



AMERICAN ACADEMY *of* ACTUARIES

**Recommended Approach for Setting Regulatory Risk-Based Capital Requirements
for Variable Products with Guarantees (Excluding Index Guarantees)**

**Presented by the American Academy of Actuaries' Life Capital Adequacy
Subcommittee to the National Association of Insurance Commissioners' Life Risk-
Based Capital Working Group**

San Diego, CA – December 2002

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The following report is a follow up to a proposal from March 2002 and was prepared by the Life Capital Adequacy Subcommittee's C-3 Work Group (chaired by Bob Brown). The work group is made up of several members of the subcommittee as well as Tom Campbell, Frank Clapper, Geoff Hancock, Regynald Heurtelou, Tim Hill, Craig Morrow, Dan Patterson, Jim Reiskytl, Link Richardson, Max Rudolph, Dave Sandberg, and Albert Zlogar. The work group would also like to thank Jan Brown, Allen Elstein, Larry Gorski, Dennis Lauzon, and Mark Tenney for their helpful suggestions and feedback.

Background

Several years ago, the NAIC's Life Risk-Based Capital (RBC) Working Group asked the American Academy of Actuaries (Academy) to take a fresh look at the C-3 component of the RBC formula to see if a practical method could be found to reflect the degree of asset/liability mismatch risk of a particular company.

The Academy's Life Capital Adequacy Subcommittee (LCAS) reviewed the request and agreed that more sensitivity to the specifics of product design and funding strategy is appropriate to advance the goal of differentiating weakly capitalized companies from the rest. We have defined C-3 risk to include Asset/Liability risk in general, not just interest rate risk.

Effective December 31, 2000, the NAIC implemented Phase I of this project. Phase I addressed interest rate risk for annuities and single premium life. For Phase I, "annuities" is defined as products with the characteristics of deferred and immediate annuities, structured settlements, guaranteed separate accounts, and GICs (including synthetic GICs, and funding agreements). Equity based variable products were not included in Phase I, but products that guarantee an interest rate index and variable annuities sold as fixed were (if they were cash flow tested). Phase I of the project recommended the determination of capital requirements for interest sensitive products by scenario testing (October 1999 report; available at www.actuary.org). When implemented by the NAIC, the requirement exempted companies from scenario testing based on a significance and stress test of C-3 risk.

In this report, the LCAS recommends implementing Phase II, to address both the interest rate and equity risk associated with variable products with guarantees (including living, death benefit, and secondary guarantees), other than index guarantees. Companies with minimum death benefit only products are given a choice of scenario testing or a factor approach. Other guarantees require scenario testing.

Recommended Approach

- Run stochastic scenarios using prudent best estimate assumptions (the more reliable the underlying data is, the closer the assumptions will be to experience and vice versa) and calibrated fund performance distribution functions on an aggregated basis.
- Calculate required capital for each scenario similar to the method used in C-3 Phase I: for each scenario, accumulated statutory surplus is determined for each calendar year-end and its present value calculated. The lowest of these present values is tabulated and the scenarios are then sorted on this measure.
- Unlike the Phase I project, we are favoring the approach introduced in the Canadian Institute of Actuaries (CIA) work and recommend the use of a modified Conditional Tail Expectation (CTE) measure to set RBC requirements. (The CIA report on Segregated Fund Investment Guarantees is attached to this report as Appendix 4).

- It is recommended that this RBC amount be combined with the $C1_{CS}$ factor for covariance purposes.
- The way grouping (of funds and of contracts), sampling, number of scenarios, and simplification methods are handled is the responsibility of the actuary. However, all these methods are subject to Actuarial Standards of Practice (ASOP), supporting documentation and justification. Section 2.1.1 of the CIA report (Appendix 4) provides a thoughtful discussion of many of these considerations.
- Actuarial certification of the work done to set the RBC level will be required. Essentially, the actuary will certify that the work has been done in a way that meets all appropriate actuarial standards. The certification should specify that the actuary is not opining on the adequacy of the company's surplus or its future financial condition. The actuary will also note any material change in the model or assumptions from that used previously. Changes will require regulatory disclosure and may be subject to regulatory review and approval.
- Realizing that capital is only part of the solution, we feel it is important to remember that the actuarial standards of practice intend to include requirements for a stress test report/analysis in the memorandum from the actuary that reviews the sensitivity of the RBC result to future changes in equity performance. Although it does not affect the capital requirement, this test provides valuable risk management information for company management and regulators.

Glossary

Gross Wealth Ratio – The gross wealth ratio is the cumulative return for the indicated time period and percentile. (e.g., 1.0 indicates that the index is at its original level.)

Variable Annuity Guaranteed Living Benefit (VAGLB) – VAGLB is a guaranteed benefit included in a variable deferred or immediate annuity providing that:

- a. One or more guaranteed benefit amounts payable to a living contractholder or living annuitant, under contractually specified conditions (e.g., upon annuitization or at contract maturity), if any, will be enhanced should the contract value referenced by the guarantee (e.g., account value) fall below a given level or fail to achieve certain performance levels; and
- b. Only such guarantees having the potential to provide benefits whose present value as of the benefit commencement date that exceed the contract value referenced by the guarantee are included in this definition.

Guaranteed Minimum Income Benefit (GMIB) – The GMIB is a VAGLB design for which the benefit is contingent on annuitization of a variable deferred annuity contract. The benefit is typically expressed as a contractholder option, on one or more option dates, to have a minimum amount applied to provide periodic income using a specified purchase basis.

Minimum Guaranteed Death Benefit (MGDB) - The MGDB is a guaranteed benefit included in a variable deferred or immediate annuity providing that the amount payable on the death of a contractholder or annuitant will be enhanced and/or will be at least a minimum amount, regardless of the performance of the underlying variable annuity funds. Only such guarantees having the potential to exceed the account value are included in this definition.

Prudent Best Estimate - The assumptions to be used for modeling are to be the actuary's "prudent best estimate". This means that they are to be set at the conservative end of the actuary's confidence interval as to the true underlying probabilities for the parameter(s) in question, based on the availability of relevant experience and its degree of credibility.

A "prudent best estimate" assumption would normally be defined by applying a margin for adverse deviation to the "best estimate" assumption. "Best estimate" would typically be the actuary's most reasonable estimate of future experience for a risk factor given all available, relevant information pertaining to the contingencies being valued. Recognizing that assumptions are simply assertions of future unknown experience, the margin for adverse deviation should be directly related to uncertainty in the underlying risk factor. The greater the uncertainty, the larger the margin. Each margin should serve to increase the liability or provision that would otherwise be held in its absence (i.e., using only the best estimate assumption).

For example, assumptions for circumstances that have never been observed require more margin for error than those for which abundant and relevant experience data are available. Furthermore, larger margins are typically required for contingencies related to policyholder behavior when a given policyholder action results in the surrender or exercise of a valuable option.

Scope

All variable annuities that contain any living benefit guarantees, whether written directly or assumed through reinsurance, must utilize scenario testing to establish capital requirements. Variable annuities with only death benefit guarantees may use scenario testing or the "Alternative Method" described below. Variable annuities sold as fixed annuities are excluded from this recommendation and are to continue to be handled as products subject to interest rate risk. Since the fixed account option of other variable annuities is included, the instructions for interest sensitive products need to be modified to remove them from that capital calculation. Variable life insurance with secondary guarantees must also be included, if doing so increases the capital requirement. (Equity indexed products are excluded from this requirement. Separate account products that guarantee an index are covered in another recommendation from the LCAS which is also being submitted to the NAIC in December 2002.)

Modeling Methodology

1. Scenarios

Scenarios will consist of a sufficient number of equity scenarios, adequate for the purpose, created by the company. If stochastic interest rate scenarios are not part of the model being used, the GMIB results need to reflect the impact of the uncertainty in interest margins (see Appendix 3). The scenarios will need to meet the calibration methodology and requirements outlined in Appendix 2.

2. Conceptual Approach

Asset/Liability models are to be run that reflect the dynamics of the expected cash flows for the entire contract, given the guarantees provided under the contract. Federal Income Tax, insurance company expenses, fund expenses, and contractual charges are to be reflected realistically. Cash flows from any fixed account options should also be included.

3. Assets

For the projections of accumulated statutory surplus, the starting value of assets included in the model should be set equal to the starting value of liabilities modeled (i.e., the initial surplus in the projections should be zero). The mix of assets between separate account and general account assets should be consistent with that used for cash flow testing. The amount of general account assets (and projected investment income thereon) should be net of amounts accrued for expense allowances reported in page 3, line 13A of the annual statement (i.e., the separate account CARVM/CRVM allowance). In many instances the initial general account assets may be negative, resulting in an interest expense.

4. Fund categorization

The funds offered on the product may be grouped for modeling. In Methodology Note C3-01, various current practices are provided. Regardless of the method chosen, fundamental characteristics of the fund should be made in relation to the required calibration points of the S&P 500. The modeling should reflect characteristics of the efficient frontier (i.e., returns generally cannot be increased without assuming additional risk).

5. Modeling of Hedges

If the insurer is following a clearly defined hedging strategy, the stochastic model should take into account the impact of hedge positions currently held, as well as the appropriate costs and benefits of hedge positions expected to be held in the future. This recognizes that a hedging strategy may not require hedge positions to be held at a particular point in time; however, allowance for the impact of hedge positions not currently held, is only permitted if the insurer is following a clearly defined hedging strategy approved by the Board of Directors, or an authorized committee. To the degree the hedge position introduces basis, gap or price risk, some reduction for effectiveness of hedges should be made.

6. Interest Rates

For discounting future surplus needs and for earnings on the general account portion of funds held, companies that do not have an integrated model are to use the implied forward rates from the swap curve. Companies that do have an integrated model may use the rates generated by that model or the swap curve, but must use the method chosen consistently from year to year. Whether from a model or from the swap curve, the discount rates need to be reduced for Federal Income Tax. The assumptions for GMIB purchase rate margins are discussed in Appendix 3.

Contracts which were not sold as fixed annuities but which are now heavily invested in the fixed option need to be evaluated in a manner that reflects the interest rate risk in an appropriate manner.

7. Liabilities

For the purposes of capital determination, “statutory surplus” is based on a liability value at time t equal to the greater of the cash surrender value or the formula reserve, which would be required if any policyholder options available at that time were exercised. For instance, if the formula reserve for a currently exercisable GMIB guaranteed annuity exceeded the cash surrender value, then that reserve value would be used.

8. Capital Need for a Specific Scenario

The total capital requirement for a particular scenario is the greatest present value of the negative statutory surplus across all future year-ends.

9. Capital Determination using CTE in the NAIC RBC framework

➤ What is CTE?

CTE is a risk measure that provides enhanced information about the tail of a distribution above that provided by the traditional use of percentiles. Instead of only identifying a value at the 95th percentile (for example) and ignoring possibly exponentially increasing values in the tail, CTE provides the average over all remaining values in the tail. Thus for traditional, linearly increasing distributions, the 95th percentile and CTE (90) will be equivalent risk measures, but for distributions with “fat tails” from low probability, high impact events, the use of CTE will provide a higher, more revealing (and conservative) measure than the traditional percentile counterpart.

➤ What is Modified CTE?

Modified CTE is the measure proposed to be used for C-3 Phase II and introduces additional conservatism into the risk standard by capping the results of any one scenario at 0 (i.e., no gains are allowed to offset the losses in the tail). For example, modified CTE 90 is the arithmetic average of the worst 10 percent of all scenarios, with no scenario being calculated as a positive value of accumulated surplus.

- NAIC RBC Framework - Three key characteristics are import to consider here:
 - ◆ Early Warning - RBC is designed to serve as an early warning system for companies that could be headed towards insolvency.
 - ◆ Percentile basis - The required capital level for individual risk elements is often set at the 95th percentile over a multi-year time horizon. Use of a percentile measure ignores the extreme tail, or assumes that there are no high impact, low probability events in the tail.
 - ◆ Volatility - The current measures used (asset rating, face amount, reserves and premium) result in stable RBC levels, from year to year, with linear relationships to changes in the measures for insurance and general business risk. The only exceptions to this are the economic related measures, the C-3 calculation for those companies required to do C-3 Phase I testing, and the C-1 calculation for bonds whose credit rating is changed.

- Shortcomings of Formulas to Assess Economic Risk

Formulas must make assumptions about product design, policyholder behavior and economic relationships and conditions. With the increasing economic volatility seen over the last few decades, attempts to use formulas for measuring economic based risk have not been successful and have led to the mandating of cashflow testing for life insurance and the exploration of an economic based modeling approach for this project.

- Economic Insolvency vs. Formula Insolvency

Cashflow testing has always been done on reserves as a supplement or check on the formula basis. In the end, cashflow testing has only certified that when the formula breaks down (i.e., is not an adequate measure) additional reserves will be set up to make today's formula basis, adequate.

This project stems from the inability to set a formula based reserve. Since there is no formula basis at the base to support, the economic testing is no longer a supplement to the formula, but is the primary basis. When we refer to the Canadian approach, it is an approach that is using the economic criteria as the primary basis, not the formula basis.

- Use of CTE for C-3 Phase II Risk Measure
 - ◆ Early Warning – Use of CTE improves over the use of percentile measures by including additional information on the tail.
 - ◆ Volatility – Since this is an economic risk that is being assessed here, it will, by its nature (unless hedged), be more volatile than insurance based risk measures and may show dramatic changes from year to year.

- **Blending Volatility with Early Warning**

The high volatility of an economic risk measure can limit the effectiveness of the early warning objective or RBC. One approach could be to mandate an even higher level of capital be held as a way of ensuring that there will be funds available. However, this confuses reserves with RBC and also ignores the economics of these tail events. For example, assume the following distribution of worst 10 outcomes +5, + 3, 0, -3, -7, -12, -22, -38, -58, -100. CTE 90 = -24 and CTE 95 = -46. If the worst case scenario occurs, neither the CTE 90 nor CTE 95 standard will be adequate. In addition, if the economic climate worsens next year, both standards will need to post additional capital and the level of this additional amount is unrevealed by the current CTE value. Yet, the information can be readily captured via sensitivity testing. The early warning effectiveness will be strengthened through the required disclosure of this sensitivity, since this allows both the company and regulators to get a better sense of the risk exposure. Duration measures show the impact of future interest rate changes on today's asset values. A demonstration of how equity growth of -20 percent, -10 percent, and 0 percent over the next few years will impact future capital levels is just as valuable to demonstrate the levels of exposures held by a company.
- **Traditional Reserve Definition vs. Economic Capital Based Definition**

Most people intuitively expect that reserves provide for a sum that, with normal accumulation of interest from year to year, will generally be sufficient to pay a future claim when it becomes due. In both health and property & casualty insurance, though, it is also recognized that each year, the reserve needs to be reevaluated "fresh" and that there may often need to be an adjustment, other than the normal accumulation of interest, to reflect additional liability information gained in the year or a changed legal or economic environment. This latter situation more closely resembles the C-3 Phase II RBC determination process. Economic testing of formula reserves has always focused on whether today's formula reserves are insufficient for future obligations. They have never tested to see if tomorrow's reserve will be sufficient when tomorrow arrives. For example, if cash flow testing indicates today's reserves are not adequate, it is very likely that next year's reserves could be even more "inadequate" in certain adverse scenarios, but the current framework does not require rerunning the test with strengthened future reserves to set up an even higher reserve today. While this is an interesting theoretical question, it is not an approach currently used in the US RBC or reserve framework.

10. Reserves

Since reserves for these benefits are primarily driven by economic events, the desire to "smooth" results through a formula reserve will only detract from the usefulness of the results. We feel it is most appropriate to set today's reserves at some CTE level below that required for capital, and recognize that the changes in reserves over time will mirror the impact of changes in the external economic environment. Trying to hold additional reserves today, for future reserve or capital changes, will lead to the inconsistency mentioned above with the rest of the RBC framework.

11. Risk-Based Capital

RBC is the 90 CTE value calculated above plus the starting value of liabilities (as defined for modeling) minus the reserve actually held.

12. C-1 Expense Allowance Elimination for Modeled Products

The current RBC formula has a charge for the expense allowance in reserves of 2.4 percent (pre-tax) if the surrender charges are based on fund contributions and the fund balance exceeds the sum of premium less withdrawals; otherwise the charge is 11 percent. This amount provides for the possible non-recovery of the full "CARVM Allowance", if the stock market performs poorly. Since this impact will be captured directly in the Phase II modeling, this separate requirement is no longer necessary for products covered by C-3, Phase II.

Alternative Method

A company may choose to develop capital requirements for Variable Annuity contracts with GMDBs, by using the tables of factors from Appendix 5 (not yet completed; specimen factors will also be included) of this report instead of using scenario testing if it hasn't used scenario testing for this purpose in previous years. Companies are encouraged to develop models to allow scenario testing for this purpose. Once this methodology is used for this purpose, the option to use Appendix 5 factors is no longer available. Living benefits must be evaluated by scenario testing.

