A PUBLIC POLICY PRACTICE NOTE

Exposure Draft

Selecting Investment Return Assumptions Based on Anticipated Future Experience

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of the American Academy of Actuaries

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The Committee gratefully acknowledges the contributions of former Pension Committee Chairperson Michael Pollack and Aaron Weindling.

The comment deadline for this exposure draft is June 27, 2016. Please send any comments to pensionanalyst@actuary.org.

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INTRODUCTION

This practice note is not a promulgation of the Actuarial Standards Board, is not an actuarial standard of practice (ASOP) or an interpretation of an ASOP, is not binding upon any actuary and is not a definitive statement as to what constitutes generally accepted practice in the area under discussion. Events occurring subsequent to the publication of this practice note may make the practices described in the practice note irrelevant or obsolete.

This practice note was prepared by the Pension Committee of the Pension Practice Council of the American Academy of Actuaries, to provide information to actuaries on current and emerging practices in the selection of investment return assumptions based on anticipated future experience. The intended users of this practice note are the members of actuarial organizations governed by the ASOPs promulgated by the Actuarial Standards Board.

This practice note may be helpful when setting assumptions, or providing advice on setting assumptions, for funding (where permitted by law), and for financial accounting in connection with funded U.S. benefit plans. It does not cover the selection and documentation of other economic assumptions or demographic assumptions.

The Pension Committee welcomes any suggested improvements for future updates of this practice note. Suggestions may be sent to the pension policy analyst of the American Academy of Actuaries at 1850 M Street NW, Suite 300, Washington, DC 20036 or by emailing pensionanalyst@actuary.org.

BACKGROUND

Actuarial Standard of Practice No. 27 (ASOP 27), *Selection of Economic Assumptions for Measuring Pension Obligations*, provides guidance to actuaries in selecting economic assumptions such as those relating to investment return, discount rates, and compensation increases.

Key provisions of ASOP 27 relating to the determination of investment return assumptions include the following:

- Assumptions should be reasonable and consistent with other economic assumptions selected by the actuary for the measurement period (Sections 3.6 and 3.12).

- Assumptions should be based on the actuary's observations of the estimates inherent in market data and/or should reflect the actuary's estimate of future experience (Section 3.6(d)).

- Assumptions should incorporate no significant bias[1] (Section 3.6(e)).
- The actuary should review appropriate current and long-term historical economic data as part of the assumption-setting process (Sections 3.4 and 3.8.1).
- Active management premiums should not be anticipated without relevant supporting data (Section 3.8.3(d)).

Complex issues arise in the determination of investment return assumptions, especially for an investment return assumption that will be used as a discount rate (i.e., as a means for determining the present values of promised benefit payments payable over long periods). In particular, the ASOP acknowledges the distinction between assumptions that reflect arithmetic versus geometric average returns (section 3.8.3(j)). Arithmetic averages generally exceed geometric averages, but some issues and concerns may arise in developing investment return assumptions based on these higher rates.

This practice note provides discussion and background information relating to this technical issue. It is divided into seven sections:

I. Definitions/Terminology: Sets forth definitions of terms that will be used frequently; some definitions introduce minor twists or insights compared to what the reader might be familiar with.

II. Numeric Example: Provides a numerical example that refreshes the reader's understanding of geometric and arithmetic computations for historical performance.

III. Forecast Models—the Effect of Uncertainty**:** Shows how these concepts are used in modeling.

IV. Relationships Among Statistics**:** Compares means and medians in the context of arithmetic and geometric models.

V. Analysis of Forecast Returns**:** Addresses stochastic simulations and the results that may be analyzed from them. This section provides the foundation of the debate related to the use of arithmetic and geometric averages.

VI. Issues/Concerns for Actuaries**:** Further amplifies the issues related to the selection of arithmetic vs. geometric averages.

VII. Conclusions: Summarizes the key points addressed in the practice note.

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[1] The ASOP contains an exception "when provisions for adverse deviation or plan provisions that are difficult to measure are included and disclosed under section 3.5.1, or when alternative assumptions are used for the assessment of risk."

The material presented in this practice note is complex and technical. Although an initial read-through may not require a major time investment, actuaries may find it beneficial to devote several hours to a more in-depth review and study of the concepts, arguments, and applications presented. The practice note offers two appendices and a bibliography to support further independent study.

I. DEFINITIONS/TERMINOLOGY

The analyses involved in setting an investment return assumption invoke issues that are highly technical and involve fairly subtle distinctions. Additional complexity arises in that different authors may reference similar concepts and terminology but employ them slightly differently. Some terms may also be used in a less technical sense in other contexts, and have developed certain connotations from this more general usage.

This practice note uses terminology in the following manner, which sometimes differs from the terminology employed in ASOP 27. (ASOP 27 terminology, where different, is indicated in parentheses.)

