at age 65 of $47,620. Under the best-estimate approach, the matching portfolio would be a bond making a single payment of $47,620 at age 65, costing $46,037 at age 63.

Example 9: Present Value of Interest-Sensitive Lump Sum—Sensitivity of Best-Estimate Obligation to Interest Rate Change

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
64	-	-	1.0000	-	-	-	-
65	1.00	0.41%	0.9959	$ 47,620	$47,425	$47,620	$ 47,425
66	2.00	0.62%	0.9877	-	-	-	-
67	3.00	1.14%	0.9665	-	-	-	-
68	4.00	1.76%	0.9325	-	-	-	-
69	5.00	2.29%	0.8929	-	-	-	-
Totals:					**$47,425**		**$47,425**

Based on the assumed movement in interest rates from the prior measurement date to the current measurement date (which is identical to Example 7), the value of the matching portfolio would grow to $47,425, which when adjusted for another year's interest would remain sufficient to pay the lump sum of $47,620 due one year from now (which continues to be based upon the assumed 2.50% single effective interest rate at age 63). But, importantly, the yield curve rates have decreased at all maturities and are now further away from the assumed 2.50% rate than they were one year earlier. If the 2.50% assumption is revised accordingly, the pension obligation would increase and would no longer match assets.

This situation can be illustrated more clearly by extending the example further one additional year until the actual payment date, when an assumption is no longer needed because there are actual rates. As seen in Example 10, if the yield curve remains unchanged over the subsequent year, the lump sum to be paid would be $48,826, which equates to a single rate of 1.20%—below the 2.50% rate assumed in developing the bond portfolio intended to match the payment. The assets provide only for a single-sum payment of $47,620 at age 65, thus producing a shortfall of $1,206. This shortfall reflects the failure of the portfolio to realize the best-estimate assumption underlying the lump sum value. It is similar to other actuarial gains and losses that arise from assumptions in annual actuarial valuations, such as actual increases in compensation differing from assumed increases.

Example 10: Present Value of Interest-Sensitive Lump Sum—Sensitivity of Best-Estimate Obligation to Interest Rate Change at Payment Date

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
65	0	-	1.0000	$47,620	$47,620	$48,826	$48,826
66	1	0.41%	0.9959	-	-	-	-
67	2	0.62%	0.9877	-	-	-	-
68	3	1.14%	0.9665	-	-	-	-
69	4	1.76%	0.9325	-	-	-	-
70	5	2.29%	0.8929	-	-	-	-
Totals:					**$47,620**		**$48,826**

This dynamic is a fundamental characteristic of a best-estimate approach. The exact assets necessary to fund the payment are accumulated as long as the actual lump sum conversion rate equals what was assumed. If the assumption is not realized, however, the accumulated assets will either be too much or too little.

Settlement Approaches

Settlement approaches perform differently. They value pension obligations in a manner that is consistent with the pricing of capital market instruments that share the same interest rate sensitivity as the lump sum. This allows for hedging in the asset portfolio so that benefit payments can, in theory, be matched more precisely. The interest-sensitive lump sum can be immunized by a portfolio of zero-coupon bonds that match the underlying annuity cash flows. The resulting portfolio is identical to that used in the first example (where the benefit is payable as an annuity) because the lump sum has been defined to be equal to the present value of the annuity based on the yield curve in effect on the payment date. Later in this practice note, modifications that might be applied when the lump sum conversion rates are related to, but not equal to, the liability discount curve will be discussed.

Because the benefit is payable as an interest-sensitive lump sum, the effective duration is the same as the annuity (as illustrated in Example 4). As a result, the market value of assets in the portfolio remains in line with the lump sum payable at age 65. Although different illustrative matching portfolios can be constructed using derivative securities, the simplest construction matches the underlying annuity payments with zero-coupon bonds. In Example 11, the matching portfolio consists of five zero-coupon bonds matching the expected annuity payments. As the current market yield curve is used by financial markets to price the bonds and by the plan to convert the annuity payments into a lump sum, the market value of the portfolio and the benefit obligation continue to align even as interest rates move. Regardless of the spot rates used in this example, the asset value and benefit obligation will match.

Example 11: Present Value of Interest-Sensitive Lump Sum—Sensitivity of Annuity Substitution Obligation to Interest Rate Change

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
64	-	-	1.0000	-	-	-	-
65	1.00	0.41%	0.9959	$10,000	$9,959	$10,000	$9,959
66	2.00	0.62%	0.9877	10,000	9,877	10,000	9,877
67	3.00	1.14%	0.9665	10,000	9,665	10,000	9,665
68	4.00	1.76%	0.9325	10,000	9,325	10,000	9,325
69	5.00	2.29%	0.8929	10,000	8,929	10,000	8,929
Total:					**$47,425**		**$47,425**

This example can also be made clearer by extending the time period under consideration to the point at which the lump sum is paid (Example 12). Again, assuming the yield curve remains unchanged from age 64 until age 65, the lump sum to be paid would be $48,826 as shown previously in Example 10. However, with the matching portfolio defined based on annuity payments, the underlying asset value is also equal to $48,826, because it increased in value along with the pension obligation when interest rates fell.

Example 12: Present Value of Interest-Sensitive Lump Sum—Sensitivity of Annuity Substitution Obligation to Interest Rate Change at Payment Date

				Assets		Benefit Obligation	
(1) Age at Payment Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) Bond Cash Flow	(6) Market Value of Bond at Measurement Date: (4) * (5)	(7) Benefit Payment	(8) Present Value of Benefit Payment: (4) * (7)
65	0	-	1.0000	$10,000	$10,000	$10,000	$10,000
66	1	0.41%	0.9959	10,000	9,959	10,000	9,959
67	2	0.62%	0.9877	10,000	9,877	10,000	9,877
68	3	1.14%	0.9665	10,000	9,665	10,000	9,665
69	4	1.76%	0.9325	10,000	9,325	10,000	9,325
70	5	2.29%	0.8929	-	-	-	-
Totals:					**$48,826**		**$48,826**

Determining Interest Cost and Year-End Pension Obligation

In theory, interest cost (a component of accounting cost in ASC 715 or IAS 19) represents the annual growth in the pension obligation due to the passage of time, excluding the portion of the pension obligation that is discharged through payments to the participant. Assuming that there are no payments in the upcoming year, as has been the case in the examples thus far, the year-end pension obligation reflecting a "no gain/loss outcome" would be the beginning-of-year pension obligation plus the interest cost.[17] As discussed in the Academy's August 2015 issue brief *Alternatives for Pension Cost Recognition*,[18] any approach to measuring interest cost implicitly defines a year-end discount rate or yield curve that is consistent with the "no gain/loss outcome" (assuming no change in the underlying cash flows).

Traditional Approach

The traditional approach to measuring pension costs calculates interest cost by multiplying the beginning-of-year pension obligation (adjusted as appropriate to reflect timing of first year cash flows) by the plan's discount rate. As discussed in the August 2015 issue brief, this approach would produce no gain or loss for traditional retirement plans (those that define the benefit as an annuity) if the discount rate remained unchanged from the beginning of year to the year-end. However, no model of changing interest rates will consistently produce this result for multiple pension plans because each plan's implied discount rate at the following measurement date will depend on the pattern of cash flows for that plan. For example, two plans with different patterns of cash flows could conceivably use the same yield curve to arrive at the same beginning-of-year effective discount rate. However, the year-end discount rates for these plans will almost certainly differ. The progression of the population, even if fully anticipated by assumptions, will affect the duration of the pension obligation, which will in turn affect the year-end discount rate. Relative to a fully defined expectation for year-end discount rates (i.e., a yield curve, rather than a single discount rate), most plans almost certainly experience a change in the discount rate from beginning to end of year and therefore would experience an "expected" gain or loss.