Actuarial Memorandum

An actuarial memorandum should be constructed documenting the methodology and assumptions upon which the required capital is determined. The memorandum should also include sensitivity tests that the actuary feels appropriate, given the composition of their block of business (i.e., identifying the key assumptions, that is those that contribute most to the RBC amount and if changed have the largest effect on RBC for the product). This memorandum will be confidential and available to regulators upon request.

Regulatory Communication

If there is a material change in results due to a change in assumptions from the previous year, an executive summary should be sent to the state of domicile communicating such change and quantifying the impact it has on the results. Such communication shall remain confidential.

Appendix 1 – General Methodology

Market scenarios are run for the book of business (in aggregate) for all contracts falling under the scope of this requirement, reflecting product features, anticipated cashflows, the parameters associated with the funds being used, expenses, fees, Federal Income Tax, hedging, and reinsurance. Cash flows from any fixed account options should also be included.

For each scenario, the C-3 measure is the most negative of the series of present values $S(t)*pv(t)$, where $S(t)$ is statutory assets less liabilities for the products in question at the end of year t , and $pv(t)$ is the accumulated discount factor for t years using the after-tax swap rates (or post-tax one year Treasury rates for that scenario, if applicable).

Appendix 2 – Scenario Requirements

This appendix outlines the requirements for the stochastic models used to simulate fund performance. Specifically, it sets certain standards that must be satisfied and offers guidance to the actuary in the development and validation of the scenario models. Background material and analysis is presented to support the recommendation. In this regard, there is a particular focus on the S&P500 as a proxy for returns on a broadly diversified U.S. equity fund, but there is advice on how the techniques and requirements would apply to other types of funds. General modeling considerations such as the number of scenarios and projection frequency are also discussed.

General Guidelines

Actuarial Standard of Practice No. 7 (ASOP 7) applies to determination of capital adequacy¹. Any conflict between ASOP 7 and the statutory requirements should be disclosed in the documentation. Specifically, such disclosure should make it clear that the analysis was performed in accordance with the requirements of the applicable law.

The calibration points given in this appendix are applicable to gross returns. To determine net returns the actuary must consider the costs of managing the investments and converting the assets into cash when necessary². Specifically, the simulations must reflect applicable fees and policyholder charges in the development of projected account values.

As a general rule, funds with higher expected returns should have higher expected volatilities and in the absence of well documented mitigating factors (e.g., a highly reliable and favorable correlation to other fund returns), should lead to higher capital requirements³.

State dependent models are not prohibited, but must be justified by the historic data and meet the calibration criteria. To the degree that the model uses mean-reversion or path-dependent dynamics, this must be well supported by research and clearly documented in the disclosures.

The equity scenarios used to determine capital levels must be available in an electronic format to facilitate any regulatory review.

¹ Actuarial Standard of Practice No. 7, Adopted by the Actuarial Standards Board June 2002, section 1.2(b).

² Ibid., section 3.4.1(d)

³ While the model need not strictly adhere to 'mean-variance efficiency', prudence dictates some form of consistent risk/return relationship between the proxy investment funds. In general, it would be inappropriate to assume consistently 'superior' expected returns (i.e., risk/return point above the frontier) for long-term capital modeling.

Equity Markets and the Calibration Points

In general, there are two probability measures for simulating investment returns. The Q -measure, or “risk neutral” distribution, is used for pricing securities and is predicated on the concept of replication under a ‘no arbitrage’ environment. Under the Q -measure, all securities earn the risk-free rate and derivatives (options) can be priced using their expected discount payoffs. The Q -measure is crucial to option pricing, but equally important is the fact that it tells us almost nothing about the true probability distribution. The Q -measure is relevant only to pricing (“fair market value” determination) and replication (a fundamental concept in hedging); any attempt to project values (“true outcomes”) for a risky portfolio must be based on an appropriate (and unfortunately subjective) “real world” probability model. This is the so-called physical measure, or P -measure.

The “real world” model should be used for all cash flow projections, consistent with the risk preferences of the market. This is the basis for the valuation of required capital and is the focus of the remainder of this appendix. However, the risk neutral measure is relevant if the company’s risk management strategy involves the purchase of derivatives or other financial instruments in the capital markets.

Short period distributions of historic equity returns have negative skewness, significant kurtosis (fat tails)⁴ with time varying volatility⁵ and increased volatility in bear markets. The measure of kurtosis declines when looking at returns over longer time horizons and successive application of a short term model with finite higher moments will result in longer horizon returns that converge towards normality.⁶ Ideally the distribution of returns for a given model should reflect these characteristics. Of course, due to random sampling, not every scenario would show such characteristics.

Unfortunately, at longer time horizons the small sample sizes of the historic data make it much more difficult to make credible inferences about the characteristics of the return distribution, especially in the tails. As such, the calibration criteria are developed from a model (fitted to historic S&P500 data) and not based solely on empirical observations. Statistics for the observed data are offered as support for the recommendations.

The required constraints were established by fitting a model to monthly historic data and then using the model to generate gross wealth ratios for a range of probabilities over various holding periods. The model used was a regime-switching lognormal model with two regimes (RSLN2). This model is not prescribed or ‘preferred’ above others, but was chosen because it captures many of the dynamics noted above, including ‘volatility bunching’⁷.

⁴ Harry H. Panjer et al., *Financial Economics* (Illinois: The Actuarial Foundation, 1998): pp438

⁵ John Y. Campbell et al., *The Econometrics of Financial Markets*, (New Jersey: Princeton University Press, 1997): pp379

⁶ John Y. Campbell et al., *The Econometrics of Financial Markets*, (New Jersey: Princeton University Press, 1997): pp0

⁷ Mary R. Hardy, “A Regime-Switching Model of Long-Term Stock Returns,” *North American Actuarial Journal*, 2001, pp1–53.

The model parameters were determined by maximum likelihood estimation applied to monthly S&P500 total return data from January 1945 to October 2002 inclusive. This period is sufficiently long to capture several economic cycles and adverse events and was thereby deemed appropriate to the fitting of a model designed for long term cash flow projections. The six fitted parameters are provided in Table 1 below. All values are monthly.

Table 1: RSLN2 Monthly Parameters (fitted to S&P500TR data, Jan 1945 – Oct 2002) for Log Returns

Regime 1			Regime 2		
Mean	Standard Deviation	Probability of Switching to Regime 2	Mean	Standard Deviation	Probability of Switching to Regime 1
0.0135	0.0351	0.0409	-0.0157	0.0642	0.2341

In the above table, the means and standard deviations are for the associated normal distribution in each regime. For example, the average (‘conditional’ or ‘marginal’) monthly log total return in regime 1 is 1.35 percent or 16.2% annualized. The annualized marginal volatilities in regimes 1 and 2 are 12.2 percent and 22.2 percent respectively.

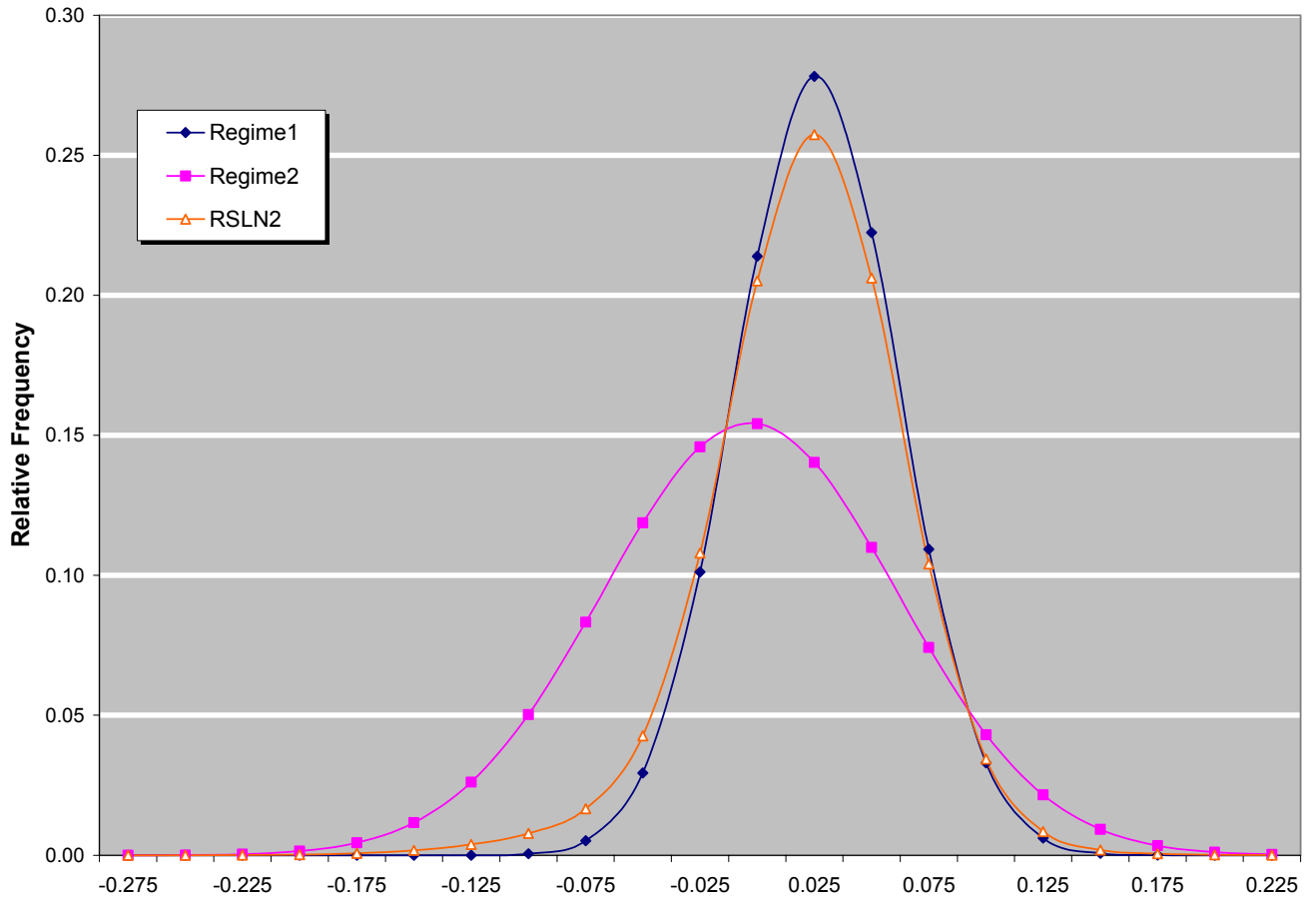
Notably, regime 2 has negative expected returns and a higher volatility that coincides well with the greater instability witnessed in adverse markets. Over the long term, the switching probabilities imply that the process spends approximately 85 percent of the time in regime 1 and the remainder in regime 2. These values are often referred to as the unconditional or ‘invariant’ probabilities.

The following chart shows the probability density function (i.e., relative frequency graph) for the monthly log total returns in regimes 1 and 2 as well as the unconditional density for the RSLN2 returns (i.e., the starting regime was randomized according to the invariant probabilities). The RSLN2 returns are not normally distributed, but exhibit negative skewness⁸ (-0.46) and positive kurtosis⁹ (1.45) characteristic of the historic data. For reference, the monthly observed data show a skewness of -0.58 and a kurtosis of 2.17.

⁸ Skewness measures symmetry about the mean. The normal distribution has a skewness of 0, indicating perfect symmetry.

⁹ Kurtosis is a measure of ‘peakedness’ relative to the tails of the distribution. By convention, the normal distribution has a kurtosis of zero, although some definitions give a kurtosis of 3. Except for this constant, the definitions are equivalent.

Probability Density Function for Monthly Log Returns



These parameters were used to generate 1, 5 and 10-year wealth factors at the calibration points. The ‘wealth factors’ are defined as gross accumulated value (i.e., before deduction of fee and charges) with complete reinvestment, starting with \$1. These can be less than 1, with “1” meaning a zero return over the holding period. The annualized means and standard deviations of these factors are shown in Table 2.

Table 2: Annualized Unconditional Mean and Standard Deviation for Gross Wealth Ratios

One Year		Five Year		Ten Year	
Mean	Std	Mean	Std	Mean	Std
1.1303	0.1755	1.8512	0.6702	3.4296	1.8168

The resulting calibration points are presented in Table 3.

Table 3: S&P500 Total Return Wealth Factors at the Calibration Points

Calibration Point	One Year	Five Year	Ten Year
0.5%	0.65	0.58	0.67
1.0%	0.70	0.66	0.79
2.5%	0.77	0.78	1.00
5.0%	0.84	0.91	1.21
10.0%	0.91	1.07	1.51
90.0%	1.35	2.73	5.79
95.0%	1.42	3.07	6.86
97.5%	1.48	3.39	7.94
99.0%	1.55	3.79	9.37
99.5%	1.60	4.10	10.48

To interpret the above values, consider the 5-year point of 0.66 at the $\alpha = 0.01$ percentile. This value implies that there is a 1 percent probability of the accumulated value of \$1 being less than \$0.66 in 5-years time, without knowing the initial state of the process. For left tail calibration points (i.e., those quantiles less than 50 percent), lower factors after calibration are acceptable. For right tail calibration points (quantiles above 50 percent), the model must produce higher factors.

For models that require starting values for certain state variables (such as the RSLN2 model which requires an assumption about the starting regime), long-term ('average' or 'neutral') values should be used for initialization. For example, the starting regime in the RSLN2 model could be 'randomized' according to the unconditional (invariant) probabilities of being in each regime.

The scenarios need not strictly satisfy all calibration points, but the actuary should be satisfied that any differences are not material to the resulting capital requirements. In particular, the actuary should be mindful of which tail most affects the business being valued. If capital is less dependent on the right (left) tail for all products under consideration (e.g., a return of premium guarantee would primarily depend on the left tail, an enhanced death benefit equal to a percentage of the gain would be most sensitive to the right tail, etc.), it is not necessary to meet the right (left) calibration points.

If the scenarios are 'close' to the calibration points, an acceptable method to true up the scenarios is to start with the lowest bucket not meeting the calibration criteria (e.g., one year factor at $\alpha = 0.5$ percent) and randomly duplicate (or re-generate) a scenario meeting this criteria until the set of scenarios meets this calibration point. If a fixed number of scenarios is required, a scenario can be eliminated at random in the first higher bucket that satisfies the calibration criteria. The process would continue until all one-year calibration points are achieved and repeated for the 5- and 10-year criteria. However, on completing the 5-year (or 10-year) 'buckets', it may be necessary to redo the 1-year (or 1- and 5-year) tests if those buckets no longer meet the calibration points. It is acknowledged that this method is not statistically correct, but it is not anticipated that the

process would introduce any material bias or distortion in the calculated capital requirements.

To analyze the reasonableness of the calibration table, it is worthwhile examining the historic data. The January 1945 to October 2002 monthly S&P500 total return data series (694 data points) allows for 693 non-overlapping end-of-month return observations; 57 non-overlapping observations of annual returns (56 if the starting month is November or December), 11 non-overlapping observations of five-year returns and only 5 non-overlapping observations of ten-year returns.

However, there are several non-overlapping series to choose from since we can select different starting points for the calculations. For example, there would be 10 sets of 57 non-overlapping returns for annual periods corresponding to different monthly starting points of January to January, . . . , October to October. The November and December ‘start months’ would give 2 sets of 56 annual return observations each. These “sets” are not independent, but provide slightly different empirical estimates of the underlying distributions.

Tables 4a through 4c (below) summarize the left and right-tail returns based on these empirical observations. To interpret Table 4a, the 1.72 percentile for the one-year return is based on the worst result of 57 independent observation periods of annual returns (56 for the November and December ‘starting months’), where $1.72 \text{ percent} = 1 \div (N + 1) = 1 \div 58^{10}$. The 3.45 percent result is based on the second worst result (i.e., $2 \div 58$). Because there are 10 possible starting points for the 57 years of non-overlapping returns and 2 sets of 56 annual returns, corresponding to the various starting months, the “empirical” range shows the minimum and maximum of the results from the possible non-overlapping series. For reference, the mid-point (average of minimum and maximum) value and certain calibration points are also included.

The ranges corresponding to the calibration criteria quantiles have been imputed (interpolated) from neighboring empirical values (the shaded rows). For the 5- and 10-year holding periods, the empirical quantiles (α -level) are provided as ranges.

¹⁰ Strictly, the lowest returns for the scenario sets with November and December ‘starting months’ only give estimates of the $1.75\% = 1 \div 57$ quantile. We have ignored this technicality in the calculations.