- *Average*: A statistic related to a sequence of values, which can be either historical returns or a single scenario of future returns. In other material, the word “average” is used to describe a calculation performed on a random variable. To avoid confusion, this practice note will use other terms to describe results that apply to random variables. The two types of average returns addressed by the practice note are:

    - *Arithmetic average return*: Calculated from a sequence of periodic returns by dividing the sum of the rates of return by the number of periods.

    - *Geometric average return*: Calculated from a sequence of periodic returns by first converting each of them to the amount that would be accumulated during the period from an investment of $1. For example, the single period accumulation that corresponds to a 10% return is 1.1, while the accumulation corresponding to a -5% return is .95. The geometric return over N periods is determined by raising the product of the N periodic single period accumulations to the power of (1/N) and subtracting 1 from the result.

- *Terminal wealth*: The amount that accumulates from an initial investment of one unit. For any value of terminal wealth at the end of N periods, the equivalent discount rate is determined by raising the terminal wealth to the 1/N power and subtracting 1.

- *Independent and identically distributed (IID):* In probability theory and statistics, a sequence of random variables is independent and identically distributed if each random variable has the same probability distribution and all are mutually independent; i.e., it is unaffected by the variables that came before. The assumption that observations be IID tends to simplify the underlying mathematics of many statistical methods. The assumption is important in the classical form of the central limit theorem, which states that the probability distribution for IID variables with finite variance approaches a normal distribution. In practical applications of statistical modeling, however, the IID assumption may not be realistic.

The following terms describe properties or results developed from the probability distribution of a random variable, such as the output from a stochastic simulation:

- *Expected value*: The average of possible values for a random variable weighted by the probability associated with each. In a stochastic simulation this outcome is not necessarily known, but is estimated as the average of the variable in question over all stochastic trials.

  Note: The word "expected" is commonly used to reference a specific anticipated outcome to the exclusion of other possibilities (e.g., it is expected to rain tomorrow). This different usage can create confusion. When charged with setting an assumption for "expected" return, some actuaries may adhere to the statistical definition implied above while others may intend the less technical usage. To ensure clarity, this practice note uses "mean" rather than "expected value."

- *Mean*: A synonym for the technical meaning of expected value as defined above. Other sources sometimes describe average returns developed from historical results or a single sequence of forecast outcomes as mean returns; e.g., arithmetic mean return or geometric mean return. To clarify the distinctions, this practice note uses mean only to describe a statistic related to a random variable; i.e., a calculation made from a simulated array of possible outcomes, not a statistic calculated from a single sequence of values.

- *Median*: A value that separates the upper 50% from the lower 50% of the distribution of outcomes for a random variable.

In a stochastic simulation, the arithmetic and geometric average returns and the terminal wealth outcomes are themselves random variables. In determining an appropriate basis for setting an investment return assumption, it is relevant to consider statistics such as:

- the mean value of arithmetic average return (forward looking expected arithmetic return);

- the mean value of geometric average return (forward looking expected geometric return);

- the mean and median values of terminal wealth; and

- the equivalent discount rates associated with the mean and median values of terminal wealth.

II. NUMERIC EXAMPLE

Much of the discussion that follows will consider these calculations as applied to a set of simulated future capital market outcomes such as those developed from a stochastic forecast. These outcomes can be visualized as a table of results, arranged with each scenario as a row and results for each simulation year as a column. The analysis of historical results or a deterministic forecast would, in contrast, entail only one set of outcomes.

Exhibit 1

	Annual Return -- for each simulation year							
Scenario	*1*	*2*	*3*	*4*	*5*	*Arithmetic Average Return*	*Geometric Average Return*	*Terminal Wealth*
A	5%	16%	20%	7%	-4%	8.8%	8.5%	1.50
B	14%	1%	6%	-12%	3%	2.4%	2.0%	1.11
C	1%	14%	26%	-3%	18%	11.2%	10.7%	1.66
D	22%	-4%	6%	11%	-3%	6.4%	6.0%	1.34
E	6%	14%	-3%	-8%	12%	4.2%	3.9%	1.21

The statistics for each scenario are determined as described above. For example, the arithmetic average for scenario A is equal to (5% + 16% + 20% + 7% - 4%) / 5 = 8.8%. The terminal wealth is (1.05)(1.16)(1.20)(1.07)(0.96) = 1.50. The geometric average for the same scenario by definition connects to the terminal wealth figure and is derived as $(1.05)(1.16)(1.20)(1.07)(0.96)^{1/5} - 1 = 8.5\%$.

The combination of model-generated scenarios makes up a collection of random variables for which additional statistics can be calculated. The mean and median of arithmetic average, geometric average, and terminal wealth are shown below. The equivalent discount rates that generate terminal wealth figures are also calculated.