This outcome can be clearly demonstrated by considering two plans: Plan A has a single expected payment three years from the valuation date. The discount rate for Plan A is equal to the three-year spot rate. Plan B has an expected payment one year from now and another expected payment 10 years from now. Its discount rate is calculated as the single rate that produces the same benefit obligation as does discounting the payment one year from now at the one-year spot rate and the payment 10 years from now at the 10-year spot rate. Assume that this calculation also produces a discount rate equal to Plan A's discount rate (the three-year spot rate). One year later, Plan A will have a single payment two years from the valuation date, while Plan B will have a single payment nine years from the valuation date. It is possible that the

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[17] The term "no gain/loss outcome" is used rather than "expected obligation" because the measurement of the obligation at the beginning of year does not define a unique corresponding year-end measurement. It might be argued that the interest cost implicitly defines a year-end expectation for discount rates. However, that result is a function of the application of the accounting standard. The discount rates used in the beginning-of-year measurement are observed in the market. They do not reflect any assumption that the actuary or plan sponsor has made about the year-end interest rate environment.

[18] http://www.actuary.org/files/Pension_Cost_Recognition_08142015.pdf

resulting discount rate for Plan A (the two-year spot rate) or the resulting discount rate for Plan B (the nine-year spot rate) could match the previous year's three-year spot rate. However, unless the yield curve is completely flat, one would not expect both Plan A and Plan B to have the same discount rate as the prior year.

The principle that no gain or loss will arise if the single effective discount rate remains unchanged at year-end is true for most, but not all, of the lump sum valuation approaches discussed. Consider the various examples already presented:

Example	Single Effective Discount Rate	Lump Sum Conversion Interest Rate	Beginning-of-Year Pension Obligation	Interest Cost (Traditional Approach)	"No Gain/Loss Outcome" Year-End Pension Obligation
Example 1—Annuity	2.42%	N/A	$45,465	$1,100	$46,565
Example 2—Fixed Lump Sum	1.70%	N/A (non-interest-sensitive)	46,404	789	47,193
Example 3—Best Estimate Lump Sum	1.70%	2.50%	46,037	783	46,820
Example 4—Annuity Substitution	2.42%	N/A	45,465	1,100	46,565
Example 5—Individual Implied Lump Sum Rates	1.70%	3.17%	45,465	773	46,238
Example 6—Aggregate Implied Lump Sum Rates	2.42%	2.42%	45,465	1,100	46,565

For each of the approaches summarized above, other than the individual implied lump sum rate approach, if there is no change to the single effective interest rate and other assumptions, there will be no gain/loss at the end of the year. That is, the year-end pension benefit obligation will equal the no gain/loss outcome shown above. Many different yield curves at year-end could produce the same 2.42% single effective discount rate as was used at the beginning-of-year. A year-end yield curve that is flat with all spot rates equal to 2.42% would do so for all populations and all payment patterns. If all spot rates are not the same, then the effective interest rate depends on both the year-end payment profile and the shape of the yield curve. If the plan is open to new entrants and provides ongoing accruals such that the pattern of projected benefit payments is exactly the same at the end of the year as it was at the beginning, an identical yield curve would produce the same effective interest rate. If the population becomes more mature, an increase in spot rates would be necessary to achieve the same effective interest rate (assuming the typical upward-sloping yield curve).

A "no gain/loss outcome" at year-end if there is no change in the effective interest rate does not hold true for the individual implied lump sum rates approach in Example 5. Continuing to use an implied lump sum rate of 3.17% produces a lump sum payment of $47,024; discounting that value from the payment date to the end-of-year measurement date using the effective interest rate of 1.70% generates the year-end value of $46,238 shown in the table above. This valuation approach produces the same result at the beginning of the year as annuity substitution in Example 4 and several other settlement approaches. But the no gain/loss outcome at the end of the year is different than the value under the other approaches that produce identical obligations at the beginning of the year.

The payments that are discounted to measure the pension obligation in Example 5 are the settlement values of annuity payments calculated based on the implied rates as of the lump sum payment date. These payment amounts have been calculated from the interest rates in effect at the beginning of the year. At the end of the year, these implied lump sum payments would be recalculated to reflect end-of-year interest rates. An interest rate progression that produces the same discount rate at the beginning and the end of the year is likely to change the rates as of the lump sum payment date and therefore change the settlement values. An infinite number of no gain/loss scenarios do exist, but they require discount rate changes to offset changes in these implied lump sum amounts. When the yield curve is upward-sloping, as is typical, achieving the no gain/loss result under the individual implied lump sum rates approach would generally require higher discount rates than for the other techniques discussed in this practice note to be consistent with the lower no gain/loss liability, and would result in an effective interest rate that would likely differ from the beginning-of-year rate.

Spot Rate Method

The spot rate method is one of the granular approaches to calculating pension cost discussed in the August 2015 issue brief. Under this method, interest cost on the present value of each year's payment is calculated using the same spot rate used to discount the value of that payment.

A gain or loss on annuity payments will be zero if the same spot rate is used at year-end for each payment as at the beginning of the year. This is, in other words, a one-year shift in the yield curve. For example, at the beginning of the year, the pension obligation for the payment expected to be made five years from now is calculated by discounting that payment five years at the five-year spot rate. Interest cost attributable to that payment is then calculated by applying that same rate to the beginning of year pension obligation. Adding the interest cost to the beginning-of-year pension obligation results in a year-end pension obligation, which equals the payment discounted for four years at the same five-year spot rate. Thus if the four-year spot rate at year-end is the same as the five-year spot rate at the beginning of the year, the no gain/loss outcome will be realized. (Other interest rate scenarios could produce the same outcome by chance if gains and losses on various payments in the plan offset each other.) With a typical upward-sloping yield curve, this one-year shift in spot rates would represent an increase in rates. The interest cost calculated under the spot rate method is typically lower than under the traditional method.

If Examples 1-6 above are modified to use the spot rate method, the results at first appear to suggest that the change has no effect on several approaches: the fixed lump sum (Example 2), best-estimate lump sum (Example 3) and forward-rate techniques (Examples 5-6). However, this is really just a function of the simple example chosen. These approaches reflect the timing of a single anticipated lump sum payment, so the spot rate for the payment date is used as the effective discount rate and to calculate interest cost.

The effect of the spot rate method can be better illustrated with multiple lump sum payment dates. The previous examples can be expanded to assume a second lump sum payable at year 7 equal to the value of an annuity of five equal payments of $10,000 at years 7 to 11. These expanded examples appear in Appendix A, the results of which are summarized in the table below.