Table 4a: Non-overlapping 1-year Accumulation Factor S&P500TR Jan 1945 – Oct 2002

Quantile(α)	Empirical Range			Cal. Point
	Minimum	Maximum	Mid	
1.72%	0.611	0.889	0.750	
2.50%	0.666	0.890	0.778	0.77
3.45%	0.734	0.891	0.812	
5.00%	0.789	0.903	0.846	0.84
5.17%	0.795	0.905	0.850	
6.90%	0.822	0.909	0.866	
8.62%	0.867	0.917	0.892	
10.00%	0.872	0.925	0.898	0.91
10.34%	0.873	0.926	0.900	
89.66%	1.324	1.389	1.356	
90.00%	1.325	1.392	1.359	1.35
91.38%	1.331	1.405	1.368	
93.10%	1.335	1.411	1.373	
94.83%	1.353	1.474	1.413	
95.00%	1.354	1.479	1.416	1.42
96.55%	1.362	1.521	1.442	
97.50%	1.380	1.570	1.475	1.48
98.28%	1.394	1.610	1.502	

Table 4b: Non-overlapping 5-year Accumulation Factor S&P500TR Jan 1945 – Oct 2002

Percentile	Empirical Range			Cal. Point
	Minimum	Maximum	Mid	
8.33% – 9.09%	0.809	1.314	1.061	
10.00%	0.855	1.356	1.105	1.07
16.67% – 18.18%	1.037	1.527	1.282	
81.82% – 83.33%	2.030	2.936	2.483	
90.00%	2.385	3.526	2.955	2.73
90.91% – 91.67%	2.473	3.673	3.073	

Table 4c: Non-overlapping 10-year Accumulation Factor S&P500TR Jan 1945 – Oct 2002

Percentile	Empirical Range		
	Minimum	Maximum	Mid
16.67%	1.230	2.195	1.713
83.33%	4.217	6.969	5.593

The calibration table can also be compared to statistics for the longer S&P500 (U.S. Equity market) time series October 1902 to October 2002 inclusive. This dataset (1201 monthly data points) permits 1200 non-overlapping end-of-month return observations; 100 non-overlapping observations of annual returns, 20 non-overlapping observations of five-year returns and 10 non-overlapping observations of ten-year returns.

Tables 5a through 5c summarize the left and right-tail returns based on the empirical observations for this time series. The format and interpretation of these values follows directly from Tables 4a through 4c presented earlier. For reference, the mid-point (average of minimum and maximum) value and key calibration points are shown.

Table 5a: Non-overlapping 1-year Accumulation Factor S&P500TR Oct 1902– Oct 2002

Quantile(α)	Empirical Range			Cal. Point
	Minimum	Maximum	Mid	
1.0%	0.324	0.659	0.492	0.70
2.0%	0.508	0.764	0.636	
2.5%	0.560	0.767	0.663	0.77
3.0%	0.612	0.770	0.691	
4.0%	0.714	0.813	0.764	
5.0%	0.733	0.825	0.779	0.84
10.0%	0.848	0.894	0.871	0.91
90.0%	1.348	1.391	1.370	1.35
95.0%	1.389	1.489	1.439	1.42
96.0%	1.390	1.528	1.459	
97.0%	1.416	1.575	1.496	
97.5%	1.424	1.701	1.563	1.48
98.0%	1.432	1.827	1.629	
99.0%	1.457	2.597	2.027	1.55

Table 5b: Non-overlapping 5-year Accumulation Factor S&P500TR Oct 1902 – Oct 2002

Percentile	Empirical Range			Cal. Point
	Minimum	Maximum	Mid	
5.0%	0.375	1.010	0.692	0.91
10.0%	0.637	1.223	0.930	1.07
90.0%	1.704	2.588	2.146	2.73
95.0%	2.475	4.490	3.483	3.07

Table 5c: Non-overlapping 10-year Accumulation Factor S&P500TR Oct 1902 – Oct 2002

Percentile	Empirical Range			Cal. Point
	Minimum	Maximum	Mid	
10.0%	0.582	1.800	1.191	1.51
90.0%	4.074	6.912	5.493	5.79

While it may be argued that the pre-WW2 data is not relevant to current and future market conditions due to fundamental changes in the economy, the above statistics clearly suggest that the calibration points are not unduly conservative or aggressive when examining the empirical data over very long timeframes.

Using the Calibration Points

The actuary may need to adjust the model parameters in order to satisfy the calibration criteria in Table 3. This can be accomplished in a variety of ways, but a straightforward approach would modify the parameters controlling ‘drift’ (expected continuous return) and ‘volatility’ (standard deviation of returns). This might be accomplished analytically, but in most practical applications would require simulation.

As a first step, the actuary should determine which tail (left, right or both) is most relevant for the business being valued and then identify those calibration points not satisfied by the current scenario set. All else being equal, lowering drift will decrease the resulting wealth factors, while raising volatility will decrease the left-tail factors (i.e., those quantiles < 50 percent) and increase the right. Changes to both drift and volatility can obviously affect the entire shape of the curve, but as a general rule ‘drift’ has less impact over the shorter holding periods (i.e., the 1-year ‘tail factors’ are more affected by volatility).

As an example, suppose the company is using the independent lognormal (“ILN”) model for equity returns. This is a two-parameter model whereby the log returns are normally distributed with constant mean μ and variance σ^2 . From the historic monthly S&P500TR data (January 1945 to October 2002, inclusive) we obtain the monthly maximum likelihood estimators of $\mu = 0.0092$ (11.05 percent annualized) and $\sigma = 0.042$ (14.56 percent annualized)¹¹.

Without adjustment, ILN scenarios generated from these parameters would not satisfy the calibration requirements. Nevertheless, lowering the drift to $\mu = 0.0077$ (9.2 percent annualized) and increasing the standard deviation to $\sigma = 0.0534$ (18.5 percent annualized) would materially satisfy Table 3. However, the resulting wealth factors would be extremely fat-tailed over the longer holding periods, indicating more conservatism than would strictly be necessary. As such, it should be clear that a two-parameter model (such as the independent lognormal) does not offer much flexibility – to obtain a ‘better fit’, it would be necessary to introduce more parameters¹².

Other Markets/Funds

The calibration of other markets (funds) is being left to the judgement of the actuary, but the scenarios so generated must be consistent with the calibration points in Table 3. This does not imply a strict functional relationship between the model parameters for various markets/funds, but it would generally be inappropriate to assume that a market or fund consistently ‘outperforms’ (lower risk, higher expected return relative to the efficient frontier) over the long term.

The actuary should document the actual 1-, 5- and 10-year wealth factors of the scenarios at the frequencies given in Table 3. The annualized mean and standard deviation of the wealth factors for the 1-, 5- and 10-year holding periods must also be provided. For equity funds, the actuary should explain the reasonableness of any significant differences from the S&P500 calibration points.

¹¹ Based on the MLEs for the independent lognormal model, the expected return on the index is 12.87% annual effective.

¹² In particular, parameters are needed to model time-varying volatility.

When parameters are fit to historic data without consideration of the economic setting in which the historic data emerged, the market price of risk may not be consistent with a reasonable long-term model of market equilibrium. One possibility for establishing ‘consistent’ parameters (or scenarios) across all funds would be to assume that the market price of risk is constant (or nearly constant) and governed by a linear relationship. That is, higher expected returns can only be garnered by assuming greater risk. Here, we use the standard deviation of log returns as the risk measure.

Specifically, two return distributions X and Y would satisfy the following relationship:

$$\text{Market Price of Risk} = \left(\frac{\mu_X - r}{\sigma_X} \right) = \left(\frac{\mu_Y - r}{\sigma_Y} \right)$$

where μ and σ are respectively the (unconditional or long-run) expected returns and volatilities and r is the expected risk-free rate over a suitably long holding period commensurate with the projection horizon. One approach to establish consistent scenarios would set the model parameters to maintain a near-constant market price of risk.

A closely related method would assume some form of ‘mean-variance’ efficiency to establish consistent model parameters. Using the historic data, the mean-variance (alternatively, ‘drift-volatility’) frontier could be constructed from a plot of (mean, variance) pairs from a collection of world market indices. The frontier could be assumed to follow some functional form¹³, with the co-efficients determined by standard curve fitting or regression techniques. Recognizing the uncertainty in the data, a ‘corridor’ could be established for the frontier. Model parameters would then be adjusted to move the proxy market (fund) inside the corridor.

Clearly, there are many other techniques that could be used to establishing consistency between the scenarios. While appealing, the above approaches do have drawbacks¹⁴ and the actuary should be careful not to be overly optimistic in constructing the model parameters or the scenarios.

Funds can be grouped and projected as a single fund if such grouping is not anticipated to materially reduce capital requirements. However, care should be taken to avoid exaggerating the benefits of diversification. The actuary must document the development of the investment return scenarios and be able to justify the mapping of the company’s variable accounts to the proxy funds used in the modelling.

¹³ Quadratic polynomials and logarithmic functions tend to work well.

¹⁴ For example, mean-variance measures ignore the asymmetric and fat-tailed profile of most equity market returns.

Discount Rates

For discounting future capital strain, the Federal Income Tax adjusted swap curve rates may be used. Alternatively, an economic model built into the scenario generator may be used to simulate 1-year Treasury rates. In the latter case, the rates must start at current levels, approximately satisfy the ‘no arbitrage’ principle (on an expected basis) and exhibit deviations from expected values generally consistent with the Phase I interest model. In addition, if interest rates are not assumed to be independent of the equity scenarios, the basis for the assumed relationship needs to be well documented.

Correlation of Fund Returns

In constructing the scenarios for the proxy funds, the company may require parameter estimates for a number of different market indices. When more than one index is projected, it is generally necessary to allow for correlations in the simulations. It is not necessary to assume that all markets are perfectly positively correlated, but an assumption of independence (zero correlation) between the equity markets would inappropriately exaggerate the benefits of diversification¹⁵. An examination of the historic data suggests that correlations are not stationary and that they tend to increase during times of high volatility or negative returns. As such, the actuary should take care not to underestimate the correlations in those scenarios used for the capital calculations.

If the projections include the simulation of interest rates (other than for discounting surplus strain) as well as equity returns, the processes may be independent provided that the actuary can demonstrate that this assumption (i.e., zero correlation) does not materially underestimate the resulting capital.

Random Number Generator

A good pseudo-random number generator provides a set of values that are statistically indistinguishable from a truly random sequence from the given distribution. There are many algorithms for generating pseudo-random numbers, but the quality varies widely between them. The user should not indiscriminately deploy a generator without first confirming (through statistical testing) that it performs adequately under the conditions for which it will be used. In particular, the generator should have sufficiently high periodicity¹⁶ and not exhibit material bias or serial correlation¹⁷.

Many stochastic simulations require the “mapping” of generated $U(0,1)$ values to the real line $(-\infty, +\infty)$ in order to obtain random samples from the Normal distribution. Such mapping can be accomplished by a variety of methods, but some routines are much more

¹⁵ If the models assume zero correlation, then the business must be valued on a ‘fund-by-fund’ basis.

¹⁶ Periodicity is defined as the number of values that can be produced by the generator before the sequence repeats itself.

¹⁷ Serial correlation of lag k occurs when values separated by k numbers exhibit significant correlation.

robust than others. In particular, the actuary should ensure that the mapping is “continuous” and 1-to-1 (within the precision of the computer)¹⁸.

Number of Scenarios

For straight Monte Carlo simulation (with equally probable “paths” of fund returns), the number of scenarios should typically equal or exceed 1000. The appropriate number will depend on how the scenarios will be used and the materiality of the results. The actuary should ensure that the number of scenarios used provides an acceptable level of precision.

Fewer than 1000 scenarios may be used provided that the actuary has determined through prior testing (perhaps on a subset of the portfolio) that the CTE results materially reproduce the values obtained from running a larger scenario set.

Suppose the number of scenarios used for simulation is N . Hence, the CTE estimator at the α -confidence level is the average of the $k = N \times (1 - \alpha)$ order statistics (i.e., sample results ordered from highest to lowest). The standard error of the estimator is a function of α , $\text{CTE}(\alpha)$ and the $(k+1)$ order statistics divided by the square root of k . So, to increase the precision of the calculations, it may be necessary to significantly increase the number of scenarios.

Variance reduction techniques are intended to improve the accuracy of an estimate more efficiently than simply increasing the number of simulations. Variance reduction can be used provided the actuary can demonstrate that they improve the results. Many of the techniques are specifically designed for estimating means, not tail measures, and could in fact reduce accuracy (and efficiency) relative to straight Monte Carlo simulation¹⁹.

A practical ‘brute force’ approach to estimating the standard error of the CTE estimator would be to generate M independent sets of scenarios (using the same model parameters) where each set contains N scenario paths. Provided M is reasonably ‘large’ (say, $M \geq 10$), a sample variance may be calculated on the M different $\text{CTE}(\alpha)$ values. A rough $(100 \times \beta)$ percent confidence interval for $\text{CTE}(\alpha)$ may then be constructed using the normal approximation: $\text{CTE}(\alpha) \pm \sigma \cdot \Phi^{-1}[0.5 \times (1 + \beta)]$ where σ is the sample standard deviation of the $\text{CTE}(\alpha)$ estimators and Φ^{-1} is the inverse cumulative density function for the standard normal distribution. If the interval appears ‘too wide’ (e.g., interval width $> 10\%$ of the CTE estimate), more scenarios may be required.

Frequency of projection and time horizon

Use of an annual projection frequency (“timestep”) is generally acceptable for benefits/features that are not sensitive to projection frequency. The lack of sensitivity to projection frequency should be validated by testing wherein the actuary should ensure that the use of a more frequent (i.e., shorter) time step does not materially increase capital

¹⁸ Small deviations in the $U(0,1)$ sample should be associated with appropriately small deviations in the Normal values.

¹⁹ However, with careful implementation, many variance reduction techniques can work well for CTE estimators.

requirements. A more frequent time increment should always be used when the product features are sensitive to projection period frequency.

Care must be taken in simulating fee income and expenses when using an annual time step. For example, recognizing fee income at the end of each period after market movements, but prior to persistency decrements, would normally be an inappropriate assumption.

It is important that the projection frequency of the investment return model be linked appropriately to the projection horizon in the liability model. In particular, the horizon should be sufficiently long so as to capture the vast majority of surplus costs (on a present value basis) from the scenarios²⁰.

Pre-Packaged Scenarios

The Academy has developed 10,000 scenarios for the following 6 asset classes.

1. Money Market
2. Bond
3. S&P500
4. Diversified international equity
5. U.S. small capitalization
6. Aggressive or specialized equity

The scenarios are available as gross monthly accumulation factors over a 40-year horizon (i.e., a 10000×480 matrix for each asset class) in comma-separated value format (*.csv). These scenarios have been appropriately correlated so that the K^{th} scenario for each asset class should be used together and considered one ‘future investment return scenario’. Hence, the scenarios can be combined (by blending the accumulation factors²¹) to create additional ‘proxy’ scenarios for the company’s funds.

For example, suppose the actuary wanted to construct scenarios for a ‘balanced fund’ which targets a 60/40 allocation between bonds and U.S. equities. If we denote $[AF^X]$ as the matrix of accumulation factors for asset class X , then the balanced scenarios would be defined by $[AF^{BAL}] = 0.60 \times [AF^{BOND}] + 0.40 \times [AF^{S\&P500}]$. Care should be taken to avoid exaggerating the benefits of diversification. The actuary must document the development of the investment return scenarios and be able to justify the mapping of the company’s variable accounts to the proxy funds used in the modelling.

²⁰ As a general guide, the forecast horizon should not be less than 20 years.

²¹ It is important to blend the accumulation factors (not the returns) in order to achieve the desired asset mix.

Appendix 3—GMIB Purchase Rate Margins

Although we would “expect” the interest rate n years hence to be that implied for an appropriate duration asset by the forward swap curve, there is a steadily widening confidence interval about that point estimate with increasing time until “maturity”. The “expected margin” in the purchase rate is less than that produced by the point estimate, since a greater proportion of contract-holders will have a CARVM reserve in excess of cash surrender value when margins are low than when margins are high. As a practical matter, this effect can be approximated by using a purchase rate margin based on an earnings rate .30 percent below that implied by the forward swap curve. If the interest rates paths used in the stochastic modeling already reflect the impact of this uncertainty, then no such approximation is needed.

Methodology Note C3-01 – Fund Categorization for Variable Accounts

An appropriate proxy for each variable account must be designed in order to develop the investment return paths. This does not mean that unique scenarios need to be developed for each individual variable fund. In most applications, this would be impractical and therefore, some grouping will be necessary. However, the development of the proxy scenarios is a fundamental step in the modelling and can have a significant impact on results. As such, the actuary must carefully and deliberately map each variable account to an appropriately crafted proxy fund. As noted, this ‘mapping’ is typically not ‘one-to-one’, but ‘many-to-several’.

It would rarely be appropriate to estimate the stochastic model parameters (for the proxy) directly from actual company data. Rather, the proxy would normally be expressed as a linear combination of recognized market indices (or sub-indices). This approach has several distinct advantages:

- A small number of well-developed data series can be used to model a wide range of funds.
- It promotes consistency in practice.
- Historic data is generally available over long time periods. This enhances the reliability of any model parameters estimated from the empirical data.