Simulation results	*Mean*	*Median*
Arithmetic average	6.6%	6.4%
Geometric average	6.2%	6.0%*
Terminal wealth	1.36	1.34
Discount rate associated with terminal wealth	6.4%	6.0%*

* Note that the median geometric average return equals the discount rate equivalent of median terminal wealth by definition.

Reporting past performance
Over a single investment period, arithmetic and geometric calculations of return are equal by definition. For multiple periods, however, the average returns will be equal only if each of the time-period returns are the same. To the extent that there is return volatility, the arithmetic average will be higher than the geometric average return, as the above example illustrates.

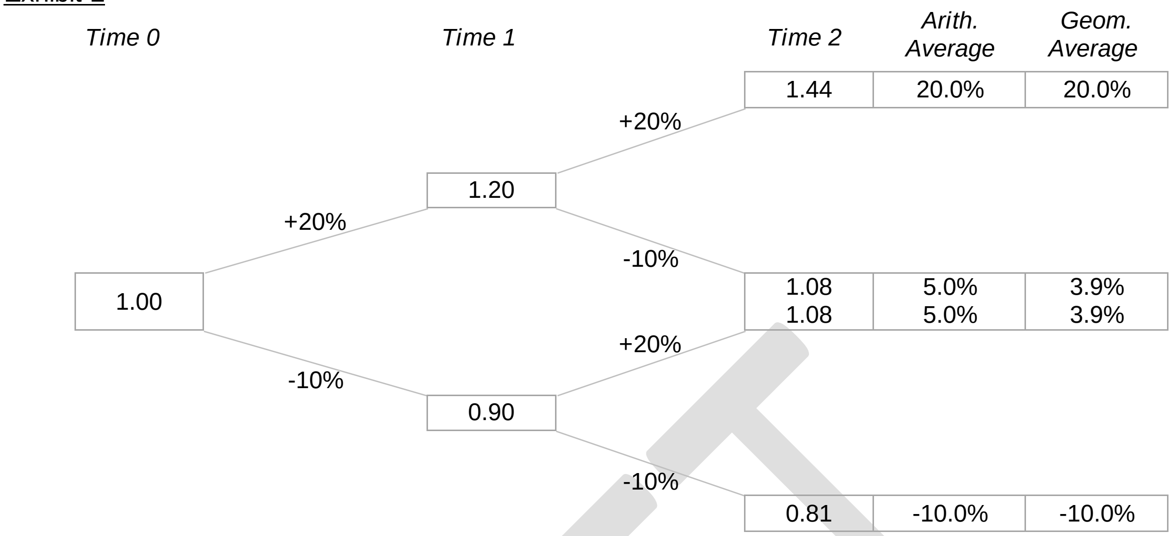
The conventional goal of performance reporting is to cite the rate of return that, given the initial asset value and subsequent cash flows, will reproduce the asset value at the end of the period. Only the geometric average return has this property. Suppose, for example, that the sequence of returns illustrated in scenario D actually came to pass, i.e., becomes "history." In that case, the terminal wealth would reconcile with the geometric average of the portfolio returns: $(1.060)^5 = 1.34$. The arithmetic average does not: $(1.064)^5 = 1.36$, a figure that exceeds the terminal wealth amount. Thus, there is little controversy that the appropriate return statistic for reporting past investment return is geometric average return.

III. FORECAST MODELS—THE EFFECT OF UNCERTAINTY

The analysis of past performance does not consider uncertain future outcomes, but forward-looking/forecast models typically do, and such analysis is critical to actuarial work. Intuitive conclusions based on conventional mathematical relationships that apply in a straightforward way to the analysis of historical results may need to be reviewed in order to understand how the implications may differ in the analysis of expected future outcomes.

Consider this highly simplified example—a distribution of outcomes based on only two potential return outcomes, +20% or -10%, with a 50% probability assigned to each. For illustrative purposes, the returns for each year are presumed to be independent, without any element of either serial correlation or reversion to mean.

Exhibit 2



The mean of each year's return is, of course, 5%. The median annual return result, imputed in this case as the midpoint of the simulation outcomes, can also be presumed to be 5%.

But when the two years' results are combined, relationships change. Asymmetry is introduced due to the dollar-denominated return increment (in excess of the median) associated with the positive outcome being greater than the dollar amount of the return shortfall (below the median) associated with the corresponding negative outcome.[2] The implication, when considering the distributions of geometric average return or terminal wealth, is that the mean outcome will typically exceed the median outcome.

	Mean	*Median*
Arithmetic average	5.0%	5.0%
Geometric average	4.5%	3.9%
Terminal wealth	1.10	1.08
Discount rate associated with terminal wealth	5.0%	3.9%

IV. RELATIONSHIPS AMONG STATISTICS

In analyzing a distribution of uncertain future returns, two noteworthy connections between the resulting return and wealth outcomes are often presumed:

- The discount rate associated with the *mean* terminal wealth equals the mean value of arithmetic average return.