Example	Equivalent Single Discount Rate	Lump Sum Conversion Interest Rate	Beginning -of-Year Pension Obligation	Interest Cost (Traditional Approach)	"No Gain/Loss Outcome" Year-end Pension Obligation (Traditional Approach)	Interest Cost (Spot Rate Method)	"No Gain/Loss Year-end" Pension Obligation (Spot Rate Method)
1A—Annuity	2.96%	N/A	$83,044	$2,454	$85,498	$2,240	$85,284
2A—Fixed Lump Sum	2.64%	N/A (non-interest-sensitive)	85,543	2,262	87,805	1,947	87,491
3A—Best Estimate Lump Sum	2.64%	2.50% / 3.50%	84,141	2,221	86,362	1,910	86,051
4A—Annuity Substitution	2.96%	N/A	83,044	2,454	85,498	2,240	85,284
5A—Individual Implied Lump Sum Rates	2.64%	3.17% / 4.24%	83,044	2,192	85,236	1,885	84,929
6A—Aggregate Implied Lump Sum Rates	2.96%	2.96% / 2.96%	83,044	2,454	85,498	N/A	N/A

The table shows results similar to the first set of examples, but with the spot rate approach consistently producing a lower interest cost. This general observation is attributable to the upward slope of the yield curve used in the examples. Additional observations include the following:

- The aggregate implied lump sum rates approach has not been shown for the spot rate method. This approach uses a single effective interest rate, and it is therefore not compatible with the spot rate method (or any other of the granular approaches).
- The settlement approaches (annuity substitution in Example 4A and individual implied forward rates in Example 5A) produce the same pension obligation at the beginning of the year, which is also the same as the pension obligation for payment of annuities in Example 1A.
- The individual implied lump sum rates approach in Example 5A produces a lower interest cost than annuity substitution in Example 4A, regardless of which interest cost approach is used.
- The relationships of the fixed lump sum (Example 2A[19]) and the best-estimate lump sum (in Example 3A) approaches to those of the annuity substitution (Example 4A) and individual implied lump sum rates (Example 5A) approaches depend on plan provisions and on the assumptions used.

As discussed above, the no gain/loss outcome for annuity payments under the spot rate method occurs when there is a one-year shift in the yield curve. Examples 2A-6A in Appendix A show the results of the same one-year shift applied to the lump sum approaches. These examples show that the same no gain/loss outcome holds for all of the lump sum valuation options when using the spot rate method except for individual implied lump sum rates in Example 5A. This is similar to the traditional approach. The forward rates that would be used at the end of the year under the individual implied lump sum rate approach, recalculated based on a one-year shift in the yield curve, are different from those calculated at the beginning of the year. As a result, the implied lump sum payments are not the same; the interest rate change environment that would produce a no gain/loss outcome for the other approaches would produce a loss for the individual implied lump sum rates approach in Example 5A.

Cash Balance Plans and Other Hybrid Plans

Cash balance plans typically define an accrued benefit by reference to a hypothetical account balance. The accounts usually grow with interest credits and pay credits (if the plan is not frozen) each year. At the time of payment, participants may elect an annuity benefit that is based upon the actuarial equivalent of the hypothetical account balance (though sometimes with an additional subsidy). Most plans also provide for an option to elect a lump sum payment equal to the account balance. The alternative valuation approaches and matching portfolios described here could be thought of as inverting the approaches used for valuing lump sums in annuity-based plans.

Note that there are other types of hybrid plans, such as pension equity plans, stable value plans, and deferred pension equity plans. These plans all define the accrued benefit by reference to a lump sum amount and thus raise many of the same issues that apply to cash balance plans as discussed in this section. They will have actuarial equivalent conversion factors from the lump sum to an annuity that are either fixed or interest-sensitive.

[19] There is no difference between settlement and best estimate approaches where the lump sum conversion factor is fixed. The fixed lump sum example continues to be shown merely for comparison purposes.

Cash Balance Plan With Fixed Interest Crediting Rate, Benefit Paid as a Lump Sum

In Example 13, the cash balance plan simply provides for a 4% fixed interest crediting rate and the payment is assumed to be made in the form of a lump sum. Under such a plan, the promise is equivalent to a fixed lump sum at the assumed retirement date and the appropriate lump sum valuation would be consistent with the previously described approach for fixed lump sums in Example 2. The cash balance account grows with interest at the interest crediting rate to the assumed payment date and then is paid in a single lump sum. The benefit obligation is the amount of the expected lump sum discounted back to the measurement date. The matching portfolio would contain a single zero-coupon bond matching this expected payment.

Example 13: Present Value of Cash Balance Lump Sum With Fixed Interest Crediting Rate

(1) Age at Payment Date	(2) Years from Measurement Date	(3) Applicable Spot Interest Rate	(4) Account Balance	(5) Lump Sum Payment	(6) Present Value Factor	(7) Present Value of Payment: (5) * (6)
63	-	-	$45,000	-	1.0000	-
64	1.00	1.43%	46,800	-	0.9859	-
65	2.00	1.70%	48,672	$48,672	0.9668	$47,054
66	3.00	2.01%		-	0.9420	-
67	4.00	2.35%		-	0.9113	-
68	5.00	2.60%		-	0.8796	-
69	6.00	2.81%		-	0.8468	-
Total:						**$47,054**
Macaulay Duration:				**2.00**		
Effective Interest Rate:				**1.70%**		
Interest Crediting Rate:				**4.00%**		

Cash Balance Plan With Fixed Interest Crediting Rate and Fixed Conversion, Benefit Paid as an Annuity

For Example 14, the cash balance plan promises a benefit that is the annuity equivalent of the account balance, where the interest crediting rate and annuity conversion rates are fixed. The benefit would be analogous to a traditional benefit defined as an annuity and converted to a lump sum using a fixed basis. Because the conversion is not interest-sensitive, the matching portfolio would simply contain zero-coupon bonds matching the timing and amount of the annuity payments and the settlement approach to valuation would be based on these zero-coupon bonds.

Example 14: Present Value of Cash Balance Annuity—Fixed Annuity Conversion and Fixed Interest Crediting Rate

(1) Age at Payment Date	(2) Years from Measurement Date	(3) Applicable Spot Interest Rate	(4) Account Balance	(5) Annuity Payment	(6) Present Value Factor	(7) Present Value of Payment: (5) * (6)
63	-	-	$45,000	-	1.0000	-
64	1.00	1.43%	46,800	-	0.9859	-
65	2.00	1.70%	48,672	$10,140	0.9668	$9,803
66	3.00	2.01%		10,140	0.9420	9,552
67	4.00	2.35%		10,140	0.9113	9,241
68	5.00	2.60%		10,140	0.8796	8,920
69	6.00	2.81%		10,140	0.8468	8,586
Total:						**$46,101**
Macaulay Duration:			**3.93**			
Effective Interest Rate:			**2.42%**			
Interest Crediting Rate:			**4.00%**			

Cash Balance Plan With Fixed Interest Crediting Rate and Interest-Sensitive Conversion, Benefit Paid as an Annuity

When the cash balance plan converts between the stated account balance and an associated annuity using a basis that depends on prevailing interest rates at the time of conversion, the plan has actually promised the equivalent of a fixed lump sum. The payment made in the form of an annuity based on the spot rates in effect at the conversion date does not change the matching portfolio, which would still contain a single zero-coupon bond matching the cash balance lump sum at the assumed date of annuity conversion. In effect, this design inverts the previously analyzed situation in which the plan pays a lump sum equivalent of a stipulated annuity based upon an interest-sensitive factor. This outcome suggests a "lump sum substitution" settlement approach to valuation. This approach would have the same properties as an annuity substitution approach would for a traditional plan that pays interest-sensitive lump sums. Example 15 shows the equivalence between the valuation of the cash balance annuity using the underlying lump sum and using an annuity conversion factor derived from implied forward rates (equivalent to the individual implied conversion rate approach).[20]

[20] The conversion between cash balance and annuity payments is assumed to use the same factor of 4.7028 used to develop the lump amount of $47,028 in the earlier, annuity-based individual implied lump sum rates illustration (Example 5).