The proxy construction process should include an analysis that establishes a firm relationship between the investment return proxy and the specific variable funds. Such an analysis can include, but would not be limited to the following:

- Portfolio objectives
- MorningStar classification
- Asset composition
- Historical returns
- Performance benchmark
- Market beta
- AG 34 classifications

When sufficient fund performance information exists, the analysis should examine the relationship of these data to the market/sector indices. Due to shifts in investment objectives, fund manager changes and tactical allocation (e.g., market timing), this comparison may not be straightforward, but would ideally include a study of serial correlations, tracking error and asset composition.

If credible historical fund data is not available, the proxy should be constructed by combining asset classes and/or employing allocation rules that most closely reflect the expected long-term composition of the specific fund given the investment objectives and management strategy. The relevant historic market data can then be used to estimate parameters. If sufficient historical market (or sub-sector) data does not exist, the return-generating process should reflect the contribution of each component by reference to some ‘efficient markets’ hypothesis.

However defined, efficient market theory generally posits that higher expected returns can only be attained by assuming greater risk. While the historic market data does not indicate a clearly defined ‘risk-return’ relationship, it would be imprudent to ignore the concept of market efficiency in establishing the proxy funds and the associated model parameters used to generate the investment return scenarios.

Methodology Note C3-02 – Interim Reserves

For the purposes of calculating capital requirements, interim reserves are used to determine surplus strains, using the greater of the cash surrender value or the reserve value of any option available at that point in time.

Methodology Note C3-03 – Model Building for In-Force Liabilities

When determining RBC requirements the actuary must comply with ASOP 7 and 23.

For large blocks of business, the actuary may employ grouping methods to in-force seriatim data in order to improve model run times. Care needs to be exercised when aggregating data for RBC purposes. Grouping methods must retain the characteristics needed to model all material risks and options embedded in the liabilities. RBC needs to cover “tail scenarios” and these are impacted by low probability, high impact scenarios. This may require more granularity (i.e., model points) in the grouping of data than what is needed for other purposes.

Actuaries may want to consider the following when grouping data;

- Various breakpoints for “in-the-moneyness”.
- Grouping funds that have similar risk/return characteristics.
- Product variations (e.g., various types of living and/or death benefit options).
- Annuitant and/or owner age.
- Duration of contract.
- Market
- Distribution channel.
- Other factors which could significantly impact the results.

It is important that adequate testing be done to validate models on both a static and dynamic basis. The model used must fit the purpose. The input data, assumptions, and formulas/calculations should all be validated. Peer review is recommended.

Methodology Note C3-04 - Policyholder Behavior

Policyholder behavior assumptions encompass actions such as lapses, withdrawals, transfers, premium suspensions, etc. Policyholder behavior is difficult to accurately predict and behavior assumptions can significantly impact the results. In the absence of empirical data, the actuary should set behavior assumptions on the conservative side for purposes of setting RBC.

In setting behavior assumptions, the actuary may want to consider the following;

1. Behavior can vary by product, market, distribution channel, fund performance, time/product duration, etc...
2. Options embedded in the product may impact behavior.
3. Options may be elective or non-elective in nature. Living benefits are often elective and death benefit options are generally non-elective.
4. Elective policyholder options may be more driven by economic conditions than non-elective options.
5. As the “value” of a product option increases the likelihood of policyholder behavior anti-selecting against the insurer increases.
6. Behavior formulas may have both a rational and irrational component. The rational component should be dynamic.
7. Options that are ancillary to the primary product features may not be significant drivers of behavior. Whether an option is ancillary to the primary product features depends on many things such as:
 - For what purpose was the product purchased.
 - Is the option elective or non-elective?
 - Is the value of the option well known?

The impact of behavior can vary by product, time period, etc. Sensitivity testing of assumptions is recommended.

Appendix 4 – CIA Report on Segregated Fund Investment Guarantees

This report has been reprinted with permission from the Canadian Institute of Actuaries and (due to size constraints) is included as a separate attachment.



REPORT

**CIA TASK FORCE ON SEGREGATED FUND
INVESTMENT GUARANTEES**

December 2001

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Document 20020

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MEMORANDUM

TO: All Fellows and Students of the CIA
FROM: Murray Taylor, Chairperson
Task Force on Segregated Fund Investment Guarantees
DATE: December 1, 2001
SUBJECT: **Final Report of the CIA Task Force on Segregated Fund Investment Guarantees**

This December 31, 2001 report replaces an earlier report dated August 1, 2000. The primary changes include:

- Re-organized material and minor changes for readability. Terminology now conforms to documents produced by OSFI and IGIF.
- Updated Task Force Work completed in 2001 and the inclusion of additional sections.
- Comments on stochastic modelling for minimum capital.
- The Appendices have been reordered and the following added:
 - A. Recommend Reading/Resources
 - B. Calibration Dataset: TSE 300 Total Return Monthly Data Jan 1956 - Dec 1999
 - F. Dec 2001 MCCSR Guideline (Sec. 9): Segregated Fund Guarantee Risk
 - G. Dec 2001 MCCSR Guideline: Documentation of Factor-Based Methodology
 - H. Aug 2001 Guidance Note: Capital Offset for Segregated Fund Hedging Programs
 - I. Guidance for the 2001 Valuation of Life Policy Liabilities: CIA CLIFR
 - J. Letter to CIA with Concluding Remarks

This report is a research paper and does not constitute a CIA Standard of Practice, although in the absence of definitive standards or guidance to the contrary, the recommendations in this report could be regarded as “best practices”.

MT

REPORT OF THE CIA TASK FORCE ON SEGREGATED FUND INVESTMENT GUARANTEES

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1. OVERVIEW

The CIA Task Force on Segregated Fund Investment Guarantees began in the fall of 1999 following numerous activities within the CIA to raise the awareness of the unique and significant risks associated with segregated fund type investment performance guarantees.

A number of research papers, presentations at CIA meetings, and practitioner research culminated in the Symposium on Segregated Funds held in September 1999. Twenty-seven papers were presented over two days to more than 300 delegates attending from all over North America.

The Task Force was initiated after this symposium.

1.1 Members of the Task Force

The members of the Task Force are listed below for your reference.

Boris Brizeli	Martin le Roux
Barry Cotnam	Christian-Marc Panneton
Simon Curtis (chairperson until June 2000)	Steven Prince
David Gilliland	G. Daniel Cooper*/Sheldon Selby (OSFI liaisons)
Stephen Haist	Louis-Marie Houde (IGIF liaison).
Geoffrey Hancock (vice-chair)	Murray Taylor (chairperson)
Mary Hardy (University of Waterloo)	

* Danny was replaced by Sheldon early in 2000 due to reassigned duties at OSFI.

Each Task Force member other than Mary Hardy can be reached at his *Yearbook* address.

Each person on the Task Force has brought a unique strength to our work based on his/her knowledge and experience in relation to the risks of segregated fund guarantees. As well, the Task Force membership represents such important areas as: academia, business, professional, and regulatory experience.

1.2 Scope of the Task Force

The Task Force was charged with developing recommended approaches for the use of stochastic techniques to measure the obligations created by segregated fund or similar fund type (e.g., mutual fund) investment guarantees. The Task Force was then to negotiate implementation of these standards with the relevant practice committees. The types of guarantees being reviewed include minimum maturity benefits, death benefits, income benefits and any other benefits where an underlying level of investment performance is guaranteed by an insurer for segregated funds or other similar assets (e.g. variable insurance and annuity contracts with guaranteed minimum benefits).

In early discussions, the Task Force reached a strong consensus that the nature of the risk associated with many of these guarantees (generally low frequency, but potentially high severity losses) meant that overall capital adequacy and consistency of practice in establishing policy liabilities and capital requirements were fundamental issues. As such, we agreed that the Task Force would extend its mandate beyond the setting of policy liabilities to include a suitable focus on minimum capital (i.e., MCCR) and capital resiliency testing (i.e., DCAT).

Having reached this consensus, the work of the Task Force immediately became influenced by the need to respond to the MCCR capital formula proposed by the Office of the Superintendent of Financial Institutions (OSFI) for segregated fund guarantees in December 1999 for implementation in December 2000. OSFI and IGIF indicated that in order for the Task Force to have meaningful input into the debate on appropriate capital requirements, it would need to have firm practical proposals that could be considered by mid-year 2000.

In response, the Task Force accelerated its work in 2000 to produce the first version of this report (August 2000) which provided a framework for all balance sheet provisions for this risk (minimum capital, policy liabilities and DCAT resiliency testing) Note: Draft versions of the August 2000 report were also made available to CIA membership, regulators and other interested parties in May and June of 2000.

In 2001, the Task Force focused on refinement and documentation of factor-based capital; the development of principles for minimum capital using stochastic modelling; and expanded educational initiatives on the stochastic modelling techniques for these risks.

In addition to the work of the Task Force, we recommend the CIA Working Group Report “The Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP” (August 2001) as a valuable reference.

1.3. Principles for Balance Sheet Management

Overview of General Framework

The Task Force based our work on the following three key principles:

- a) Stochastic modelling techniques are superior to other techniques for measuring the risks/costs of segregated fund type investment guarantees.
- b) Consistency of actuarial practice is important; and,
- c) Policy liability and minimum capital requirements should be considered together based on the concentration of the risk in the tail of the net cost distribution.

Based on our endorsement of stochastic techniques as the superior way to quantify and measure guarantee risk exposure, the “ideal theoretical” framework would involve moving to stochastic methods immediately to determine both minimum capital and policy liability requirements.

However, because the use of stochastic techniques to establish policy liabilities and/or minimum capital within the life insurance industry is still in its developmental stages, we believe that to move to a pure stochastic basis to determine total balance sheet coverage of these risks is not practical nor desirable at this time if we are to maintain public confidence and consistency of practice.

The scope (Section 1.2) of our Task Force’s work acknowledges the need to move gradually to a more fully stochastic, integrated assessment of these risks for the combination of policy liabilities and minimum capital.

We believe this approach satisfies the principles we have established and provides a framework that can be supported by all constituents. The departure from an independent policy liability/capital approach is certainly contemplated in recent comments expressed by OSFI (supported by IGIF) and is appropriate for this type of risk.

The use of a factor-driven approach to establish minimum total balance sheet provisions, where the factors themselves are stochastically determined and reflect all major risk drivers, directionally supports the move to stochastic techniques. However, it tempers this move to stochastic techniques with the need to maintain confidence that there will be consistency in application between companies and appropriate overall balance sheet coverage.

A detailed discussion on each of minimum capital, policy liabilities and DCAT resiliency testing appears in various sections of this report.

1.4 Key Recommendations and Conclusions

The following recommendations of the Task Force are expanded upon in the report: Effective for year-end 2000, that practitioners be encouraged to use stochastic investment modelling techniques on the basis outlined in this paper to establish policy liabilities for segregated fund investment guarantees. To ensure that the range of actuarial practice is sufficiently narrow, actuaries should be required to calibrate the underlying models to specified target measures and values, and to follow specific written guidance on assumptions and methodology. To move actuarial practice rapidly to stochastic techniques, practical education on how to apply these techniques should be made available to practitioners. The Task Force does not endorse a deterministic framework as an appropriate approach to establishing policy liabilities.

- Effective for year-end 2000, we recommended that regulators introduce a factor driven approach for the minimum capital for these risks that reflects the product types and current risk profile of a company's inforce business. The recommended factors are derived from extensive stochastic analysis based on a number of models currently being used by practitioners in Canada. The factors would be used to establish minimum overall balance sheet requirements, with minimum capital equal to the difference between this amount and the policy liabilities. While the Task Force strongly supported stochastic techniques, we believed that use of a more prescribed basis to determine a minimum level of total balance sheet coverage for 2000 and 2001 was appropriate while these techniques become more refined.
- Effective for year-end 2001, we recommend refinement to the factors/formulas used in 2000 based on feedback and experience at year-end 2000.
- Effective for year-end 2002, a framework for minimum capital be established using stochastic techniques. The stochastic approach would logically build on the stochastic policy liability calculations and would be introduced on a basis that did not endanger the integrity of the overall balance sheet coverage.
- More emphasis be placed on the role of DCAT to test specific catastrophic deterministic scenarios, such as the exposure of the company to a "Japan" type equity correction, rather than having such deterministic testing be embedded directly in policy liability and minimum capital calculations.
- Opportunities for the education of actuaries and other interested parties in stochastic techniques continue to be developed as they apply to segregated fund type risks. This focus will extend beyond CIA activities to possibly include other actuarial audiences, academic liaisons, and other professional or regulatory bodies.

1.5 Future Responsibilities for Development

The Task Force work and mandate is completed as of December 2001. However, continued development of theory and practice in the modelling of segregated fund investment guarantees is encouraged. A formal handoff of responsibilities has occurred in the following areas:

- *Education Issues:* to the CIA Committee on Investment Practice
- *Liability Issues:* to the CIA Committee on Life Insurance Financial Reporting (CLIFR)
- *Capital Issues:* to the CIA Committee on Risk Management and Capital Requirements and the
CIA Committee on the Appointed/Valuation Actuary

See Appendix J for the letter written on behalf of the Task Force to the CIA indicating potential effort needed in the future.

2. MODELLING

Policy liabilities for segregated funds, as for other policy liabilities, should be based on an explicit prospective analysis of asset and liability cash flows. Because of the uncertainty of the underlying investment returns on which the liability costs and revenues are based, a stochastic approach is required to estimate these values.

Due to the complex nature and diversity of the segregated fund guarantee, there are generally no closed form solutions available to calculate the policy liabilities according to real-world (i.e., not risk neutral) probabilities. A more flexible and pragmatic approach to calculating policy liabilities is first to use stochastic simulation to generate multiple paths of investment returns based on a selected investment return model, and then to evaluate the liability costs and revenues using the generated path set. Each path is also commonly referred to as an investment scenario or a “scenario”.

In the pages that follow we have separate sections that deal with the investment return models and liability cash flow models, as well as a separate discussion on the modelling of any hedging of these features.

2.1 Investment Return Models

A key component of the stochastic modelling of the future costs associated with segregated fund investment guarantees is the model used to determine investment return paths.

The Task Force does not support mandating specific models for establishing investment return paths. We can find no precedent for mandating specific models and we believe that such an approach would risk failure because of resistance from the membership. Instead, the Task Force believes that a framework which requires mandatory calibration of equity based models to specified criteria plus specific guidance/prescription that addresses certain model building items (including assumptions) can acceptably narrow the range of practice and ensure appropriate policy liabilities.

Specifically:

- Guidance is given to narrow the range of practice on setting investment assumptions.
- Investment scenario models used for the generation of equity returns will need to produce investment path results that calibrate to certain statistical criteria that measure items such as dispersion of paths and thickness of the distribution’s tail.

2.1.1 Key Considerations in Selection/Development of Investment Return Models

There are a large number of investment return models and no single model can currently be identified as superior to all others. Due to the large amount of ongoing research in actuarial science, finance, econometrics, statistics and mathematics, stochastic modelling is constantly evolving. Also, due to the increasing power of computers, models that were once considered too complex to be practical can now be used. This evolution will surely continue in the future.

Notwithstanding this diversity of models, there are some requirements that need to be met in the context of using stochastic models to calculate the policy liabilities and minimum capital.

a) *Random Number Generator*

The random numbers generated by computer algorithms are called pseudo-random because they are not truly random. Knowing the algorithm and the seed to the sequence is sufficient to predict the next random number that will be generated. A good pseudo-random number

generator provides a sequence that is statistically indistinguishable from a truly random sequence from the given distribution.

Before using random sequences for stochastic simulation, the generator should be validated to ensure it performs adequately. That is, it should produce a sequence that follows the assumed distribution within reasonable sampling variability. This can be verified by statistical testing.

The generator should have sufficiently high periodicity where “periodicity” is defined as the number of values that can be produced before the sequence repeats itself. The periodicity should be substantially higher than the number of distinct random numbers required for the projection.

Results from stochastic modelling should be reproducible. This would ordinarily be accomplished by using a repeatable pseudo-random number generator and priming the algorithm with a known “seed” value. Note that the periodicity of the generator will vary for different seeds. Furthermore, some commercial software applications include built-in pseudo-random number generators with very low periodicities for certain seeds.

Variance reduction techniques can be used provided it can be demonstrated that they improve the results. It should be noted that most variance reduction techniques are designed to improve efficiency of an estimate of the mean. Where the objective is a measure of the risk arising from one tail of a distribution, some methods may in fact reduce efficiency relative to straight Monte Carlo simulation.

Many stochastic simulations require random samples from the *Normal* distribution. If the random number generator produces values from the standard *Uniform* distribution, then the Inverse Normal function is required to “map” (0,1) values to the real line $(-\infty, +\infty)$. Such mapping can be accomplished by table lookups, polynomial evaluation and a variety of other methods (e.g., Box-Mueller). Some of these routines are much more robust than others. It is important to ensure that the mapping is “continuous” (within the limitations of the computer) and that small deviations in the $U(0,1)$ sample are associated with appropriately small deviations in the resulting *Normal* values. Special treatment may be required for $U(0,1)$ samples that are indistinguishable from 0 or 1 due to internal machine representation.