[2] In the above example, this effect can be viewed as a simple comparison between the high outlier result (1.44) and the low outlier result (0.81). The high outlier exceeds the median result (1.08) by 0.36, while the low outlier falls below it by only 0.27; i.e., the outlier gain outcome exceeds the amount associated with the outlier loss. The impact of this imbalance can be seen in the fact that the mean/average terminal wealth figure exceeds the median terminal wealth.

- The discount rate associated with the *median* terminal wealth outcome equals the mean value of geometric average return (but only over a long projection period).

In the context of the above simplified two-year projection, the first result is obviously borne out; i.e., the discount rate that generates the mean terminal wealth is the same as the arithmetic average return, 5.0% in both cases. In regard to the geometric average in the above example, however, the mean result exceeds the discount rate equivalent of median terminal wealth. This connection is borne out only over longer projection periods; i.e., the mean geometric average return declines as the number of simulated periods increases, and ultimately converges to the rate equivalent of the median terminal wealth.

The relationships among statistics are easiest to evaluate when future years' distributions of returns are considered to be independent and identically distributed (IID)—as was presumed in the example above. While this assumption forms the basis of many statistical models and conclusions, this treatment is oversimplified in that it does not incorporate the dynamics of observed "real world" return patterns.

Nonetheless, the IID assumption allows for straightforward application of statistical concepts, and permits the representation of portfolio return as a normally distributed random variable. This facilitates the assertion of certain numerical relationships that will be discussed further below. Note that the relationships may be valid even when prospective returns are not IID; at least some of these same relationships will be found in the output from any scenario generation model when applied over sufficiently long periods of time.

Arithmetic average and geometric average returns:

- Over a single period, arithmetic and geometric measures of return are identical by definition.
- Over multiple periods, the mean arithmetic average return will equal the mean geometric average return only if all periodic returns are equal. If there is any return volatility, arithmetic average return will exceed geometric average return.
- Mean geometric average return results will tend to decrease as the projection period increases (given some level of return volatility). This might be viewed as implying that the wealth-reducing effect of return volatility increases over time. There are a number of estimates for the relationship between mean arithmetic (A) and mean/median geometric average (G) returns. The most common approximation, although not fully precise, is $G \approx A - \text{Variance}/2$, where variance is that related to single period returns.[3]

Arithmetic average return and terminal wealth:

- Mean arithmetic average return relates to mean terminal wealth. In other words, accumulating assets at the mean arithmetic average rate is expected to produce the mean terminal wealth.

[3] See the referenced Mindlin paper for a more complete discussion of this formula, along with an array of alternative estimation approaches.

Geometric average return and terminal wealth:

- Median geometric average return equates to median terminal wealth; again, this is by definition. Because mean geometric average return is asymptotic to median geometric return as the projection period increases, mean geometric average return ultimately equates to median terminal wealth.

In considering the above relationships, remember that some can only be expected to apply over long periods of time. Thus, not all of the relationships were fully borne out over the two- and five-year simulation periods that were shown in the earlier simplified examples.

An actuary referencing forecast results should also consider the need for review and testing of simulation outcomes from the particular capital market model being relied on in order to determine which relationships do and do not hold. In other words, the actuary should consider evaluating, rather than presuming, connections such as the critical linkage between the mean values for arithmetic average and terminal wealth.

V. ANALYSIS OF FORECAST RETURNS

The actuary's determination of an expected return assumption may be based on simulated future capital market outcomes along with, or in place of, a review of actual/historical capital market results. A stochastic forecast model will generate an array of possible results that can be characterized as arithmetic or geometric average returns. Some characteristics associated with each statistic may be of interest; the actuary should consider which of these characteristics are more desirable for a given application. These properties may include the following:

No expected gain/loss
This is a traditional actuarial objective. If the expected return assumption is set equal to the discount rate equivalent of mean terminal wealth, the expected gain or loss on assets in the future, in dollar terms, will be zero. Appendix 3 of ASOP 27 asserts that the mean arithmetic average (forward looking expected arithmetic) return will produce no gains or losses. This is certainly the result that would be expected from a model based on IID-type parameters, but may not be found in other models that incorporate implied mean reversion.[4] In such cases, it may be appropriate to determine the discount rate equivalent of the mean terminal wealth result rather than to approximate that outcome by use of the arithmetic average.