Example 15: Present Value of Cash Balance Annuity—Interest-Sensitive Annuity Conversion and Fixed Interest Crediting Rate

(1) Age at Pmt. Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Two-Year Forward Rate	(5) Discount to Conv. Date at Forward Rate	(6) Account Balance	(7) Annuity Payment: (6) / ∑ Column (5)	(8) Lump Sum Payment	(9) Present Value Factor	(10) Present Value of Annuity: (7) * (9)	(11) Present Value of Lump Sum Payment: (8) * (9)
63	-	-			$45,000	-		1.0000	-	-
64	1.00	1.43%			46,800	-		0.9859	-	-
65	2.00	1.70%	1.98%	1.0000	48,672	$10,350	$48,672	0.9668	$10,006	$47,054
66	3.00	2.01%	2.63%	0.9744		10,350		0.9420	9,749	-
67	4.00	2.35%	3.37%	0.9426		10,350		0.9113	9,432	-
68	5.00	2.60%	3.60%	0.9099		10,350		0.8796	9,104	-
69	6.00	2.81%	3.88%	0.8759		10,350		0.8468	8,764	-
Totals:				**4.7028**					**$47,054**	**$47,054**
Macaulay Duration:			**2.00**							
Effective Interest Rate:			**1.70%**							
Interest Crediting Rate:			**4.00%**							

Analyzing the sensitivity to movement in interest rates shows that an investment that aligns with the defined lump sum payment hedges this type of design, in the same way as for Examples 11 and 12 where the annuity was fixed and the lump sum was a function of interest rates. In the annuity example, the hedging portfolio would be sold to pay for the lump sum, exchanging payments aligned with the annuity payments for their market-equivalent value to be paid as a lump sum. In this cash balance example, the proceeds from the zero-coupon bond maturing on the expected payout date would be used to purchase a portfolio of bonds matching the annuity payments at prices prevailing on the conversion date. In both cases, the market rate conversion between lump sum and annuity means that the lump sum amount has the same present value as the bond portfolio that matches the annuity payments. Examples 16 and 17 below show that this equality continues to hold as the benefit commencement date approaches and interest rates change.

Example 16: Present Value of Cash Balance Annuity—Interest-Sensitive Annuity Conversion and Fixed Interest Crediting Rate After One Year

						Assets		Benefit Obligation	
(1) Age at Pmt. Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) One-Year Forward Rate	(6) Discount to Conversion Date at Forward Rate	(7) Bond Cash Flow (Projected Account Balance)	(8) Market Value of Bond at Meas. Date: (4) * (7)	(9) Annuity Payment: Projected Balance / ∑ Column (6)	(10) Present Value of Benefit Payment: (4)* (9)
64	-	-	1.0000	-		-	-	-	-
65	1.00	0.41%	0.9959	0.41%	1.0000	$48,672	$48,473	$10,150	$10,109
66	2.00	0.62%	0.9877	0.83%	0.9918	-	-	10,150	10,026
67	3.00	1.14%	0.9665	1.51%	0.9704	-	-	10,150	9,810
68	4.00	1.76%	0.9325	2.22%	0.9363	-	-	10,150	9,465
69	5.00	2.29%	0.8929	2.77%	0.8965	-	-	10,150	9,063
Totals:					**4.7950**		**$48,473**		**$48,473**

Example 17: Present Value of Cash Balance Annuity—Interest-Sensitive Annuity Conversion and Fixed Interest Crediting Rate After Two Years

						Assets		Benefit Obligation	
(1) Age at Pmt. Date	(2) Years from Meas. Date	(3) Applicable Spot Interest Rate	(4) Present Value Factor	(5) 0-Year Forward Rate (Spot Rate)	(6) Discount to Conversion Date at Forward Rate	(7) Bond Cash Flow	(8) Market Value of Bond at Meas. Date: (4) * (7)	(9) Annuity Payment: Balance / ∑ Column (6)	(10) Present Value of Benefit Payment: (4) * (9)
65	-	-	1.0000	-	1.0000	$48,672	$48,672	$9,969	$9,969
66	1.00	0.41%	0.9959	0.41%	0.9959	-	-	9,969	9,928
67	2.00	0.62%	0.9877	0.62%	0.9877	-	-	9,969	9,846
68	3.00	1.14%	0.9665	1.14%	0.9665	-	-	9,969	9,634
69	4.00	1.76%	0.9325	1.76%	0.9325	-	-	9,969	9,295
Totals:					**4.8826**		**$48,672**		**$48,672**

It is worth noting that in the above cash balance examples, the matching portfolio is only valid if the payment date is accurately estimated. Because the assets are unlikely to provide a return equal to the plan's fixed interest crediting rate, a change in the payment date could result in a shortfall or surplus depending upon the direction of the change in timing and the relative yield of the assets compared to the interest crediting rate. A similar mismatch would occur with the annuity examples if payments commencing at other possible start dates are not actuarially equivalent to the assumed payment stream, based on the yield curve used to measure obligations.

Just as the individual implied lump sum rates approach in Example 5 results in a shorter duration and lower interest cost than does the annuity substitution approach in Example 4 when converting a fixed annuity to a lump sum, the opposite is true when converting a fixed lump sum to an annuity.

Variable Interest Crediting Rates

Additional complexity arises with a plan that provides benefits with a variable interest crediting rate. Many U.S. qualified cash balance plans use a variable interest crediting rate based on long-term bond yields. However, bond yields do not represent the annual return on a bond because they do not capture capital appreciation or depreciation when interest rates change. As a result, if the cash balance account is credited with bond yields, rather than market asset returns, there is unlikely to be an asset available that can reproduce the behavior of the cash balance account, and thus there is an increased risk of mismatch due to assumptions not being met. Although valuation issues are similar to those applicable cash balance plans with fixed interest credit rates, a bond yield-based variable interest crediting rate makes the lump sum substitution approach discussed above more dependent on the assumption of the interest-crediting rate prior to payment date and therefore less like a true settlement approach.

For a market-rate cash balance plan, the interest crediting rate is based on the actual return on plan assets. If the preservation-of-capital requirement is disregarded, a settlement approach to measuring the pension obligation would simply be to set the obligation equal to the current account balance at the measurement date. The current balance (plus actual asset return) will be sufficient to fully meet the obligation, as long as the benefit ultimately paid is equal to the account balance or an alternative payment form that is actuarially equivalent based on market rates as of the payment date. Factoring in the preservation-of-capital requirement (the requirement that the payment be no less than the sum of the pay credits without interest) would add additional cost. Assessing the additional cost is beyond the scope of this practice note.

'Greater of' Plan Designs

Plans that pay a lump sum based on the greater of two factors present another valuation challenge. To convert from a traditional annuity benefit to a lump sum, these plans may use a minimum lump sum basis stated in the plan that is compared to the statutorily required minimum basis. The lump sum amount to be paid is equal to the larger of the results of these two calculations.

Continuing the example from the previous sections, suppose that the plan provides a lump sum that is equal to the greater of 4.8 times the annual payment (as in the fixed lump sum from Example 2) and the present value of the annual payments using the valuation spot rates (as in the annuity substitution example, Example 4). It is difficult to find a traditional asset portfolio that would reproduce this payment pattern, which makes it challenging to apply a settlement approach to the valuation of the obligation.

Under this plan design, the two lump sum factors would be equal when the effective interest rate is 2.08%.[21] The participant will receive 4.8 times the monthly payment whenever the effective interest

[21] $4.8 = \sum_{n=0}^{4} (1 + \text{i})^{-n}$ where i = 2.08%

rate is greater than or equal to 2.08% and will receive the present value of the annuity stream using the valuation spot rates whenever the effective interest rate is less than 2.08%. That is equivalent to the participant having the right to receive a lump sum equal to 4.8 times the annual payment plus an option that provides an additional payment if interest rates fall below 2.08%.