The generation of pseudo-random numbers should be at the highest precision capable by the machine (e.g., double precision).

b) *Number of Scenarios*

To offer some guidance as to the number of scenarios that need to be generated, recall that the standard error of the result can be expressed as a function of the square root of the number of observations. To increase the precision of the policy liability calculation, it may be necessary to significantly increase the number of scenarios.

The number of scenarios should be at least 1,000. The appropriate number will depend on how the scenarios will be used (e.g., calculating percentiles will generally require more scenarios than calculating expected values), and the materiality of the results. The actuary should test that the number of scenarios used provides an acceptable level of precision.

c) *Frequency*

Use of an annual projection frequency is generally acceptable for benefits/features that are not sensitive to projection frequency. The lack of sensitivity to projection frequency should be validated by testing.

A more frequent time increment should always be used when the product features are sensitive to projection period frequency (e.g., many older age death benefits, most reset options, etc.).

It is important that the projection frequency of the investment return model be linked appropriately to the projection period in the liability model.

Care must be taken in simulating the fee income as a percentage of the segregated fund to avoid any over-appraisal of revenue.. For example, recognising fee income at the end of each period after market movements, but prior to persistency decrements, would normally be an inappropriate assumption.

d) *Model*

Investment returns would normally be generated on a gross basis: before the application of any fees or consideration of specific product features. The objective is to model the asset returns independently of any product features. However, care must also be taken to assess if total returns (including reinvestment of income) or price returns are required for the specific segregated funds that will be modelled.

There are a large number of potential models available and we do not want to restrict the use of any model that reasonably fits the historical data. The calibration criteria are defined below.

The model should be based on a *P*-measure (real world experience based valuation) as opposed to a *Q*-measure (risk-neutral capital markets valuation).

The *P*-measure approach produces a distribution of outcomes based on a real world view of outcomes for the actual assets/liabilities on the balance sheet. It generally uses historic returns to model future investment paths. It is, therefore, consistent with the overall Canadian valuation approach that values liabilities in the context of the cash flow outcomes on the assets currently held and anticipated to be held in the future.

The *Q*-measure is appropriate for the valuation of the liabilities where a market value or hedging cost is required, using the hedging principles of financial engineering. In this case, an expected value of the liability under the *Q*-measure will give the hedge cost.

The Task Force believes it is appropriate to use the *P*-measure to project asset and liability cashflows and thereby obtain a net cost distribution for the Canadian GAAP balance sheet provisions.

The model should not generate negative stock prices nor should it produce negative interest rates for any sustained periods.

State dependent models relate the change from one period to the next to current market levels or recent market performance. For example, a mean-reverting process is state dependent because the future scenarios depend on how the current market variables relate to long-term historical values. State dependent models are not required, but are acceptable if they are justifiable based on the historical data and meet the calibration criteria.

A related issue that receives a significant amount of discussion is whether the model should explicitly allow for recent market experience (e.g., reflect an assumption that following significant appreciation, a higher provision for a correction is appropriate and vice versa). This is another form of a state dependent model so such behaviour assumptions are permitted provided they continue to meet the calibration criteria.

e) *Stochastic Model Parameters Estimation*

Different models may require more or fewer parameters and use different statistical distributions. A typical model should have at least two parameters relating to the drift and volatility of the stochastic process.

These model parameters should be estimated based on historical market data as opposed to recent market performance. Due to the long-term nature of the segregated fund guarantee, as a rule of thumb, historical data should cover at least two times the projection span. However, when historical data are not available or it is not justifiable to use it, then some adjustments may be required.

Generally, market indices should be modelled rather than the specific fund performance. There will be more credible data available for the market index and the specific fund performance can depend on additional factors that may not be consistent over time (for example the fund manager can quit or be replaced).

Parameter estimates for a number of different market indices may need to be included in the generated scenarios so they can be combined to model a specific segregated fund portfolio. When more than one index needs to be projected, it is necessary to allow for correlations between different markets. It is not necessary to assume that all markets are perfectly positively correlated, but it would be appropriate to use correlations other than zero. The actuary should consider that correlations are not stationary, and that they tend to increase during times of high volatility or negative returns.

If making *ad hoc* adjustments to observed correlations, care should be taken to ensure that the resulting correlation matrix is internally consistent. (Technically, a correlation matrix should be positive semi-definite).

Also, when foreign indices are used to establish the benchmark index, the foreign exchange rate must also be considered. In some situations, it may be appropriate to have separate parameters for the market index and for the foreign exchange rate. The fact that a currency has depreciated or appreciated significantly in the historical period should be scrutinised before assuming that the trend will continue in the future. In such cases, it may be more appropriate to include an explicit currency exchange model or use local currency data to estimate the model parameters.

If required, these parameters must be adjusted to reflect the skewness and the tail fatness observed in the historical data. This required adjustment is discussed below as part of the calibration process.

The model parameters are not required to be constant over the projection horizon.

f) Selecting Investment Return Assumptions for Specific Funds

To develop investment return paths for a specific fund, an appropriate proxy for the segregated fund must be constructed. The specific fund's investment policy, its asset allocation implied by the fund performance objective, the history of fund performance and trading activities must be examined prior to proxy construction and then reflected in the proxy asset composition.

The proxy may take the form of a linear combination of recognised market indices or economic sector sub-indices or, less commonly, as a well-defined set of trading rules in a specified asset universe. Using combinations of recognised market indices or economic sector sub-indices facilitates using a limited number of well developed and researched data-sets to model a wide range of funds.

The proxy construction process should involve analyses that confirm a close relationship between the investment return proxy and the specific segregated funds.

The specific analyses can include, but are not limited to:

- Comparison of the serial long-term and short-term historical returns of the proxy and the specific fund.
- Analysis of serial correlations between the proxy and the specific fund.
- Comparison of asset composition over time of the proxy and the specific fund.
- Comparison of the systematic risk between the proxy and the specific funds' assets.
- Comparison of the specific risks between the proxy and the specific funds' assets.
- Comparison of the source-of-return attribution between the proxy and the specific fund.
- Comparison of the volatility and risk-adjusted return between the proxy and the specific fund.
- Comparison of the long-term expected asset composition of the proxy and the specific funds.

When sufficient historical information about specific funds' performance is not available, the proxy should be constructed by combining asset classes and/or allocation rules that most closely reflect the expected long-term asset composition of the specific fund. The proxy return-generating process can then be modelled by mapping this asset composition to the historical performance of market indices or economic sectors that most closely reflect the proxy long-term asset composition. Where sufficient historical information for a specific market index or sub-sector does not exist, the return-generating process would reflect the contribution of this component to the specific funds total return by reference to the efficient markets risk-return relationship, as described below.

Investment managers may seek to generate incremental returns by short-term changes in fund allocation to individual assets or asset classes/sectors. As described below, such incremental returns may only be achieved at an increased level of risk. This risk component must be reflected in the return-generating process of the specific fund.

A well-established tenet of the modern portfolio theory is that, over the long term, additional return can only be achieved by undertaking additional risk. If the specific fund investment policy expects to generate excess return by pursuing active portfolio management, a risk-return relationship must be reflected in the specific funds return-generating process. This relationship can be captured from efficient frontier construction, the capital market pricing

model or arbitrage pricing theory. The final proxy for the return-generating process of the specific fund should conform to this risk-return relationship.

2.1.2 Calibration of Investment Return Models Used for Generating Returns

The calibration tests are to ensure that the model is able to generate scenarios that take into account the skewness and fatness of the tail observed in historical data. The emphasis of these tests is placed on fitting the left-tail of the distribution (downside events) as opposed to fitting the entire data set or some other measure such as the mean.

Calibration requirements are included only for equity return models, since this is the primary source of risk with respect to segregated fund investment guarantees.

For equity return models, the model should be calibrated using a prescribed data set. The recommended data set is the TSE 300 Total Return monthly data from January 1956 to December 1999 inclusive (see Appendix B). Once the model has been calibrated with this data set, the “fitted” model should be used for all indices as described below (in other words, the basic model is only “fitted” once).

Note that it is the *model* which must pass the calibration tests, *not* the actual scenarios used for valuation. It is important to emphasize that a calibrated model used with parameters estimated from data series different from the prescribed dataset (i.e., different market and/or historical period) will produce scenarios that may or may not meet the calibration criteria. However, even with a calibrated model, it remains the actuary’s responsibility to ascertain the reasonableness of the parameters used to generate scenarios for valuing the policy liabilities.

For models which are a function of recent history (e.g., market levels, current volatility, mean-reversion process, etc.), calibration tests must be done using the long-term trend of these parameters as recent history. In other words, the model calibration should be done using long-term trends in values for the recent history, and not use the actual history of only the past immediate periods. Once the model is calibrated, the forward projection (from the valuation date) can start with the actual values for the immediate past periods.

The Task Force proposes the following prescribed calibration process for stochastic models of total returns on equity investments.

1. All model calibration for equity return models should be done with a single data-set. The data-set we are proposing is the TSE 300 total return data from January 1956 to December 1999 inclusive (end-of-month values). This data series is provided in Appendix B. The parameters should be estimated by maximum likelihood techniques or by similar statistical methods. No allowance should be made for expenses in the parameters.
2. The calibration is applied to the total return 1-year, 5-year and 10-year accumulation factors generated by the asset model. For models which assume correlation between returns in successive periods, the accumulation factors should be calculated using neutral starting values.
3. Table 1 provides maximum returns for the 2.5th, 5th, and 10th percentiles for the accumulation factors (Appendix E provides a description of the analysis undertaken to establish these calibration points). As an example of how to interpret the table, for a five-year holding period, the total return must be –25% or lower at least 2.5% of the time.

TABLE 1

Accumulation period	2.5 th percentile	5 th percentile	10 th percentile
1-year	0.76	0.82	0.90
5-years	0.75	0.85	1.05
10-years	0.85	1.05	1.35

4. The model with the initially determined parameters (i.e., uncalibrated parameters) might not satisfy the calibration criteria in Table 1. In this case the parameters may be adjusted until a set of calibrated parameters that meet the calibration criteria are determined. Alternatively, a different model may be selected.
5. The final calibrated parameters for the TSE data-set should be extrapolated to other data-sets using the formula that follows. If $k(\text{TSE})$ is the uncalibrated parameter for the TSE data-set, and $kI(\text{TSE})$ is the calibrated parameter, then for any other data-set, the calibrated parameter $kI(\text{DATASET})$ is defined as $kI(\text{DATASET}) = k(\text{DATASET}) + [kI(\text{TSE}) - k(\text{TSE})]$. This approach should be followed for each fitted parameter.
6. Each of the maximum return criteria must be met. This means that the model used must produce return values for the accumulation factors that are no larger than the appropriate table values, for each holding period/percentile combination.
7. For some models the percentiles may be calculated analytically; if simulation is used, the random number generator must be reliable. A sufficient number of simulations should be performed to ensure that the criteria are met with a high degree of confidence (95% certainty would not be unreasonable). Instructions on how to achieve this level of certainty are given in Appendix D.
8. In addition to the percentile criteria in Table 1, the mean of the one-year accumulation factor should lie in the range 1.10 to 1.12. The standard deviation of the annual accumulation factor should be at least 0.175.

Appendix C provides an example of how a common simple fixed volatility lognormal model can be calibrated to meet these criteria.

Other models are equally acceptable, and indeed may be preferable if they do a better job of capturing the characteristics of actual market returns (such as fat tails and time varying volatility). Appendix D provides a brief overview and further references for how other models may be calibrated (e.g., regime switching lognormal, stochastic volatility lognormal, stable model).

2.2 Modelling of Liabilities

This section outlines the key issues surrounding the development and use of “liability models” in assessing and quantifying the risks associated with the guarantees offered on segregated funds and other similar contracts. The term “liability models” is used to refer to those elements of the simulation model other than the investment performance components. These other elements generate paths of cash flows associated with the segregated fund guarantees for each stochastically generated investment scenario. The liability models need to be integrated with the stochastic investment return models in a reasonable and consistent manner.

At a minimum, the models that are used to generate liability cash flows should appropriately reflect the following:

1. Product features
2. Inforce population
3. Policyholder behaviour
4. Investment assumption for additional assets supporting the investment guarantees

Each of the above items will be discussed in terms of key modelling issues and selection of assumptions.

Ideally, any risk assessment under a stochastic framework would simulate the portfolio on a contract-by-contract (seriatim) basis and accommodate all policy elements according to the terms of the contract(s) being valued. However, practical considerations may necessitate certain approximations and compromises due to data insufficiency and/or computational efficiency (e.g., grouping of similar contracts). Such approximations are acceptable provided the actuary has conducted prior sensitivity testing and is satisfied that the approximations do not materially affect the results.

2.2.1 Product Features

Within materiality considerations, the liability models should attempt to realistically accommodate all significant product features including, but not limited to, the following:

a) Management Expense Ratios (MERs)

Total MERs (including all taxes charged to the fund such as GST) should vary by fund according to the terms of the contract and recent company practice. The actuary should not assume a change in MERs in the future unless there is a clear and justifiable reason for doing so, taking into account past practices, competitive pressures and reasonable policyholder reactions.

b) Maturity Date

Contracts should be projected to at least the earliest possible maturity date. If policyholders have the option to renew in the future, the actuary should test the effect of allowing a proportion of such customers to “rollover” their contracts. If policyholders have the option to change their maturity dates after contract issue, the actuary should assume some proportion of policyholders will elect the shortest possible maturity.

c) Contract Guarantees

Investment guarantees should be modelled according to the terms of the contract. For example, the model should calculate the guaranteed amount at the “level” at which it actually applies (e.g., fund-by-fund, deposit level, by policy year, family-of-funds, etc.). The level of the guarantees at the valuation date should reflect the actual guaranteed amounts in effect at that date (i.e., appropriately adjusted for prior withdrawals, resets, transfers, deposits, etc.)

d) Ratchets & Resets

The model should reflect both elective and automatic resets/ratchets in the guaranteed amounts according to the terms of the contract. If resets are discretionary, the actuary should assume that some proportion of contract holders will elect to exercise the reset option when it is in their financial best interest to do so. The actuary need not assume that all policyholders act with 100% efficiency in an economically rational manner. However, the model should allow the frequency of elective resets to vary according to the current and/or historical economic environment.

e) *Fund Transfers (Switching)*

The actuary should consider building a model that allows contract holders to transfer monies between investment options to the extent that such transfers increase the policy liabilities and can reasonably be justified. In this case, the actuary should assume that some proportion of contract holders would elect to transfer monies when it is in their financial best interest to do so. The actuary need not assume that all policyholders act with 100% efficiency in a rational manner. Ideally, the model should allow the frequency (timing) and/or amount of transfers to vary according to the current and/or historical economic environment and past policyholder behaviour. In most circumstances, it would be acceptable to ignore the impact of future fund transfers, but there are situations where it would be appropriate to make some allowance for a shifting asset mix. For example, if the actuary believes that the current investment mix is not representative of long-term policyholder behaviour (e.g., the portfolio is over-weighted in fixed income funds), it would be appropriate to test the sensitivity of results to changes in the investment profile.

f) *Elective Partial Withdrawals*

The model should allow policyholders to withdraw monies without surrendering their contracts to the extent that such withdrawals can reasonably be anticipated and should justifiably be treated differently from surrenders. In this case, the actuary should assume that some proportion of contract holders will elect to withdraw monies in a financially sensible way. The actuary need not assume that all policyholders act with 100% efficiency in a rational manner. Ideally, the model should allow the frequency (timing) and/or amount of withdrawals to vary according to the current and/or historical economic environment and past policyholder behaviour. Care must be taken in setting the withdrawal assumptions so that they interact with surrenders in a reasonable manner and that the overall level of fund depletion is appropriate.

i) *Systematic Withdrawals and Income Distributions*

If applicable, the model should make allowance for pre-authorized (systematic) withdrawals consistent with client instructions and any restrictions imposed by the contract or legislation. The model should also reflect the periodic distribution of investment income from the fund if such amounts are not automatically reinvested.

h) *Investment Options*

The actuary should assume that existing investment options will continue to be offered unless there is a clear reason to justify otherwise (e.g., the company has already declared its intention to close the fund).

i) *Future Deposits*

Unless future premiums are required to maintain the contract in force, it would ordinarily be acceptable to ignore the impact of future deposits. However, the actuary may wish to adjust the model to allow policyholders to make additional deposits to their contracts to the extent that such deposits increase the policy liabilities (e.g., deposit-level guarantee and a fixed maturity date) and can reasonably be expected. In this case, the actuary should assume that some proportion of contract holders will elect to contribute additional monies to their contracts in a financially sensible way and in manner consistent with their most recent instructions. The actuary need not assume that all policyholders deposit additional amounts. Ideally, the model should allow the frequency (timing) and/or amount of deposits to vary according to the current and/or historical economic environment and past policyholder behaviour.

j) Surrender Charges & Transaction Fees

The model should take into account surrender charges and transaction fees only to the extent that such charges can reasonably be anticipated to affect the costs projected in the valuation. In general, such charges would be earmarked for other purposes (e.g., recovery of unamortized acquisition costs or payment of administrative expenses) and would not be considered available for the payment of guaranteed policy benefits.