On the other hand, because geometric average return corresponds to the median terminal wealth outcome, the incidence of gain and loss outcomes would tend to be equal when

[4] In models with mean reversion tendencies, the mean arithmetic average return result is likely to exceed the discount rate equivalent of mean terminal wealth. This imbalance arises from such models' tendency to pull outlier results within a given sequence of simulated returns back toward the median over the successive years. Doing so effectively suppresses "longitudinal" volatility (the range of accumulated wealth outcomes over time) while leaving "cross-sectional" volatility (the range of return outcomes for any one simulation year) unaffected.

using this assumption, while reference to the mean wealth outcome or arithmetic average return implies a greater incidence of loss outcomes. As noted earlier, this seeming anomaly is caused by the fact that the simulated array of forecast scenarios generally includes high outlier outcomes with larger dollar gains than the amount of dollar losses associated with low outlier outcomes. Put another way, the average terminal wealth outcome is more affected by outlier results than is the median terminal wealth.

Also, note that if the assumed expected return is set to the expected geometric average, and that geometric average return is realized over a given experience period, no gain/loss will result. However, if the assumed return is set to the expected arithmetic average return, and that arithmetic average return is realized, unless that return is realized as a constant rate, there will be an experience loss. This implies that, given that the experienced return amounts will almost certainly be returned as a constant rate, an arithmetic average return that is *greater* than the investment return assumption must be realized in order to avoid a loss.

Credibility/robustness[5]

As already noted, the mean of a random variable is much more sensitive to outlier values than is its median, because the mean value is affected by the existence of a few large outlier values, while the median is not. Because geometric average return corresponds to the more robust median terminal wealth statistic, it is also considered to be a more robust outcome from a capital market simulation model than is the arithmetic average return.

This characteristic becomes important if the actuary believes that outlying scenarios in a probability distribution are not fully credible. Certain statistical techniques can also be used to address this situation; e.g., the outlying scenarios may be truncated, or their values may be replaced with threshold values.

Conservatism

Because mean arithmetic average return will almost always exceed mean geometric average return, the use of arithmetic average for discounting purposes would be viewed as a less conservative assumption.

[5] These terms are related in the sense that they connect to the level of confidence that may be attributed to a given modeling result.

-- The term *robustness* relates to (1) the sensitivity of a given result to outlier data in the distribution from which it is derived, and (2) the ability of a test or result to provide valid insight even if the model presumptions are altered or violated.

-- The term *credibility* as employed in this context relates to the level of reasonableness/validity associated with a given simulation result; it seems rational to assert that reliance on a less robust forecast result would be considered less predictive of actual future outcomes.

VI. ISSUES/CONCERNS FOR ACTUARIES

The actuary should carefully consider the issues involved in setting an investment return assumption, whether based on a review of capital market history or simulated future return scenarios generated by a forecast model.

As noted earlier, it is generally accepted that the geometric average best represents investment outcomes associated with historic investment returns; i.e., this result exactly equates to the actual amount of wealth generated during the historic period. If historical returns are referenced in assumption-setting, the actuary should consider the need to address issues related to differences in economic conditions—which may be viewed as significantly different from those during the historical period. Critical factors to evaluate would normally include inflation levels, expectations for productivity gains, the level of interest rates, and the level of stock market valuation.

This paper has described a number of special considerations that relate to the review of projected results. Note that even if a stochastic forecast model were calibrated to fully align with historical results—asset class means, standard deviations and correlations that exactly match historical statistics—it would still produce a range of outcomes rather than the deterministic/single outcome represented in the historic record.

The implications of these two different approaches for assumption-setting (capital market history vs. forecast/simulation) may seem inconsistent, in that the same historical information could, at least in theory, be used to support two significantly different assumptions, depending on whether that history is evaluated deterministically or used as a basis for developing an array of projected future outcomes. It is challenging, but necessary, to understand the underlying rationale for this difference, which directly relates to the arithmetic/geometric return considerations previously discussed.

The generation and calibration of economic scenarios involves a host of assumption-related decisions, and simplifications are inevitably necessary as part of the process. The effect of these simplifications is an important consideration when assessing the credibility of simulated results. For example, the cyclical qualities of capital markets may not be accurately simulated in modeling. It may be reasonable to presume that mean reversion tendencies exist in capital market outcomes over time.[6] If so, a model that does not capture this mean reversion quality—e.g., one based on an IID presumption for the generation of annual outcomes—would be expected to produce a range of outcomes that is unrealistically broad. Because mean wealth outcomes are disproportionately affected by high outlier results, the actuary should carefully consider the credibility associated with return/wealth outcomes that are heavily dependent on these high outlier results.

[6] Mean reversion tendencies would presumably result from constraints on the range of economic activity and capital market results, e.g., those imposed by resource/workforce/productive capacity limitations in the overall economy, current or simulated levels of interest rates vs. presumed normative levels, the level of equity pricing in comparison to historic mean price levels, and through the operation and underlying objectives of government fiscal and monetary policies.