One approach to the valuation of such a lump sum would be to determine which lump sum factor would apply at each payment date, if assumptions are realized. This would treat the benefit like a fixed lump sum if the effective interest rate was projected to be above 2.08% at the time of payment, or like an interest-sensitive lump sum if the effective interest rate is below 2.08%. This aligns with a best-estimate valuation philosophy (similar to Example 3). However, the characteristics of greater-of plan design features may warrant additional consideration.

The benefits payable according to such plan designs typically change asymmetrically if assumptions are not realized. In the example of this section, lump sum payments increase as the conversion interest rate falls below 2.08%. But the opposite is not true: Lump sum amounts do not decrease as the conversion interest rate climbs above 2.08%. The benefit payable at these higher interest rates is defined by the fixed conversion factor of 4.8 times the annuity payment, and additional increases do not affect the lump sum benefit amount. It may be appropriate for a plan design where benefits can increase but not decrease to have a larger pension obligation than one in which benefits can vary in either direction. ASOP No. 4 addresses this issue in Section 3.5.3, "Plan Provisions that are Difficult to Measure." The extent to which this benefit asymmetry should affect the pension obligation is not clearly addressed by accounting guidance.

A pure settlement approach would be to value the benefit according to the interest-sensitive hypothesis using a hypothetical portfolio backing the underlying annuity and to include an additional value associated with purchasing the embedded option inherent in the plan design. (This option permits the participant to exchange the interest-sensitive lump sum being valued for the fixed lump sum when it is advantageous to do so.) Such an approach could be daunting to incorporate rigorously in an actuarial valuation; the current state of pension actuarial practice seldom includes explicit pricing of such options. Thus, it is often more practical to consider loading the pension obligation for the value of such an option either directly (by a multiple of the benefit amount) or indirectly (by using a somewhat lower discount rate).

A plan that provides a lump sum equal to the greater of an account balance or the present value of an annuity benefit (such as a grandfathered traditional benefit) calculated using a variable interest rate factor would present similar valuation challenges when applying a settlement approach.

Subsidies

In general terms, a subsidy is present when one benefit option (e.g., optional form of payment or commencement date) is more valuable than another. This can include plan provisions such as early retirement subsidies that are provided in the annuity form of payment but are not included in the corresponding lump sum payments. This section of the practice note, however, focuses on subsidies introduced when benefits are converted from their normal form using an actuarial equivalence basis that relies on interest or mortality rates that differ from the valuation assumptions. Such a subsidy may either be positive, when the pension obligation for someone electing the optional form exceeds the pension obligation for someone electing the normal form, or negative, when the pension obligation for someone electing the normal form is greater.

As an example, consider a plan that offers a lump sum converted from an underlying annuity benefit. This conversion takes place using Treasury rates rather than the (generally higher) corporate rates used to value the pension obligation. This generates a subsidy, as the pension obligation measured assuming a lump sum payment would be greater than the pension obligation measured assuming an annuity.

The best-estimate technique described in Example 3 accommodates valuing such subsidies in a straightforward manner, as the expected payment amounts can be determined in a way that captures the subsidies. In the example, the projected lump sum amounts can be calculated by converting anticipated annuity payments to lump sums using the relevant expected Treasury rates in determining the pension obligation. It can be considerably more complicated to reflect these variable subsidies on a settlement basis.

One general approach is to calculate the pension obligation and service cost without reflecting the subsidies and to include a load for their effects. This sort of load is often based on an analysis of results for representative participants. Other possible approaches directly adjust the annuity benefits that are used in the valuation of the subsidized payment form. These and similar methods may represent a more rigorous approach when the valuation uses annuity substitution (Example 4) or individual implied lump sum rates (Example 5), because the adjusted annuity stream may reflect more precisely the interest-rate sensitivity of the lump sum payment.

Subsidies resulting from differences in mortality may be valued under the annuity substitution approach by using post-commencement mortality that reflects the lump sum conversion basis rather than the valuation basis (as is the case for funding valuations governed by IRC Section 430). When using the individual implied lump sum rates approach described in Example 5, converting to lump sums using the plan's expected mortality conversion basis will properly reflect any inherent mortality subsidy.

Reflecting interest rate subsidies requires first deciding whether the valuation should reflect the current gap between interest rate bases or should reflect a longer-term expectation of the difference. Accounting guidance does not address this point. When emphasizing settlement objectives, actuaries may use rates inferred from current market conditions. If avoiding anticipated gains or losses is a higher priority, actuaries may opt for a longer-term expectation. In the example, the difference between future Treasury and corporate rates might be set on a best-estimate basis by analyzing historical relationships. On a settlement basis, the assumed future relationship would be determined by the forward rates inherent in yield curves on the measurement date.

If calculated precisely, the subsidy may be captured by applying a load to each individual annuity payment equal to

$$\left(\frac{1+valf_{i,j}}{1+convf_{i,j}}\right)^{j-i}, \text{ where}$$

i is the number of years until the lump sum will be paid
j is the number of years until the specific annuity payment would have been made
$valf_{i,j}$ represents forward rates from the valuation interest rate basis
$convf_{i,j}$ represents forward rates from the conversion interest rate basis

Although this may appear daunting when written algebraically, the formula can be understood intuitively. The subsidy is a measure of the relationship between the plan's lump sum conversion factors and the factors based on the valuation interest rates. Each of these calculations relies on the rates used to discount payments between times i and j, which are forward rates. Depending on the specific circumstances of a valuation, other approaches may be devised that produce similar results.

Special Considerations for Plans That Use Bond Matching to Determine Accounting Discount Rates

Bond matching approaches select a portfolio of bonds that generate cash flow sufficient to pay benefits when due. The discount rate is then determined based on this portfolio. The application of bond matching to interest-sensitive payments is relatively straightforward when best-estimate approaches are used, such as in Example 3. The matching portfolio can be chosen to match the anticipated payment amounts. However, bond matching can introduce additional intricacies for settlement approaches such as those represented by Examples 4-6.

These complexities are conceptually similar to those relating to subsidies discussed earlier. Both result from differences between the interest rates used to convert optional forms and the interest rates used to discount projected payments. In the case of bond matching, however, the discount rate used to determine the pension obligation is implicit in the portfolio of selected bonds. Spot rates and forward rates that correspond to the valuation interest rate basis are generally unavailable. In addition, auditors reviewing the matching portfolio may expect an explicit demonstration that the pension obligation is settled.

If annuity substitution as in Example 4 is used, the projected benefit payments used in the valuation are in the form of annuities for a traditional plan even when lump sums are expected to be paid. A portfolio of bonds selected to match the annuity payments will provide coupon and principal payments that align with those annuities, but not the lump sums. (In fact, the lump sums may not even have been specifically calculated in the valuation process.) To generate the necessary cash to pay lump sum benefits, bonds would need to be sold before they mature. The annuity substitution approach assumes that changes in the interest rate that influence both bond pricing and lump sum payments will result in the sale price of the bond matching the lump sum to be paid. But future prices of individual bonds in the portfolio will depend on both the prevailing level of general interest rates and the pricing of those specific bonds, which may not move precisely as indicated by general changes in the yield curve. Many bond models do not reflect this latter factor and implicitly assume that bonds can be sold on a pricing basis equivalent to the lump sum conversion rates. In other words, they assume that the sale price of the bond is the discounted value of future bond cash flows measured using the lump sum conversion interest rates. Where the selected bonds are reasonably expected to differ in pricing from those used to determine future lump sum conversion rates, the actuary may want to consider making an explicit adjustment for the anticipated cost of this difference, such as a load to the annuity cash flows being valued. Such an adjustment will likely be appropriate if, for example, only the highest-yielding AA bonds are used to construct the matching portfolio.