2.2.2 Inforce Population

Within materiality considerations, the liability models should reflect the characteristics of the actual portfolio as of the valuation date. While a contract-by-contract assessment is preferred (i.e., reflecting all policy elements at the valuation date on a seriatim basis), some approximations and a certain amount of grouping may be necessary for practical reasons. The actuary should be satisfied that such approximations do not materially affect the policy liabilities.

To the extent possible, the valuation should use all relevant policyholder data by contract. The following list gives some indication of the information that may be required for valuation. This list is not meant to be exhaustive and is for illustration purposes only as some items may not be relevant for certain products nor needed in a given situation.

- Fund value by investment option
- Applicable guaranteed benefit amounts
- Gender
- Attained age
- Issue age or contract duration
- Expected maturity date
- Earliest maturity date
- Expected annuitization date (if guaranteed income benefits and/or annuity purchase rates)
- Earliest annuitization date (if guaranteed income benefits and/or annuity purchase rates)
- Tax status (registered versus non-registered)
- Deposit plan (e.g., DAC), including allocation instructions
- Deposit/withdrawal/fund transfer/reset history
- Systematic withdrawal options

“Applicable guaranteed benefit amounts” should be calculated according to the terms of the contract. Specifically, this would involve determining the guaranteed benefit amounts at the appropriate “level” (by fund, by contract, by deposit, by policy year, etc.) and adjusting for past withdrawals, resets, transfers and deposits.

2.2.3 Policyholder Behaviour

Within materiality considerations, the model should accommodate all relevant forms of policyholder behaviour and persistency, including but not limited to the following:

- Mortality
- Surrenders

- Partial Withdrawals (Systematic and Elective)
- Fund Transfers (Switching/Exchanges)
- Resets/Rachets of the Guaranteed Amounts (Automatic and Elective)
- Elective Resets of the Maturity and/or Annuitization Date(s)
- Future Deposits

As mentioned previously, it would ordinarily be acceptable to ignore the impact of fund transfers and future (optional) deposits, but the actuary should exercise caution in assuming that the status quo will be indefinitely maintained. Under some circumstances, it would be appropriate to test the impact of a shifting asset mix and/or consider future deposits to the extent they can reasonably be anticipated and increase the policy liabilities. Normally, the underlying model assumptions would differ according to the attributes of the policy being valued. This would typically mean that policyholder behaviour and persistency might be expected to vary according to such characteristics as:

- Gender
- Attained Age
- Issue Age
- Contract Duration
- Time to Maturity
- Tax Status
- Fund Value
- Investment Option
- Guaranteed Benefit Amounts
- Surrender Charges and/or Transaction Fees

Unless there is clear evidence to the contrary, behaviour should be consistent with past experience and reasonable future expectations. Ideally, policyholder behaviour would be modelled dynamically according to the current/prevaling and/or historical economic environments. However, it is reasonable to assume a certain level of non-financially motivated behaviour. The actuary need not assume that all policyholders act with 100% efficiency in a financially rational manner. Neither should the actuary assume that policyholders will always act irrationally.

The liability model should incorporate margins for adverse deviations (MFADs) for all risk factors which are non-dynamic (i.e., the non-scenario tested assumptions) and are assumed not to vary according to the financial interest of the policyholder. Margins would normally fall in the standard range of 5 to 20% unless there is clear evidence to justify otherwise. For example, a higher margin may be applicable to the lapse assumption in the absence of credible and relevant experience data.

Risk factors which are not scenario tested, but could reasonably be expected to vary according to (a) a stochastic process, or (b) future states of the world (especially in response to economic drivers) may require additional margins and/or signal a need for higher margins for certain other assumptions.

Behaviour that is modelled dynamically should incorporate margins by reflecting potential adverse experience in a reasonable manner. Care should be taken to ensure that any modelled

dynamic behaviour reasonably reflects the possible range of future experience and is consistent with the other variables in the model, including the non-scenario tested assumptions.

Companies should attempt to track experience by collecting and maintaining the data required to conduct credible and meaningful studies of policyholder behaviour.

a) Mortality

The mortality assumption should be based on actual past and expected future experience to the extent that credible data can justify such an assumption. Otherwise, the actuary should assume mortality consistent with that used in valuing similar contracts, reflecting the form of underwriting (if any), and other policy attributes.

b) Surrenders

In general, the policy liabilities for the investment guarantees on segregated funds are very sensitive to the assumed lapse rates. In many circumstances, the products may be considered “lapse supported” to the extent lower lapse rates increase the policy liabilities. Unless the actuary has relevant and credible experience data to support different assumptions, he/she should not assume lapse rates that materially differ from industry experience and/or any guidance offered by the CIA and its practice committees.

In particular, the actuary would normally assume that surrenders will decrease when the Fund Value/Guaranteed Amount ratio decreases (i.e., when the maturity guarantee is deeper in the money), although some minimum non-zero rate of surrenders would normally be appropriate (i.e., terminations unrelated to maturity guarantee value). However, when the Fund Value/Guaranteed Amount ratio is very high, surrender experience may be comparable to investment funds that do not offer a guaranteed benefit (e.g., most mutual funds). Furthermore, surrenders would typically be expected to temporarily spike once surrender charges (back-end loads) wear off and decline as the time-to-maturity decreases.

Care must be taken to ensure that the overall rate of fund depletion (i.e., the combined effects of surrenders and partial withdrawals) is appropriate and not excessive.

c) Fund Transfers (Switching/Exchanges)

Transfer rates would typically contain both fixed (non-dynamic) and variable (dynamic) components. Usually, transfers would be modelled only when the level and terms (notably, the maturity date) of the investment guarantee do not change when monies are “switched” between investment options. If the maturity date and guaranteed amount are reset upon a transfer, the risk to the carrier is very similar to that of a withdrawal and new deposit.

The variable (dynamic) component of the transfer rate can reasonably be expected to vary according to the degree to which the investment guarantee is “in-the-money” and the expected performance differential between the “source” and “destination” funds.

d) *Elective Resets of the Guaranteed Amounts*

The actuary should assume that some proportion of contract holders elect to exercise the discretionary reset option when it is in their financial best interest to do so. The actuary need not assume that all policyholders act with 100% efficiency in a rational manner.

The “reset utilization rate” (fraction of policyholders who choose to reset) should vary over time according to the relationship between current guaranteed amount (before reset) and fund value. The utilization rate can be expected to rise whenever the investments guarantees are “out-of-the-money” (e.g., when the ratio of fund value to guaranteed amount is greater than one). The rate of utilization should recognize the MV/GV ratio (option value component), historical returns (performance component) and the remaining term-to-maturity (time component).

To the extent that relevant and credible data are available, the base reset utilization rate should be consistent with company and industry experience. The actuary should allow for both rational (financially motivated) and irrational behaviour by assuming that some threshold (trigger point) needs to be reached before resets occur. The actuary should further recognize that a certain proportion of policyholders will not reset even when it may be a financially optimal decision.

If the maturity date is also re-established upon reset, the utilization rate will likely fall as attained age increases and the term-to-maturity shortens. The model should also constrain the frequency of resets according to the terms of the contract (e.g., only on policy anniversaries, to a maximum of two resets per year, etc.)

e) *Future Deposits*

The actuary may wish to assume some level of future deposits in the short term if the fund is open to new money and additional premiums are not treated the same as “new contracts” (e.g., deposit level or policy year guarantee and a fixed maturity date). In this case, the level of future deposits should be consistent with recent experience and reasonable policyholder behaviour. Monies should be allocated to the various investment options in a manner consistent with past behaviour and recent client instructions.

2.2.4 Investment Assumption for Additional Assets Supporting Guarantees

The discount rates (or accumulation rates) used to determine the asset balance necessary to support the liabilities should be appropriate to the assets that will be allocated to support the policy obligations created by the investment performance guarantees.

These rates would not normally be the same returns as generated for the stochastic paths that project the segregated fund balances since the assets supporting the liabilities will not normally be held in units of the segregated funds. However, the rates should be consistent with the economic scenarios underlying the projections. For example, government bond yields would be appropriate if the assets allocated to back the cost of the guarantees consist of government bonds, but the paths of government bond returns should consistently reflect any economic parameters that underlie the stochastic paths used to establish the segregated fund growth.

2.3 Modelling of Hedges

If the insurer is following a clearly defined hedging strategy, the stochastic model should take into account the cash flows resulting from the hedge positions currently held and expected to be held in future.

Hedging strategies may include static strategies based on long-dated derivative contracts and dynamic strategies based on trading of short-dated contracts. The considerations in this section may also be relevant to modelling of reinsurance strategies.

Modelling of dynamic hedging strategies poses special challenges. In general, such strategies involve the use of risk-neutral valuation to estimate the liabilities' fair value (i.e., a valuation intended to be consistent with current prices of traded securities) and their sensitivities to changes in various market parameters (the "greeks"). The hedge portfolio is rebalanced periodically so as to remain matched based on the "greeks." In principle, in order to take account of such a strategy, the actuary will need to incorporate the insurer's risk-neutral model in the realistic stochastic simulation model, and re-run the risk-neutral model at each time step under each simulated "real world" scenario. In this way, it will be possible to model the "greeks" at each point, and hence, the rebalancing that would take place over the course of that scenario.

The stochastic simulation model should be specifically designed to take account of potential weaknesses in the hedging strategy. At a minimum, the following should be reflected:

- Basis risk between the underlying segregated fund assets (typically mutual fund units) and the hedge positions (e.g., stock index futures and options).
- Non-normal asset returns ("fat tails") and uncertain future realised volatility. This will be a particular issue if the hedging strategy depends mainly on linear instruments such as futures.
- Uncertain future implied volatility. This will be an issue if the hedging strategy depends on future purchases of short-dated options.
- Effect of bid-ask spreads and transaction costs.
- Finite intervals between portfolio rebalancing.

Ideally, the simulation model should also take account of the following:

- Uncertain future interest rates.
- Uncertain future correlations between different asset classes. This will be a particular issue if guarantees apply on a "family of funds" basis.
- Liquidity risk, in that it may not be possible to rebalance quickly in volatile market conditions. However, extreme illiquidity is a risk that would more appropriately be covered by capital as opposed to policy liabilities.

Even very detailed modelling is unlikely to capture accurately all potential risks in a hedging strategy, and PFADs should be established on a conservative basis. Institutions seeking to obtain credit for hedging programs in establishing minimum required capital for segregated fund investment guarantees should refer to OSFI Guidance Note "*Capital Offset for Segregated Fund Hedging Programs (MCCSR)*" for instructions and requirements (August 2001). See Appendix H. IGIF also has comparable requirements.

It will generally be impractical to perform full stochastic modelling of a hedging strategy on a production basis. The following approximations may be used, but should be justified by reference to stochastic modelling of a representative sample of contacts.

- If it can be demonstrated that the hedging strategy results in a distribution of hedging errors with a mean close to zero, then the “best estimate” policy liability could be approximated by the initial value of the hedge portfolio (i.e., the risk-neutral value) adjusted for the difference, if any, between the market value of the hedging assets and their associated balance sheet values. PFADs could be estimated by re-running the risk-neutral valuation model using more conservative volatility and correlation parameters.

Margins in these parameters should be calibrated by reference to stochastic modelling of sample contracts.

- Alternatively, PFADs could be estimated by simulating the hedging strategy over a relatively short period, ignoring future rebalancing, but revaluing all assets and liabilities on a mark-to-market basis at the end of the simulation period. The results could then be extrapolated in order to estimate the required PFADs over the full term of the contract. The extrapolation factors would be determined by reference to stochastic modelling over the full term of a representative sample of contracts.

3. MINIMUM CAPITAL (MCCSR)

3.1 Background

From a theoretical perspective, the Task Force endorses the idea of setting the total balance sheet requirement (“TBSR”) for policy liabilities plus minimum capital using a similar stochastic analysis to that used for determining the policy liabilities.

However, for the reasons outlined earlier in this note, the Task Force recognizes that moving immediately to a fully stochastic method for setting all balance sheet provisions for investment guarantee risks will likely prove unacceptable initially to key audiences. Therefore, we believe that the total balance sheet requirement should initially be determined using a prescribed factor approach where the methodology and factors are broad enough to reflect appropriately the risk characteristics of a company’s specific block of business.

The factor-based methodology proposed by the Task Force in August 2000 was developed from stochastic analysis using models and methodologies described in this paper.

While the Task Force prefers the use of stochastic methods to set all balance sheet provisions for investment guarantee risks, we believe the proposed factor based approach is vastly superior to the deterministic scenario test as outlined in the December 1999 letter from OSFI for introduction in December 2000. Most notably, the factor approach has been calibrated to produce reasonable results for a wide variety of benefit types. This was a significant shortfall of the deterministic “drop/recovery” methodology which could produce penalistic capital levels for certain limited risk guarantees. The factor methodology should also be more stable than the deterministic approach from period to period, while still adapting appropriately to changing conditions and risk profiles (e.g., investment market performance).

The Task Force also proposes a fully stochastic basis for minimum capital for use in coming years (see section 3.3), subject to specific criteria established by OSFI/IGIF.

3.2 Factor-Based Capital Approach

3.2.1 Goals of Factor Table Approach

In developing the proposed factor approach, the Task Force considered a number of key goals/objectives for minimum capital that have been enunciated by OSFI/IGIF.

- *A risk based approach that reflects the nature of the benefits, the product structure, and the underlying risks apparent in each company.* There are many ways in which the segregated fund guarantee risk is either magnified or diluted significantly based on product design combined with investment type of fund; relative relationship of market value and guaranteed value; availability of margins to offset cost; distribution of business by years of issue/maturity, degree of hedging or reinsurance, etc. Identification of these factors separately and in combination is necessary to create a dynamic approach that is responsive to evolving product and policyholder situations.
- *An approach that complements prudent actuarial standards and practices:* The proposed factor approach reflects varying product and investment structures. It will be populated by factors determined based on rigorous stochastic actuarial testing using the same underlying stochastic standards as will be applied to the determination of policy liabilities. A wide set of practitioners involved in the testing process achieved reasonably convergent results. This has provided the

means to develop a credible set of factors based on a number of models used to assess these risks.

- *An approach that provides incentives for prudent risk management decisions:* The recognition of different product designs, the adjustment factors for different funds/markets and term exposures, and credits for hedging/reinsurance will serve to increase or decrease requirements as companies' risk reduction strategies result in changes in exposure.

3.2.2 Key Drivers of Factor Table Approach

Under the proposed factor approach, many detailed aspects of risk exposure have been taken into account. The key drivers in choosing appropriate factors are based on:

a) Guarantee Type and Investment Type

The choice of appropriate factors based on guarantee type will generally be straightforward, but interpolation may be necessary in some cases. The slotting of investment type will often be more challenging:

- for investment type, guidance will be given based on general descriptions of each of the investment categories as well as with ranges of volatility measured by the expected standard deviation based on long-term analysis of the fund index or proxy;
- where the guarantee is not applied uniquely at the investment type level, care is needed to consider proper investment slotting based on the fund make-up at a client level exposed to the guarantee; for example, if there is a "family of funds" guarantee, the actual mix of specific client accounts would be reflected; also, if the guarantee applies separately to all deposits in a particular policy year for a given client, then each year's exposure and its related asset mix would be considered separately for investment categorization.

b) Inforce Business Adjustment Factors

With the business identified and categorized by product type and investment class, there are other necessary adjustments based on the following variables evidenced in the block of business:

- Market Value to Guarantee Value relationship ("MV/GV ratio");
- "Short" or "long" period to next (or final) maturity date; and
- MER adjustment to the extent fees are significantly different from those assumed.

c) Credits Available

Credits are then available for:

- Margins available to support the risk;
- Existing hedges or reinsurance.

d) Balance of Complexity and Practicality

In determining the product designs to include in the table, a list of product features commonly available in both Canada and the United States were considered. Key cornerstone benefit types were chosen to develop factors that would help to maintain simplicity and provide a good basis for interpolation and limited extrapolation.

Where a product includes two or more of the benefit types, the factors are designed to be additive. This is generally the case with respect to maturity and death benefits. The rationale for this approach is as follows:

- The separation of the death and maturity benefits is not always appropriate from a risk measurement perspective. However, this approach facilitates the creation of a “simpler” table for mixing and matching of many of the product components that could be offered.
- The most conservative legal benefit is one cornerstone, which led to the factors for the 75% death and 75% maturity benefit guarantees based on final maturity date provisions.
- The most prevalent inforce benefits are represented (i.e., a large inforce block of traditional product with 75% maturity and 100% death benefit and a large emerging block of 100% death and ten-year maturity benefits with elective resets).
- Both mortality and lapse decrements were reflected in the development of the factor tables. However, the failure of this process to integrate the benefits together for a single factor is believed to provide a small element of conservatism.

For investment fund types, several key fund classes have been identified, with detailed guidance (both subjective and statistical) given on how to classify funds into these different types.