One might also consider whether any type of mean outcome is a reasonable basis for decision-making. When considering events that are repeatable, gains from one iteration are available to offset losses that occur in other iterations. For example, presume a bet of one dollar on the numbers (selecting a single integer from 1 to 1,000) with a payoff of 1,000:1; the expected value of the wager is the same one dollar. It seems reasonable in this case to offset the highly likely but relatively small losses with the relatively unlikely but very large gain associated with a win. As long as the one-dollar bet is a small portion of the bettor's overall wealth, the game can be repeated often enough that the few favorable outcomes can be expected to offset the effect of the more numerous unfavorable outcomes.

However, if the number of expected incidences of betting is reduced for any reason (e.g., the bet amount is a large portion of the bettor's wealth), the situation changes. If there will be only a few betting opportunities, the more appropriate focus is the distribution of expected outcomes, with greater focus on likely as opposed to mean outcomes. This recognizes that gains from the improbable but extremely favorable outcome are unlikely to be available to offset losses from the much more probable unfavorable outcomes. Of course, this does not necessarily imply that the midpoint of the distribution of outcomes is necessarily the most appropriate choice. Depending on objectives, a 50% chance of achieving the targeted result may or may not be sufficient.

Similarly, although many simulations of a pension fund's financial experience may be run, there will ultimately be only one outcome. Gains from other favorable simulations will not be available to offset losses from unfavorable realized results. Thus, averaging the results from an array of potential outcomes may result in a measure that has limited practical value, especially in situations where it is more likely that actual experience will fall short of that average outcome. For this reason, a focus on the distribution of results, such as the median and various percentile outcomes, may be more appropriate as a guide to decision-making.

VII. CONCLUSIONS

The conclusions from this practice note can be briefly summarized as follows:

- Two approaches are commonly used in developing an investment return assumption for use as a discount rate—a review of historical statistics and the evaluation of simulated outcomes from a stochastic forecast model.

- In evaluating historical return statistics, the use of geometric average return results is generally appropriate, whereas in the review of simulated future outcomes, consideration may be given to both mean geometric and arithmetic average results, along with other related statistics such as the discount rate equivalent of mean or median terminal wealth.

- In the context of simulated future outcomes, the actuary might expect that the use of an assumption based on the mean arithmetic average, or the return rate that generates the mean terminal wealth outcome, will produce no expected future gain or loss.[7] However, the gain/loss parity results from the greater dollar gain associated with high outlier outcomes vs. the loss associated with low outlier outcomes. Thus, the use of this assumption implies a greater-than-50% chance of a loss being incurred.

- The use of a mean geometric average is consistent with the approach used for assessing historical data, i.e., the amount of accumulated wealth actually produced by capital markets in the past. In the context of simulated future results, over long periods it will align with the median wealth outcome, thus balancing the expected incidence of gains and losses in the future. The mean geometric average is less sensitive to the influence of outlier results than is the arithmetic average, which means that it is the more robust outcome from capital market modeling.

The actuary will want to understand the rationale and implications associated with any statistical outcome selected for use. An actuary considering the use of an arithmetic average rate for discounting over long periods of time should find it helpful to consider the issues and concerns raised in this practice note.

APPENDIX 1

Application to Return Assumption Used in U.S. Accounting (ASC-715 and GASB67)

For the most part, this practice note has focused on investment return assumptions developed for use as discount rates and applied over long periods of time. However, U.S. corporate accounting rules applicable to pension and retiree welfare plans call for a different application for an investment return assumption. In corporate accounting, the investment return rate is applied to a market or market-related value of assets to develop an expected return amount for only the current year, which is then used as an offsetting item in the determination of annual pension (or OPEB/retiree welfare) cost.

Corporate accounting rules specify “the average rate of earnings expected on the funds invested or to be invested to provide for the benefits included in the projected benefit obligation.” However, because the rate is applied to a current asset value and serves as an estimate of the current year’s investment return, its application implies a very different meaning from an investment return assumption that is applied as a discount rate or used as an estimate of asset accumulation over long time horizons. It is only when return assumptions are applied as a constant rate over multiple time periods that many of the issues discussed earlier arise.

[7] As noted earlier, the presumed equality in these two forecast outcomes might not be found in models that incorporate significant mean reversion tendencies; i.e., calculated mean arithmetic average returns might exceed the level implied by mean terminal wealth.

Based on the principles already discussed, it would appear that a mean arithmetic average return figure—whether based on historical data or derived from a forecast model—would be more compatible with the function of estimating a single year's investment return.

On the other hand, the asset value to which the rate is applied may be a highly smoothed value rather than an actual market value. This substitution acts to reduce the recognized effect of investment return volatility in the calculation of expected return amounts. The gap between arithmetic and geometric average return statistics is higher when the return results are more volatile; this implies that the rationale for the use of mean arithmetic average return is not fully consistent with an application in which mechanisms act to reduce the recognition of gains and losses and thus dampen the volatility effects.