The individual implied lump sum rates approach as in Example 5 does not require selling bonds before they mature. The bond portfolio can be selected to match the timing of lump sum payments and the amounts of the implied lump sums. This portfolio, in conjunction with hypothetical zero-cost transactions that exchange these amounts for the specified benefit payments, would effectively settle the pension obligation. Adjustments for any material subsidies may also need to be considered in this calculation, as described in the previous section. Although a precise application of these techniques can be quite complicated, no assumptions about the pricing of specific bonds at future dates would be required.

Summary

Plans with interest-sensitive cash flows present particular valuation challenges. This practice note has provided a discussion of the differences between various methods for valuing lump sums and other interest-sensitive benefit payments. Differences in the equivalent single effective interest rate and duration highlight how two methods that might produce the same beginning-of-year pension obligation can produce different outcomes for interest cost, gain/loss, and sensitivity to future changes in interest rates. While this discussion pertains mainly to traditional plans that pay interest-sensitive lump sums, similar issues apply (in reverse) to lump sum-based plans where the annuity conversion basis is interest-sensitive.

Some of the key conclusions from this practice note include:

- The best-estimate approach to calculating pension obligations measures the cost of the expected lump sum cash flow using static lump sum conversion rates that represent a best estimate as of the measurement date. Any bond portfolio whose cash flows are structured to match the resulting pension obligation cash flows will precisely meet the pension obligation only if the actual lump sum interest rate at each payment date exactly matches the assumed lump sum interest rate.
- In the absence of significant subsidies, settlement approaches (e.g., annuity substitution and individual implied lump sum rates) produce a benefit obligation similar to the obligation associated with the underlying annuity.
- The aggregate implied lump sum rates approach is a simplified settlement approach that uses a lump sum rate equal to the single effective interest rate derived from the annuity cash flows. It is therefore not compatible with granular approaches to pension cost recognition.
- The individual implied lump sum rates approach produces a lower interest cost than other settlement approaches, despite producing the same pension obligation. This holds true regardless of which interest cost approach is used (traditional or spot rate), as long as the yield curve exhibits a typical positive slope.
- While the individual implied lump sum rates approach results in a shorter duration and lower equivalent single effective interest rate than the annuity substitution approach when converting a fixed annuity to an interest-sensitive lump sum, the opposite is true when converting a fixed lump sum (e.g., from a cash balance plan) to an annuity using market (rather than fixed) rates.
- Plan designs that provide for “greater of” formulas or subsidies on selected payment forms, or that have a difference in the interest rate basis for lump sum calculations relative to pension obligation measurement, may require adjustments to the basic approaches discussed in this practice note to reflect items such as the potential asymmetry of benefit payments as interest rates rise or fall, optionality and anti-selection risk, the expected utilization of a the subsidy, differences between conversion rates and

discount rates, and the degree to which current interest rates vary from a longer-term expectations.

The information in this practice note should be helpful to an actuary evaluating the various approaches to measuring the pension obligation for interest-sensitive cash flows under the relevant accounting standards.

Appendix A: Spot Rate Method Examples with Two Separate Lump Sum Payments

The column for age has been removed as now there are multiple lump sum payments that may reflect multiple participants. There is no separate display for the annuity substitution approach as the entries are the same as for the annuity approach shown in Example 1A below. The example numbers reflected in Appendix A correspond to the examples in the main paper, with, for example 1A being the beginning-of-year pension obligation illustration associated with Example 1 and 1B being the corresponding year-end pension obligation illustration (thus, because the calculations for the annuity substitution method are not shown there is no Example 4A or 4B).

Example 1A: Present Value of Annuity

(1) Years from Meas. Date	(2) Applicable Spot Interest Rate	(3) Payment Amount	(4) Present Value Factor	(5) Present Value of Payment: (3) * (4)	(6) Interest at Equivalent Rate: EIR * (5)	(7) Interest at Spot Rate: (2) * (5)
0						
1	1.43%		0.9859	-	-	-
2	1.70%	$10,000	0.9668	$9,668	$286	$164
3	2.01%	10,000	0.9420	9,420	278	189
4	2.35%	10,000	0.9113	9,113	269	214
5	2.60%	10,000	0.8796	8,796	260	229
6	2.81%	10,000	0.8468	8,468	250	238
7	2.96%	10,000	0.8153	8,153	241	241
8	3.12%	10,000	0.7821	7,821	231	244
9	3.23%	10,000	0.7512	7,512	222	243
10	3.33%	10,000	0.7207	7,207	213	240
11	3.45%	10,000	0.6886	6,886	203	238
Totals:				**83,044**	**2,454**	**2,240**
Macaulay Duration:			**6.19**			
Effective Interest Rate (EIR):			**2.96%**			

No gain/loss liability at year-end under the spot rate method = $83,044 + $2,240 = $85,284.

Example 2A: Present Value of Fixed Lump Sum

(1) Years from Meas. Date	(2) Applicable Spot Interest Rate	(3) Payment Amount	(4) Present Value Factor	(5) Present Value of Payment: (3) * (4)	(6) Interest at Equivalent Rate: EIR * (5)	(7) Interest at Spot Rate: (2) * (5)
0						
1	1.43%		0.9859	-	-	-
2	1.70%	$48,000	0.9668	$46,409	$1,227	$789
3	2.01%	-	0.9420	-	-	-
4	2.35%	-	0.9113	-	-	-
5	2.60%	-	0.8796	-	-	-
6	2.81%	-	0.8468	-	-	-
7	2.96%	48,000	0.8153	39,135	1,035	1,158
8	3.12%	-	0.7821	-	-	-
9	3.23%	-	0.7512	-	-	-
10	3.33%	-	0.7207	-	-	-
11	3.45%	-	0.6886	-	-	-
Totals:				**85,543**	**2,262**	**1,947**
Macaulay Duration:			**4.29**			
Effective Interest Rate (EIR):			**2.64%**			

No gain/loss liability at year-end under the spot rate method = $85,543 + $1,947 = $87,490

For the best-estimate lump sum example shown below, the actuary assumes different best-estimates rates for different payment dates. The discounting is shown for each lump sum payment in columns 3-5.

Example 3A: Present Value of Interest-Sensitive Lump Sum—Best-Estimate Approach

(1) Years from Meas. Date	(2) Applicable Spot Interest Rate	(3) Annuity Payment	(4) Best Estimate Discount to Lump Sum Payment Date	(5) Lump Sum Payment	(6) Present Value Factor	(7) Present Value of Lump Sum Payment: (5) * (6)	(8) Interest at Equivalent Rate: EIR * (7)	(9) Interest at Spot Rate: (2) * (7)
0								
1	1.43%				0.9859	-	-	-
2	1.70%	10,000	1.0000	47,620	0.9668	46,041	1,215	783
3	2.01%	10,000	0.9756		0.9420	-	-	-
4	2.35%	10,000	0.9518		0.9113	-	-	-
5	2.60%	10,000	0.9286		0.8796	-	-	-
6	2.81%	10,000	0.9060		0.8468	-	-	-
7	2.96%	10,000	1.0000	46,731	0.8153	38,100	1,006	1,128
8	3.12%	10,000	0.9662		0.7821	-	-	-
9	3.23%	10,000	0.9335		0.7512	-	-	-
10	3.33%	10,000	0.9019		0.7207	-	-	-
11	3.45%	10,000	0.8714		0.6886	-	-	-
Totals:						**84,141**	**2,221**	**1,910**
Macaulay Duration:			**6.11**					
Effective Interest Rate (EIR):			**2.64%**					
Best-Estimate Lump Sum Rate: 2.50% for 1st payment / 3.50% for 2nd payment								

No gain/loss liability at year-end under the spot rate method = $84,141 + $1,910 = $86,051.