The Task Force believes this approach can closely track the actual change in balance sheet risk profile. A key compromise that has been struck is the need to balance completeness (and, hence, very elaborate tables) and practicality (more concise guidance). Based on feedback during May and June 2000, the Task Force believes an appropriate balance has been reached whereby there is limited demand to add any other major adjustment tables and/or remove any adjustment factors currently recommended.

However, some of the simplifying steps used in 2000 have been expanded for 2001 based on feedback that some unexplainable results were emerging due to oversimplification of some factors.

3.2.3 Application of Proposed Factor Approach

The general approach to developing the minimum total balance sheet provision is to apply factors to the market value of the segregated fund assets based on various characteristics of risk exposure. Interpolation and extrapolation criteria are provided for those cases when data do not conform to the factor modelling points provided.

A methodology is also provided to determine appropriate credit for hedging and/or reinsurance strategies.

Appendix F provides a description of the factor-based methodology as implemented by OSFI in its December 2001 MCCSR Guideline (section 9) – Segregated Fund Guarantee Risk. Appendix G includes the MCCSR documentation as provided by the external consulting firm engaged by OSFI to assist in the factor development methodology for December 2001.

Although not attached as an Appendix, IGIF has introduced comparable requirements.

3.2.4 Development of Task Force Proposed Factors (August 2000)

The base factors and the adjustments were developed using the results of stochastic analysis from a number of models. The Task Force members, along with an external group of practitioners, ran their own stochastic models for a variety of benefit types and fund category combinations in order to develop both direct guarantee costs and the impact of offsets from available margins.

For equity funds, these stochastic models were all calibrated using the stochastic calibration criteria and methodology for the TSE 300 outlined in this report. For fixed income funds, to recognize the diversity of models used to produce bond type returns, the contributors were allowed to use uncalibrated models, but were to base the model input on observations of bond prices/returns over an established time horizon (1956-1999). For Balanced Funds, contributors were encouraged to model a 60% equity, 40% bond mix based on the models previously developed for bonds and equities.

Each contributor provided a list of summary statistics for the model output. This information was collected and reviewed for anomalies. Having vetted the information, the average of the results for each summary statistic was determined with due regard to outlying results (of which there were very few). The Task Force was very pleased with the degree of convergence in the submitted results. This was achieved after several clarifying rounds of testing which were required to ensure that our testing instructions were clear and could readily be interpreted by participants. These values were used to produce the table factors and adjustments to the factors proposed in the August 2000 report of the CIA Task Force.

Lapse and mortality decrements were specified for testing. The development of a true dynamic lapse model was outside the timeframe for development. As a result, a level 8% per annum lapse rate was generally assumed, but 10% per annum was assumed for the “to age 90” guaranteed minimum death benefits commonly offered on variable annuities in the United States. These rates are acknowledged to be below observed experience and were deliberately set to be conservative.

The mortality assumption was CIA 1986-1992 Age Last, aggregate, ultimate table. Attained age 50 was used for testing to be representative of most inforce blocks of business in Canada. Attained age 60 was assumed in testing the “U.S. GMDB” product forms.

The discount assumption used for the net cash flows of the guarantees was 6% per annum, reflecting a conservative long-term investment grade bond return.

The factors recommended to OSFI/IGIF by the Task Force in August 2000 were based on a 90% conditional tail expectation or “CTE” approach. A conditional tail expectation set at the $x\%$ level, denoted $CTE(x)$, is the average cost of the highest $(100-x)\%$ of the results. It should be noted that $CTE(x)$ is generally greater than a $(x + \frac{1}{2}(100-x))$ percentile coverage (i.e., $CTE(90)$ is generally greater than the 95th percentile). For comparison purposes, testing was also requested at the 80% CTE (90%+ coverage), and 95% CTE (97.5%+ coverage). Factors were then developed using the $CTE(90)$ for the recommended approach to setting total balance sheet coverage.

The CTE approach provides a more stable result than simply selecting a “percentage of scenarios” coverage approach (i.e., a quantile measure). This is because the CTE measure uses an average of all scenario results beyond the selected point, while the percentile approach by definition selects a single scenario to establish amounts.

An industry impact analysis was then prepared through the CLHIA and OSFI/IGIF to allow companies to assess the effects of the proposed capital standards on their own organizations and to allow OSFI/IGIF to form a preliminary view of the overall impact on the industry. Preliminary meetings have been held where members of the Task Force have met with OSFI/IGIF to review a summary of the company responses.

3.2.5 Rationale For CTE(90%) Balance Sheet Coverage

As detailed previously, the August 2000 factors were chosen to provide 90% CTE balance sheet coverage, which should generally cover over 95% of the total scenarios modelled.

An appropriate level of minimum capital may be a point of some debate. Our considerations in recommending the 90% CTE level are summarized as follows:

- Balance sheet provisions maintained through minimum capital levels should lie somewhere between the policy liabilities (developed for a block of inforce policies) and the overall company protection determined through DCAT measurement.
- Relative minimum capital should be based on the nature of the underlying risk, but with some regard to relative levels of protection with other aspects of MCCSR coverage.
- Inherent conservatism in the methodology and assumptions needs to be considered in determining the appropriate CTE level.

Our selection of the CTE(90%) coverage level was strongly influenced by our belief that there are considerable additional layers of conservatism embedded when these resulting factors are used in the MCCSR formula:

- The factors are pre-tax factors, and to the extent that the requirement is held in capital rather than policy liabilities, there is no explicit MCCSR credit for the fact that, at the margin, losses would generate an income tax offset in Canada. We considered inserting a specific tax adjustment, but the complexities involved (e.g., non-Canadian jurisdiction adjustments) and the broader overall issues within the existing MCCSR (i.e., none of the existing MCCSR factors have explicit tax adjustments) led us to avoid taking this path. However, the cost of minimum capital for this benefit will often be a much greater proportion of the cost charged to policyholders for this product compared to many other products covered by MCCSR formulae. For this reason, we believe care needs to be taken in considering the tax impact. Recoverability of tax losses may be diluted in value during extreme economic scenarios, but it should be recognized that tax losses can be sold to an acquiring company in a buyout or insolvency situation.
- Companies are expected to maintain MCCSR ratios well above 100%. OSFI has indicated that it expects companies to maintain ratios above 150% and many companies typically target ratios that are 175% or higher to satisfy expectations of shareholders, investment analysts and rating agencies. This effectively may double the amount of capital held against this risk, increasing the effective CTE and percentile coverage further into the extremes of the tail.
- Both mortality and lapse decrements were reflected in the separate development of the factor tables for maturity and death benefits. However, the failure of this process to integrate the benefits together for a single factor is believed to provide a small element of conservatism.
- Not all contingent risks for which a company is establishing minimum capital will occur concurrently (or occur at all). Therefore, by independently establishing minimum capital for each individual risk exposure, a considerably greater aggregate level of protection of the total company balance sheet is established. Several elements of diversification have not been explicitly considered in the development of the proposed factors:
 - A company may have exposures spread out over time such as multiple deposit year level guarantees for a single client and/or spread out maturity dates within a year.

- Although investment diversification has been considered at the contract and guarantee level, a company may have a wide range of clients with different investment exposures. The proposed factors do not give any credit for any of this type of diversification.

These additional levels of conservatism will mean that the net result of these factors could create coverage in excess of 99% levels on a percentile basis. We believe these implicit levels of conservatism should be reflected in setting the minimum MCCSR requirement.

In comparing the MCCSR minimum capital factors for other elements (e.g., C1, which is the most significant single MCCSR requirement for most companies), it is clear that these other requirements do not provide for loss levels for individual risk categories that extend into the extreme tails of the loss distributions.

3.2.6 Final Developments Based on OSFI/IGIF Conclusions (August 2000)

Following the publication of the August 2000 version of this report, OSFI/IGIF decided to embrace the key principles established by the Task Force, but required the coverage level to be set at CTE(95%) rather than CTE(90%).

OSFI privately engaged an external consulting firm to assist in finalizing the factor-based MCCSR methodology for December 2000 and December 2001. This work was predicated on the general framework proposed by the Task Force in August 2000 and is fully documented in Appendices F (“MCCSR 2001 Guideline – Section 9: Segregated Fund Guarantee Risk”) and G (“MCCSR 2001 Guideline: Documentation”). This approach increased the amount of time and level of detail that could be brought to the analysis, which we believe improves the usefulness of the results.

IGIF has decided to harmonize its approach with OSFI’s and to apply the same factor-based approach, except that some administrative requirements may differ slightly.

3.3 Stochastic Modelled Capital Approach

3.3.1 Total Balance Sheet Approach

Historically, capital has been approached from a different theoretical framework than policy liabilities. Liabilities were established by methodologies which inherently reflected only a limited number of risk factors. Capital was then the additional funds that might be needed to cover other (unknown) risks, or to provide for unusually adverse events in the risk factors that were reflected in the liabilities. Capital tended to be determined by a factor-based approach with limited refinement in how the factors reflected potentially different underlying risks.

The development of comprehensive balance sheet models, as outlined in this paper, makes such treatment inappropriate. The limitations of factor-based capital approaches have become more apparent as products have become increasingly complex. The comprehensive models discussed in this paper provide a meaningful and practical alternative whereby the same framework can be used to calculate (a) the policy liabilities; and (b) the total of liabilities and minimum required capital.

3.3.2 Capital Levels

The amount of minimum regulatory capital (MCCSR) to support the risks of the investment guarantees on segregated fund contracts (and other similar insurance and annuity policies) is

determined by applying the same liability valuation model to a higher statistical confidence level. The process is as follows:

- i) The model is run on the policy liability valuation basis. $CTE(X\%)$ and $CTE(95\%)$ are calculated, where $X\%$ establishes the appropriate balance sheet provision for the policy liabilities and $CTE(95\%)$ is the total of liabilities and minimum capital.
- ii) The model is also run using assumptions without the explicit valuation margins for adverse deviations (MfADs) on the non-scenario tested risk factors.

All CTEs are subject to a minimum of zero. We will distinguish the CTEs “without explicit MfADs” by marking them with an asterisk. Then, the minimum capital required is defined by: $\max[CTE(95\%), CTE^*(95\%)] - CTE(X\%)$. Note: $CTE(95\%)$, $CTE^*(95\%)$ and $CTE(X\%)$ are all calculated using the same investment scenarios.

The actuary is already required to have tested that the valuation margins produce an increase in the liabilities at the $CTE(X\%)$ confidence level. However, it should be noted that the margins which produce an increase in $CTE(X\%)$ may not necessarily produce an increase in the $CTE(95\%)$ value.

In concept, this approach tests the scenario-tested variables (those experience assumptions which vary by scenario) to a higher confidence level, but makes no additional provision for the non-scenario tested risk factors. The comparison of $CTE(95\%)$ to $CTE^*(95\%)$ simply ensures that the inclusion of the underlying valuation margins does not otherwise decrease total required capital. That is, the dollar impact of the explicit valuation MfADs at $CTE(95\%)$ cannot be less than zero.

The Task Force considered whether additional margins should be included in the non-scenario tested assumptions when setting capital levels. It was decided this was not necessary for a few reasons:

- i) As noted, it is not even certain that the sign of margins should be the same in $CTE(95\%)$ as in $CTE(X\%)$. If the sign of the margins is uncertain, there seemed little point in debating the magnitude of them.
- ii) In terms of materiality, the non-scenario tested assumptions tend to be the least volatile assumptions in the calculation. Hence, any additional margins would have less impact than the provisions established for the scenario tested assumptions.
- iii) If there is a non-scenario tested assumption which is particularly volatile, the preferred approach is to enhance the model to reflect that assumption on a scenario tested basis. If that is not practical, the actuary is already required to provide relatively larger margins in the valuation basis because of this uncertainty or volatility. We could find no compelling reason to further increase those margins in the capital calculations.

The decision about whether any particular assumption should be scenario tested or not depends on whether variations in that assumption:

- a) have a material impact on reserves or capital levels; AND
- b) the extent to which the risk factor can be assumed to vary in a logical manner with the stochastic investment returns; OR

- c) can be realistically modelled using a dynamic process (stochastic or otherwise) independent of, but not inconsistent with, the stochastic investment return model.

If a risk factor has a material impact on the liabilities, but is:

- a) generally stable; OR
- b) volatile, but has a (presumed known) distribution tightly centred about its mean; OR
- c) can be assumed not to vary significantly with the other scenario tested variables

then it generally does not need to be scenario tested.

A more complete and general discussion of stochastic methods for policy valuation can be found in the CIA research paper "Use of Stochastic Techniques to Value Actuarial Liabilities Under Canadian GAAP" (August 2001) by the CIA Working Group on the Use of Stochastic Valuation Techniques. Section 3. of this paper specifically addresses the issue of "when to use stochastic methods for actuarial liability valuation" and is particularly relevant to the valuation of segregated fund investment guarantees.

3.3.3 Minimum Capital Using Stochastic Models

The Task Force hopes that the suggested working principles for stochastic capital are helpful in moving the industry towards a model-based approach to minimum regulatory capital. However, implementation will depend upon final approval and directives from the appropriate regulatory authorities (i.e. OSFI and/or IGIF).

3.4 Control of Model and Auditability

Liabilities and capital for segregated fund investment guarantees represent material items on the financial statements for many insurance companies. As with any material item, the determination of their amounts must be controlled and audited.

With stochastic models, construction of an audit trail becomes more complex than for traditional valuation environments. The actuary must demonstrate that sufficient controls are in place so that changes to the model are made only by qualified personnel. An audit trail of changes and the impact on financial statements must be produced as well. Such an audit trail includes verification that the desired changes occurred, plus confirmation that no undesired changes also resulted. Where several changes have occurred, the actuary should be able to demonstrate the incremental impact of each change or related group of changes.

The professional judgement of the actuary has special significance in applying the models discussed in this paper. The long term nature of most investment guarantees makes it impossible to empirically "back test" the models against history as would generally be done in other financial institutions for short dated exposures. As well, there is little to no experience data on how policyholders behave in various economic climates in the presence of long-term investment guarantees. Lacking this historical record, great reliance is placed on the professionalism of the actuary in validating and prudently applying the models.

4. POLICY LIABILITIES

4.1 Background

The desired approach of the Task Force to setting the policy liabilities for the risks of the investment guarantees on segregated funds is one based entirely on the use of stochastic modelling techniques. We recognize that in order to realize this objective, an adequate framework must be put in place to promote consistency in application and result in an appropriately narrow range of practice. That is, two actuaries determining the policy liabilities for substantially similar blocks of business should not produce materially different results solely because of differences in application and best estimate assumptions. Similarly, any proposed framework should be structured to ensure that the use of stochastic methods does not lead to inadequate reserving (i.e., does not inappropriately weaken policy liabilities).

The Task Force believes that the following measures will provide the necessary framework for moving to stochastic methods in determining the policy liabilities:

- Requiring that stochastic models for investment returns satisfy the calibration criteria and other guidelines established in this paper (see Section 2.1).
- Where appropriate, providing clear guidance to narrow the range of accepted actuarial practice in setting liability assumptions for valuation of segregated fund guarantees as outlined in this paper (see Section 2.2).
- Offering additional guidance in the following areas:
 - Setting margins for adverse deviations (MfADs) and establishing appropriate overall provisions for adverse deviations (PfADs).
 - Prudent recognition of attributed fee income to offset guarantee costs.
 - Credit for sound risk management practices, including hedging and reinsurance.
 - Use of approximations.
 - Providing a framework for how to allocate expected revenue between components of a contract when the valuation considers these components separately (i.e., segregated fund investment guarantees valued independently of the base/host contract).

The Task Force believes that prescribed deterministic methods to establish a floor (minimum) value for the policy liabilities are generally flawed and inappropriate. Deterministic methods are typically incapable of addressing the wide array of product forms in the marketplace and do not always respond reliably to changing market conditions. Previous approaches, such as the “drop/recovery” scenarios in place until year-end 2000, were calibrated to provide a reasonable policy liability at only one point in time for a specific form of benefit (i.e., newly issued ten-year 100% guaranteed maturity benefits with no significant differential between market and book value). As such, the Task Force believes such methods should immediately be withdrawn from accepted actuarial practice.

4.2 Establishing the Policy Liabilities

In making an appropriate provision for the liabilities associated with segregated fund investment guarantees, the primary issue is how the valuation should incorporate margins for adverse deviations (“MFADs”) and ultimately what the resulting dollar provisions (“PFADs”) are intended to cover. In selecting appropriate MFADs, it is important to keep in mind that standards of practice suggests such provisions should reflect mis-estimation/deterioration of the mean and the degree of uncertainty

in projecting future contingent events. MFADs are not intended to cover statistical fluctuations in the underlying assumptions nor provide for catastrophic losses. For further discussion of appropriate levels of margins for adverse deviations, see section 7 in the November 2000 *Standards of Practice for the Valuation of Policy Liabilities of Life Insurers* (LSOP).