This may imply that the application of the general principles under this situation provides a rationale for a rate somewhere between the arithmetic and geometric average return. Such a modified rate might be more consistent with its application to an asset value that recognizes some but not all of the return volatility actually present in capital markets.

APPENDIX 2

Varying Attributes of Simplified vs. Complex Statistical and Forecast Models

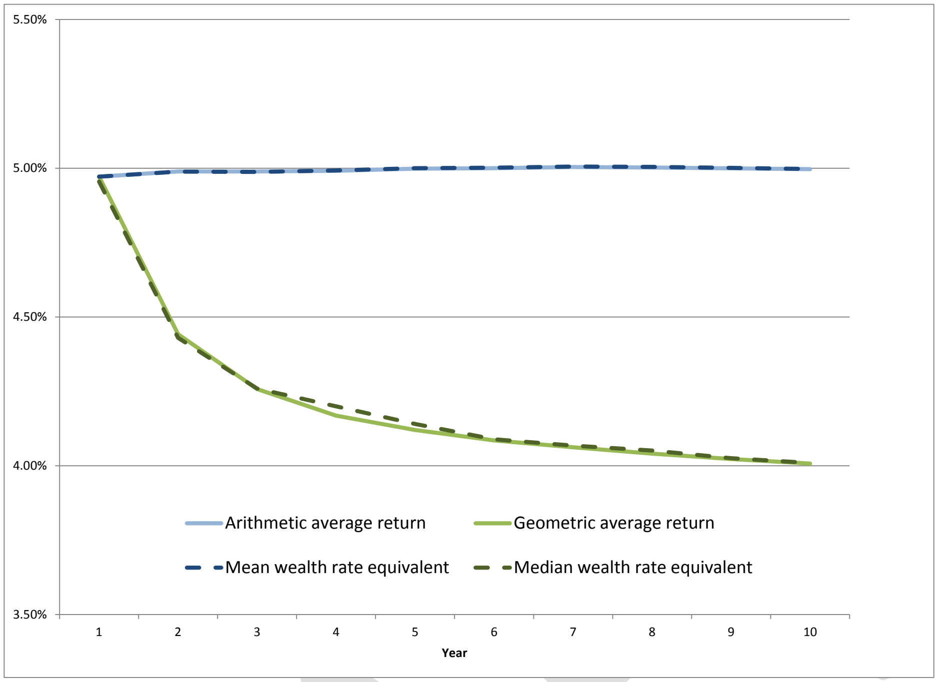
As noted earlier, the relationships among statistics are easiest to evaluate when future year's distributions of returns are considered to be independent and identically distributed (IID). While this assumption forms the basis of many statistical models and conclusions, this treatment is oversimplified in that it does not incorporate the dynamics of observed "real world" return patterns.

More simplified statistical models based on IID principles, however, exhibit a number of useful and noteworthy relationships. For a projection covering N investment periods, mean arithmetic average return, mean geometric average return, and the discount rate equivalents of mean and median terminal wealth may be referenced. Those statistics are shown in the graph in Figure 1, and exhibit the following relationships:

- Mean arithmetic average return is constant (independent of N) and is equal to the expected or mean value of the single period return.

- Mean geometric average return equates to the arithmetic average for a single-year period, and then decreases over time (as N increases).

Figure 1[8]



Terminal wealth

The objective in pension plan funding is not to achieve a particular level of investment return, but rather to accumulate an amount over time that is sufficient to provide for the payment of pension obligations. For that purpose, the most relevant statistics are those that relate to wealth accumulation, and similarly, the equivalent discount rates corresponding to those wealth statistics. In the simplified statistical model, these statistics will exhibit the following characteristics:

- Mean terminal wealth has an equivalent discount rate that is constant independent of N, and equates to mean arithmetic average return.
- Median terminal wealth has an equivalent discount rate that, by definition, equates to median geometric average return.

Mean geometric average return decreases over time as N increases; over long projection periods, it asymptotically approaches the equivalent discount rate that equates to median terminal wealth.

Relationships Referenced in ASOP 27 – Appendix 3

Some expected relationships between various statistical outcomes are referenced in the ASOP's Appendix 3. These references are essentially the same as those quoted above, i.e., statistical connections that an actuary would expect to see in statistically based models incorporating IID-type principles.

[8] Results of a return simulation based on IID presumption, normally distributed returns, 5% mean return, and 15% standard deviation.