For the individual implied lump sum rates approach, implied forward rates were calculated at each lump sum date and then the lump sum using those rates were calculated.

Example 5A: Present Value of Interest-Sensitive Lump Sum—Individual Implied Lump Sum Rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Years from Meas. Date	**Applicable Spot Interest Rate**	**Annuity Payment**	**Implied Spot Yields at Lump Sum Date**	**Discount at Implied Spot Yields to Lump Sum Payment Date**	**Lump Sum Payment**	**Present Value Factor**	**Present Value of Lump Sum Payment: (6) * (7)**	**Interest at Equivalent Rate: EIR * (8)**	**Interest at Spot Rate: (2) * (8)**
0									
1	1.43%					0.9859	-	-	-
2	1.70%	10,000		1.0000	47,024	0.9668	45,465	1,200	773
3	2.01%	10,000	2.63%	0.9743		0.9420	-	-	-
4	2.35%	10,000	3.00%	0.9425		0.9113	-	-	-
5	2.60%	10,000	3.20%	0.9097		0.8796	-	-	-
6	2.81%	10,000	3.37%	0.8759		0.8468	-	-	-
7	2.96%	10,000		1.0000	46,091	0.8153	37,578	992	1,112
8	3.12%	10,000	4.25%	0.9593		0.7821	-	-	-
9	3.23%	10,000	4.18%	0.9214		0.7512	-	-	-
10	3.33%	10,000	4.20%	0.8839		0.7207	-	-	-
11	3.45%	10,000	4.31%	0.8446		0.6886	-	-	-
Totals:							**83,044**	**2,192**	**1,885**
Macaulay Duration:			**6.19**						
Effective Interest Rate (EIR):			**2.64%**						

Implied Lump Sum Rate: 3.17% for 1st payment / 4.24% for 2nd payment

No gain/loss liability at year-end under the spot rate method = $83,044 + $1,885 = $84,929.

For the aggregate implied lump sum rates approach, there is no interest cost calculation under the spot rate method. As discussed above, there is no logical application of the spot rate method to this lump sum approach.

Example 6A: Present Value of Interest-Sensitive Lump Sum—Aggregate Implied Spot Rates

(1) Years from Meas. Date	(2) Applicable Spot Interest Rate	(3) Annuity Payment	(4) Discount at EIR for Lump Sum	(5) Lump Sum Payment	(6) Present Value Factor (at EIR)	(7) Present Value of Lump Sum Payment: (5) * (6)	(8) Interest at Equivalent Rate: EIR * (7)	(9) Interest at Spot Rate: N/A
0								Method derives single equivalent rate for all purposes from underlying annuity cash flows. This is not consistent with applying spot rates to a portion of this calculation.
1	1.43%				0.9713	-	-	
2	1.70%	10,000	1.0000	47,211	0.9434	44,540	1,316	
3	2.01%	10,000	0.9713		0.9163	-	-	
4	2.35%	10,000	0.9434		0.8900	-	-	
5	2.60%	10,000	0.9163		0.8645	-	-	
6	2.81%	10,000	0.8900		0.8397	-	-	
7	2.96%	10,000	1.0000	47,211	0.8156	38,504	1,138	
8	3.12%	10,000	0.9713		0.7922	-	-	
9	3.23%	10,000	0.9434		0.7694	-	-	
10	3.33%	10,000	0.9163		0.7473	-	-	
11	3.45%	10,000	0.8900		0.7259	-	-	
Totals:						**83,044**	**2,454**	
Macaulay Duration:			**6.26**					
Effective Interest Rate (EIR):			**2.96%**					

No gain/loss liability at year-end under the spot rate method = $83,044 + $2,454 = $85,284.

Rollforward of Results Under the Spot Rate Method

The no gain/loss outcome under the spot rate method is a one-year shift in the yield curve. The examples below apply this one-year shift to test whether the resulting pension obligation is identical to the implied year-end pension obligation derived by adding the interest cost to the beginning of year pension obligation (and adjusting for benefits expected to be paid in the intervening year, which are $0 in the example).

Example 1B: Annuity Payment Stream—No Gain/Loss Year-End Pension Obligation

(1) Years from Measurement Date	(2) Applicable Spot Interest Rate	(3) Payment Amount	(4) Present Value Factor	(5) Present Value of Payment: (3) * (4)
0				
1	1.70%	10,000	0.9833	9,833
2	2.01%	10,000	0.9610	9,610
3	2.35%	10,000	0.9327	9,327
4	2.60%	10,000	0.9024	9,024
5	2.81%	10,000	0.8706	8,706
6	2.96%	10,000	0.8394	8,394
7	3.12%	10,000	0.8065	8,065
8	3.23%	10,000	0.7754	7,754
9	3.33%	10,000	0.7447	7,447
10	3.45%	10,000	0.7124	7,124
Total:				**85,284**
Macaulay Duration:		**5.20**		
Effective Interest Rate (EIR):		**3.00%**		

Example 2B: Present Value of Fixed Lump Sum—No Gain/Loss Year-End Pension Obligation

(1) Years from Measurement Date	(2) Applicable Spot Interest Rate	(3) Payment Amount	(4) Present Value Factor	(5) Present Value of Payment: (3) * (4)
0				
1	1.70%	48,000	0.9833	47,198
2	2.01%	-	0.9610	-
3	2.35%	-	0.9327	-
4	2.60%	-	0.9024	-
5	2.81%	-	0.8706	-
6	2.96%	48,000	0.8394	40,293
7	3.12%	-	0.8065	-
8	3.23%	-	0.7754	-
9	3.33%	-	0.7447	-
10	3.45%	-	0.7124	-
Total:				**87,491**
Macaulay Duration:		**3.30**		
Effective Interest Rate (EIR):		**2.76%**		

Example 3B: Present Value of Interest-Sensitive Lump Sum—Best-Estimate Approach—No Gain/Loss Year-End Pension obligation

(1) Years from Measurement Date	(2) Applicable Spot Interest Rate	(3) Annuity Payment	(4) Best Estimate Discount to Lump Sum Payment Date	(5) Lump Sum Payment	(6) Present Value Factor	(7) Present Value of Lump Sum Payment: (5) * (6)
0						
1	1.70%	10,000	1.0000	47,620	0.9833	46,824
2	2.01%	10,000	0.9756		0.9610	-
3	2.35%	10,000	0.9518		0.9327	-
4	2.60%	10,000	0.9286		0.9024	-
5	2.81%	10,000	0.9060		0.8706	-
6	2.96%	10,000	1.0000	46,731	0.8394	39,228
7	3.12%	10,000	0.9662		0.8065	-
8	3.23%	10,000	0.9335		0.7754	-
9	3.33%	10,000	0.9019		0.7447	-
10	3.45%	10,000	0.8714		0.7124	-
Total:						**86,051**
Macaulay Duration:			**5.16**			
Effective Interest Rate (EIR):			**2.75%**			
Best estimate lump Sum Rate: 2.50% for 1st payment / 3.50% for 2nd payment						

Example 5B: Present Value of Interest-Sensitive Lump Sum—Individual Implied Lump Sum Rates Approach—No Gain/Loss Year-End Pension obligation

(1) Years from Meas. Date	(2) Applicable Spot Interest Rate	(3) Annuity Payment	(4) Implied Spot Yields at Lump Sum Date	(5) Discount at Implied Spot Yields to Lump Sum Payment Date	(6) Lump Sum Payment	(7) Present Value Factor	(8) Present Value of Lump Sum Payment: (6) * (7)
0							
1	1.70%	10,000		1.0000	47,290	0.9833	46,500
2	2.01%	10,000	2.32%	0.9773		0.9610	-
3	2.35%	10,000	2.68%	0.9485		0.9327	-
4	2.60%	10,000	2.90%	0.9178		0.9024	-
5	2.81%	10,000	3.09%	0.8854		0.8706	-
6	2.96%	10,000		1.0000	46,202	0.8394	38,784
7	3.12%	10,000	4.09%	0.9608		0.8065	-
8	3.23%	10,000	4.04%	0.9238		0.7754	-
9	3.33%	10,000	4.07%	0.8871		0.7447	-
10	3.45%	10,000	4.19%	0.8486		0.7124	-
Total:							**85,284**
Macaulay Duration:			**5.20**				
Effective Interest Rate (EIR):			**2.75%**				

For all of the options except for the individual implied lump sum rates approach, the present value calculated assuming a one-year shift in the yield curve matches the rollforward. As shown above, the implied spot yields, recalculated based on a one-year shift in the yield curve, are different than those calculated the prior year. The single equivalent lump sum rates are now 2.87% and 4.11%, compared with 3.17% and 4.24%. In other words, a one-year shift in the yield curve is not compatible with keeping the lump sum rates constant. As a result, the recalculated pension obligation under the spot rate method is higher than the rollforward of the prior-year amount (and in fact, still matches the calculation using annuity substitution) under the individual implied lump sum rates approach.

Appendix B: Interest Rate Fundamentals

Spot rates are used to calculate the present value of a single future cash flow. Spot rates are represented as s_t, where t represents the time of the future cash flow. Time will be expressed in years, and all calculations in the examples of this practice note use compound rather than simple interest.

If the five-year spot rate is 3%, the present value of $100 to be paid in five years is therefore

$$100(1 + s_5)^{-5} = 100 \times 1.03^{-5} = 86.26$$

Although spot rates are essential to time-value-of-money calculations, they typically cannot be directly observed in the capital markets. A bond typically represents a set of future cash flows rather than a single cash flow, and cash flows paid at different points in time have values that depend on different spot rates. The single interest rate that relates multiple payment amounts to their total price or present value is often called an *effective interest rate*. In the context of bonds, such an interest rate is typically referred to as a *yield*.[22] Although the yield of a bond can be observed, it does not provide a unique solution for the underlying spot rates. Suppose that a three-year bond with a face value of $100 pays 5% annual coupons at the end of the year. This bond sells for $104. The yield on the bond, represented with the letter i, is 3.57%. The yield can be verified by noting that

$$104 = 5(1 + i)^{-1} + 5(1 + i)^{-2} + 105(1 + i)^{-3}$$

when i is 3.57%. An iterative approach is required to solve for i, but a unique solution does exist and can be found readily enough. In fact, the yield on a bond is very frequently published by the same source as its price. But this is not the case for spot rates. For the same bond, all that is known is that

$$104 = 5(1 + s_1)^{-1} + 5(1 + s_2)^{-2} + 105(1 + s_3)^{-3}$$

There are many spot rates that can solve this equation, and additional techniques are necessary to develop spot rates from pricing data. Such techniques are outside the scope of this practice note, which assumes that all spot rates are known and available to the actuary.

Another type of interest rate can be useful to actuaries—*forward rates*. Spot rates describe the relationship between a present value and a future value. Forward rates are derived from spot rates and describe the relationship between future values as of any two points in time. The notation $f_{i,j}$ is used to represent the forward rate of interest between times i and j where $i < j$. They can be calculated as

$$f_{i,j} = \left[\frac{(1 + s_j)^j}{(1 + s_i)^i}\right]^{1/(j-i)} - 1$$

Suppose that the five-year spot rate is 3% and the 12-year spot rate is 4%. The rate of interest that is used to discount payments from 12 years to five years is then

[22] Several variations of yields exist, including yield to maturity, yield to worst and yield to call. Further complicating matters, yields can be cited using payment frequency conventions that vary by market. This practice note does not address these nuances.

$$f_{5,10} = \left[\frac{(1+s_{10})^{12}}{(1+s_5)^5}\right]^{1/(12-5)} - 1 = \left[\frac{1.04^{12}}{1.03^5}\right]^{1/(12-5)} - 1 = .0472$$

The 4.72% forward rate can then be used to determine that, for example, a $100 payment in year 12 has the same value as a $72.41 payment in year 5:

$$100(1+f_{5,12})^{(5-12)} = 100 \times 1.0472^{-7} = 72.41$$

This can be verified as follows:

$$100(1+s_{12})^{-12} = 100 \times 1.04^{-12} = 62.46$$

and

$$72.41(1+s_5)^{-5} = 72.41 \times 1.03^{-5} = 62.46$$

Appendix C: Duration

The term *duration* takes on several related meanings. In many situations, these alternative meanings are essentially equivalent. In the context of interest-sensitive payments, however, the alternative definitions can have significant implications.

The most fundamental definition of duration is the *Macaulay duration* of a series of cash flows. This is simply a weighted-average term until payment in which the weights are the present values of each cash flow. A mathematical expression for this is

$$MacDur = \frac{\sum_j \left(t_j PV_j\right)}{\sum_j PV_j}$$

where t_j is the time until the j^{th} payment and PV_j the present value of the j^{th} payment. (Present values are sometimes developed using a single equivalent discount rate and sometimes use each payment's individual spot rate.) The Macaulay duration is simply the time until that payment is made.

The greater the duration of a series of fixed cash flows, the more sensitive its present value is to interest rates. This observation leads to a variant of duration more tailored to estimate price sensitivity considerations: *modified duration.* The formula is very similar to that of Macaulay duration:

$$ModDur = \frac{\sum_j \left(t_j PV_j\right)}{(1+i)\sum_j PV_j}$$

where i represents an annually compounded effective rate. If i decreases by a very small amount k, the present value of the series of payments will increase by $ModDur * k$. For example, suppose that a bond with a duration of 10 yields 5%. If the yield decreases to 4.99%, the present value of the bond will increase by approximately 0.01% x 10 = 0.1%. This relationship is not exact, and it is less accurate for larger interest rate changes. It also does not work well when the interest rates at different maturities change by different amounts. Despite these limitations, modified duration is a useful and widely used concept in both actuarial and investment management contexts.

The close relationship between duration and sensitivity to interest rate changes exists only if the payment amounts remain constant as interest rates change. If payments vary based on prevailing interest rates (that is, the cash flows are interest-sensitive), modified duration will be an inaccurate measure for the interest sensitivity of the present value of the payments. In such situations, it is more accurate to describe the interest rate sensitivity of the present value as an *effective duration*—a duration that reflects the changes in underlying payment amounts. Unfortunately, there is not a general closed-form expression for effective duration. It must often be computed by shocking the interest rates, recalculating the new payment amounts, determining their present value in the new interest rate environment, and comparing this to the initial present value. In the case of an interest-sensitive lump sum payment, this practice note has demonstrated that the effective duration of the lump sum is often the same as the effective duration of the underlying annuity payment (which is, in turn, the same as the modified duration of the annuity payment).