In particular the Appendix references two key expected relationships and, as noted, uses somewhat different terminology than is employed in this practice note:

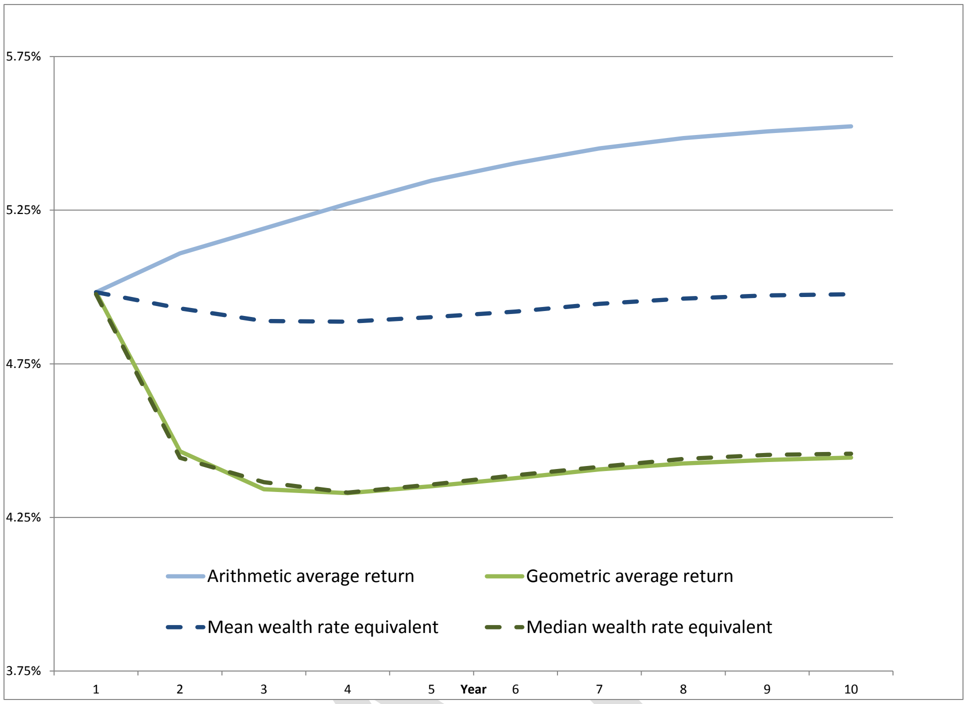
- The use of a forward-looking expected geometric return as a discount rate will produce a present value that generally converges to the median present value as the time horizon lengthens; i.e., if the actuary uses this rate to discount the obligation, there is an expectation that there will be sufficient funds to defease the obligation 50% of the time.
- The use of a forward-looking expected arithmetic return as a discount rate will generally produce a mean present value; i.e., there will be no expected actuarial gains and/or losses.

Use of more complex models should be expected to change some relationships
It is critical to note that more complex capital market/forecast models often referenced by actuaries will generally not adhere to the IID convention. At a minimum, almost all models in current use will have provisions to address differences between initial capital market conditions and "normative" conditions; e.g., current interest rates may be considered lower than the long-term norm and thus future rates will have a tendency to rise. Similarly, equity valuations could be viewed as out of sync with long-term valuation levels and have a tendency to rise or fall over time to compensate.

In addition to trends related to initial-normative capital market conditions, some models may also incorporate tendencies toward mean reversion within the generated scenarios, which implies that when return results in a given scenario are simulated to fall extremely far from the normative trend, those extreme outcomes will have a tendency to be reversed over time. For example, extremely favorable equity returns may be presumed to imply levels of economic growth, P/E ratios, and utilization of workforce, resource, and production capacities that are higher than normal. Given modeled constraints on these parameters, the result may be a bias toward unfavorable equity returns in successive periods that act to suppress prospective returns and push accumulated results closer toward the more typical range. Even more obviously, simulated high fixed-income returns generally result from decreases in yields that will tend to be reversed over time.

These types of model characteristics will tend to disrupt some of the relationships that were evidenced in the simpler statistical model reviewed earlier, as illustrated in Figure 2.

Figure 2[9]



As the above example illustrates, results from more complex models may create disconnects in at least two critical relationships:

- a trend in rates rather than constant emerging rates for mean arithmetic average and mean terminal wealth; and
- a gap rather than equality between emerging results for mean arithmetic average and mean terminal wealth.

The first outcome is a result of the tendency for initial capital market conditions to revert to normative levels over time. The second outcome is caused by the tendency for mean reversion within the capital market simulation, so that the emergence of extremely high or extremely low return/wealth outcomes creates a tendency for offsetting outcomes in successive periods—which acts to pull extreme wealth outcomes back toward median levels.

[9] Results of a return simulation based on normally distributed returns, 5% mean return in year 1 grading up to 6% mean return in year 7, and 15% standard deviation. The model also incorporates some reversion of simulation outcomes to the mean, so that a high percentile outcome in a given year leads to an increased likelihood for a low percentile outcome in a successive year; this acts to suppress the distribution of accumulated returns (e.g., the gap between mean and median wealth outcomes).

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