In general, there are two distinct approaches that could be taken in setting the policy liabilities using stochastic models:

a) *Expected Value With Margins in all Assumptions*

The policy liabilities are set equal to the expected result (mean) using “conservative” (after margins) assumptions. In particular, this would involve incorporating MFADs into the investment model parameters.

b) *Tail Measure Using “Calibrated” Assumptions for the Investment Model Parameters*

The policy liabilities are set equal to some measure from the sample distribution of results, taking into account the mean (expected) value as well as tail dispersion. Margins for adverse deviations would be incorporated into all non-scenario tested risk factors. In general, the primary scenario tested assumptions relate to investment (market) risk. However, in order to ensure an appropriate sampling from the tail of the distribution, the investment model parameters would be set so that the model simulations satisfy the calibration criteria described elsewhere in this report.

Scenario tested assumptions are those contingencies that vary by scenario according to a stochastic process (e.g., investment returns) or through a defined relationship with one or more stochastic variables (e.g., dynamic lapses). By implication, the non-scenario tested assumptions are those risk factors whose randomness is understood (or assumed known) and not simulated.

The first approach is consistent with traditional discounted cash flow techniques (such as the Policy Premium Method), in which the liability is set at the expected value using assumptions that explicitly (sometimes implicitly) incorporate some degree of conservatism (i.e., MFADs) in each assumption, including investment returns. The degree of conservatism for each assumption (size of the margins) is related to the degree of uncertainty in predicting the amount and timing of future cash flows affected by that assumption. It is also consistent with how non-investment related assumptions are handled in the CALM approach in the Life SOP for Valuation of Policy Liabilities of Life Insurers.

The second approach is essentially an extrapolation of the CALM Method of Valuation for establishing interest risk margins as outlined in the LSOP (sections 6.1 and 6.2). This approach establishes the PFAD for interest rate risk based on coverage of a plausible range of scenarios (interest rate risk being considered the primary investment risk for liabilities covered by the method).

The Task Force recommendations follow the second approach (i.e., the “calibrated tail measure” approach) for assessing the investment risk and establishing the appropriate provision for the scenario-tested assumptions. It believes this approach maintains the most consistency with evolving Standards of Practice and is a superior way to assess investment risk in a stochastic environment. Another advantage of this approach is that it allows for the use of the same set of stochastic scenarios to be used for other risk assessment exercises, including pricing, stress testing, value at risk analysis, etc.

Under the “calibrated tail measure” approach, the non-scenario tested assumptions would normally reflect explicit margins for adverse deviations consistent with the draft Consolidated Standards of Practice (“CSOP”) and guidance from the CIA practice committees. For the stochastic elements (e.g., the investment return paths), provisions for adverse deviations will be established by requiring the policy liability to cover a range of the stochastic results based on statistical measures (where the return paths are generated using “best estimate” assumptions, subject to the calibration criteria outlined in this paper).

The starting point for the valuation would be the actual policy data as at the valuation date (e.g., current fund value and guaranteed amounts). The policy liability would then be determined from the sample (empirical) distribution of results by considering one or more statistical measures. We believe this method overall maintains consistency with the CALM method of valuation in the Life SOP.

The Task Force believes that a conditional tail expectation (“CTE”) is the most appropriate measure for setting the policy liabilities and is consistent with preferred stochastic approaches for determining minimum total balance sheet provisions (policy liability plus minimum capital). CTEs are robust, easy to calculate, readily interpretable and possess many desirable statistical properties. Furthermore, CTEs are ideally suited to a “tolerance of loss” perspective (i.e., establish a balance sheet provision such that a loss greater than $\$X$ is less than Y).

The conditional tail expectation (“CTE”) is a conditional expected value based on downside risk and can be defined as the average of outcomes that exceed a specified value such as the Q^{th} percentile. The outcomes are probability-weighted according to the likelihood of occurrence. For example, $\text{CTE}(Q\%)$ would be the weighted-average of outcomes exceeding the Q^{th} sample percentile when results are ordered from best (lowest net cost) to worst (greatest net cost). In this context, $\text{CTE}(Q\%)$ can simply be calculated as the weighted-average of the worst $(100-Q)\%$ of results from stochastic modelling. Note that, by definition, $\text{CTE}(0\%)$ is the sample mean.

Admittedly, there are other measures that could be used to establish liabilities from stochastic scenario analysis. Two obvious examples include “empirical percentile values” or the “mean plus some multiple of deviation.”

For example, one might look at the $\text{CTE}(80\%)$, the 90th percentile and the sample mean plus 1.25 times downside deviation as roughly (in a non-rigorous statistical sense) equivalent risk metrics. While each of these statistics provides a meaningful measure of exposure, the CTE is more robust and displays the most desirable characteristics.

Like conditional expectations, percentile (or quantile) estimates are easily interpreted and naturally lead to confidence statements, but may be unduly sensitive to small fluctuations in the economic environment (e.g., market value to guaranteed benefit ratio) and the number of scenarios tested.

Measures which are a function of the sample mean and the spread of risk about the mean (e.g., standard or downside deviation) are easily calculated and familiar to most actuaries. Unfortunately, these measures do not always work well for distributions that are decidedly asymmetrical such as the often highly skewed sample distributions for net cost on segregated fund investment guarantees. Furthermore, it is difficult to appreciate the degree of coverage (confidence) provided by a metric such as “mean + $K \times$ downside deviation” without reference to some other sample measures such as quantiles or CTEs.

For these reasons, conditional expectations are more useful in setting policy liabilities and should lead to more appropriate income emergence consistent with reporting under Canadian GAAP. Furthermore, in focusing on downside risk, conditional tail expectations can be used to naturally integrate the various levels of risk provisioning in a company’s balance sheet (i.e., liabilities versus minimum capital).

4.3 Selecting the CTE% for the Policy Liabilities

Establishing the appropriate CTE% for the policy liabilities is an important consideration. The difference between this measure and the mean result (with margins for adverse deviations applied to all non-stochastic variables) establishes the dollar provision for the scenario-tested assumptions (most notably, investment model uncertainty).

The acceptable range for the CTE measure can notationally be written as $CTE(X+Y+Z)$. Notwithstanding the other margins incorporated into the valuation, the three primary components underlying the investment model uncertainty, and thereby affecting appropriate values for “X”, “Y” and “Z”, are:

a) *Random Component (X):*

Given the investment model and associated parameters, there may still be considerable uncertainty regarding future experience. At a minimum, the liability should make sufficient provision for this source of randomness. The Task Force believes that $X=0.60$ is an appropriate lower bound on the policy liabilities as it establishes a result that is generally in excess of one standard deviation above the mean. Note that $CTE(60\%)$ generally provides a level of coverage exceeding the 80th percentile and provides the minimum liability in terms of incorporating a provision for market uncertainty when the stochastic investment models have been calibrated according to the criteria prescribed elsewhere in this report. It does not provide a margin for parameter uncertainty, basis risk or other model risks.

b) *Parameter Uncertainty (Y):*

The parameters used in the stochastic model are necessarily estimates. The degree of uncertainty associated with these estimates can be quantified using modern statistical techniques such as Markov Chain Monte Carlo methods. Further uncertainty is introduced by several factors including the reliability of historical experience data to predict plausible future outcomes in terms of frequency and severity, the number of scenarios being run, the quality of the stochastic model/processes and the “fit” of the benchmark used to establish parameters. As a practical guide, $Y=0.10$ would provide an appropriate range for the policy liabilities in most circumstances.

c) *Model Risk (Z):*

Notwithstanding the provisions incorporated for the stochastic model and parameter uncertainty, some residual uncertainty may remain (e.g., tracking error between actual and simulated fund returns and the ability of the model to capture all materially relevant investment and market related elements). As such, a range for Z of 0 to 0.10 is generally appropriate to provide for additional uncertainty.

In deciding what values are appropriate to account for the various components of uncertainty (i.e., the values of “Y” and “Z” above), the actuary should be particularly mindful of the margins in the valuation assumptions (explicit or otherwise) and the degree of conservatism in the model itself (including the parameters). For example, while it would generally be difficult to justify using a value of 0 for either “Y” or “Z”, this could be appropriate if the model parameters or approximations were deliberately conservative to account for uncertainty. Such conservatism should be verified through testing.

“Y” and “Z” should also vary according to the risk profile of the business being valued since this will greatly amplify or depress the impact of parameter mis-estimation. Parameter uncertainty is most sensitive to the average “term-to-maturity” of the guarantees, with the largest effects arising from the longer-term contracts. Generally, the impact of parameter uncertainty is *smaller* for closer-to-expiry, in-the-money guarantees.

The aforementioned considerations lead to a standard range of practice for establishing the policy liabilities between CTE(60%) and CTE(80%). Setting the liabilities in excess of CTE(80%) would not normally be an acceptable practice as the resulting coverage would be excessive (above the 90th percentile) and inconsistent with GAAP. Provision for more catastrophic, implausible or unknown events is done through required capital.

Ideally, the liability model should attempt to incorporate dynamics for those contingencies which can reasonably be expected to vary according to future states of the world (may be path dependent) in a measurable way. This would include agent and policyholder behaviour as well as company management practices.

Care should be taken to ensure that any modelled dynamic behaviour reasonably reflects the possible range of future experience and is not inherently biased (i.e., not overly optimistic). The actuary need not assume the extremes of completely rational or irrational behaviour based purely on financial considerations, but should ensure that the *dynamics* of the behavioural relationship naturally incorporate appropriate margins under adverse conditions and that the projected range of future experience is consistent with the other variables in the model, including the other non-scenario tested assumptions.

For dynamically modelled assumptions, the underlying “fixed” (non-variable) rate of behaviour should be based on long-term best estimate assumptions. Practical considerations, such as an absence of reliable and pertinent experience data, may dictate some level of conservatism.

Further guidance on establishing provisions for scenario tested variables may be found in section 6 of the CIA research paper *Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP* (August 2001).

It may also be instructive for the actuary to reconcile the results produced by the above methodology to the results that would have been produced by applying appropriate margins directly to the investment model parameters (e.g., to the volatility and drift parameters). These margins in the parameters could be based on statistical measures such as standard error derived from historical benchmark data. Such an analysis will allow the actuary to better understand the nature and scope of the provisions embedded in the policy liabilities.

4.4 Establishing Margins for Adverse Deviations for Other Assumptions

All other cash flow assumptions (i.e., the so-called “static” or non-scenario tested assumptions that do not vary by scenario) should explicitly incorporate MFADs consistent with current accepted actuarial standards of practice and related guidance from practice committees.

Risk factors which are not scenario tested, but could reasonably be expected to vary according to (a) a stochastic process, or (b) future states of the world (especially in response to economic drivers) may require additional margins and/or demand higher margins for certain other assumptions.

One of the challenges in applying MFADs to the non-scenario tested assumptions is the fact that, *a priori*, it may not be possible to know the appropriate signs of the margins at future points in time

due to the complex interactions in the model. Indeed, the sign of the margin may change over time or from one scenario to the next. Since it is generally not practical to model dynamically the sign and magnitude of the margin along each stochastic scenario path, it is critical that the actuary conduct prior sensitivity testing to ensure that both the sign and level of the resulting PFADs are appropriate for the valuation.

4.5 Attribution of Revenue

Under basic CIA valuation principles, the policy liabilities are determined by an explicit prospective analysis of asset and liability cash flows. Therefore, it is appropriate that, when establishing policy liabilities for segregated fund guarantees, the revenues that may potentially offset the costs of these guarantees be considered within the stochastic analysis.

For the valuation of a contract with segregated fund investment guarantees, we believe that application of the CIA Valuation Standards will generally result in contracts being valued as two separate components. The investment guarantee feature, because of the non-symmetric distribution of the cost profile and the concentration of loss exposure in the tails of the distribution, will be valued using stochastic techniques as outlined in this paper. The remainder of the contract has a more symmetric risk profile and generally a more “normal” distribution of results, which is the risk profile envisioned by the general method of valuation and, therefore, can appropriately be valued directly using the more deterministic CALM methodology outlined in the Life SOP. This can essentially be thought of as unbundling the financial option/derivative (the segregated fund guarantee) from the base (host) underlying contract.

While the separation of the “costs” within a contract between the components of the valuation is relatively straightforward, the allocation of the revenue side that this separate valuation implies may be less clear-cut.

The most robust approach for making this allocation is for the actuary to establish the unitized amount of the revenue stream that is to be earmarked as a “risk charge” for this feature. This unitized amount is then considered as an expense in the base product valuation, similar to how third party fund management expenses that are a unitized charge would be treated. This “risk charge” would then be treated as the unitized revenue available to offset the cost of the segregated fund guarantees in the stochastic valuation of the segregated fund guarantees.

The “risk charge” that is designated does not necessarily have to match the risk charge assumed in pricing, although this could be a good starting frame of reference. It may be more, or may be less than this amount, but the key requirement is that it reflect a level of unitized revenue that the actuary expects to forego *permanently* in the base product valuation.

The concept of permanence is important. The stochastic valuation of segregated fund guarantees is naturally focussed on results in the tail of the loss distribution. Therefore, the unitized revenue that is reflected in these scenarios should be the unitized revenue that the actuary truly believes will be available in an adverse fund performance scenario. The corollary is that this revenue will not be available to offset costs in the base valuation in an adverse fund performance scenario.

If the actuary were to base the “risk charge” solely on an analysis of excess margins in the *current* base valuation, in an environment where market performance has been favourable, there is the potential to overstate the “risk charge” to be used in the stochastic valuation of the segregated fund guarantees.

The following is an example of how this could occur. The actuary believes that company practice is that some level of unitized margins/charges would be re-allocated to the base product valuation in an adverse investment performance scenario (e.g., to ensure DAC recoverability). In this scenario, the

available unitized “risk charge” in the stochastic valuation of the investment guarantee risk should be net of the projected re-allocation that would occur. In such circumstances, the true margin available as an offset could be materially lower than the amount implied by a simple analysis reflecting only the *current* environment.

The actuary should maintain documentation of the analysis supporting the allocation of the unitized revenue between the different components. The key point is that all sources of revenue may only be used once, and that treatment of this revenue is internally consistent in both pieces of the valuation.

Should the statistics used to determine the policy liabilities result in a net negative liability (i.e., the capitalized value of the revenues exceeds the capitalized guarantee cost), a zero net liability should be reported. This will be consistent with the Life SOP, which does not anticipate profit capitalization from this line of business. Some degree of offsetting would be acceptable between product forms, but the total reported net policy liabilities for segregated fund investment guarantees should not be negative.

4.6 Risk Management Practices

The policy valuation should reflect risk management practices to the extent that such practices can reasonably be expected to have a measurable impact on financial results and are currently in place. This would ordinarily mean the company has a well-defined written policy in respect of risk management and has demonstrated an ability and willingness to enter into or execute such control strategies. Risk management practices could include many forms of reinsurance, hedging and capital markets protection. Risk management will also involve controls on product design, funds offered, product availability, pricing constraints and other factors managed by the insurer. It is incumbent on the actuary to ensure that any recognition of such practices in the policy valuation be justified through appropriate consideration of the degree of risk transfer, especially in those adverse market scenarios which give rise to the greatest guarantee costs.

As described in Section 2.3, there are significant uncertainties associated with hedging strategies, and the actuary should be cautious in determining the amount of credit to be taken for such strategies. While no minimum PFADS are recommended, it would be appropriate to consider hedging to be a high margin situation. Generally, it would be inappropriate to take any credit for a hedging strategy that is not actively in place.

4.7 Use of Approximations

CIA Standards of Practice for the Valuation of Policy Liabilities of Life Insurers do allow for the use of approximations based on materiality, and factoring in the time, cost and other pragmatic considerations of producing a more refined result.

We anticipate that many actuaries will argue that the obligation to use stochastic models to establish policy liabilities is an onerous requirement, and that for the pragmatic reasons specified previously, acceptable approximation methods should be clearly laid out and that materiality limits be set at relatively high levels.

However, the Task Force believes the use of stochastic techniques should be encouraged in all but the most limited circumstances for the following reasons:

- From a risk management perspective, a company cannot properly manage and measure exposure to this risk without stochastic modelling capabilities, therefore, the inability to do stochastic modelling suggests an inability to truly manage the risk.

- Recently, a wide volume of literature and education has become available to actuaries in how to develop and apply stochastic techniques. Through continuing education, actuaries should clearly be becoming more familiar with the design and application of stochastic models.
- A number of professionals are available in Canada who can provide expertise and support in developing stochastic analysis on a consulting basis.
- We believe that the calibration criteria and other requirements we have specified are not unduly complicated, and that for a knowledgeable professional, implementation of these requirements is not overly difficult.
- Lastly, approximation techniques themselves will need to be relatively complicated in order to produce reasonable results.

Notwithstanding this rationale, should an actuary be able to justify the use of approximation techniques (for example, on the grounds of materiality), the Task Force strongly believes that a realistic “factor” driven approach (using the method for minimum capital proposed in this note as a broad approach) is a preferred alternative.

While this approach would take some initial development (i.e., the factors would need to be derived, the actuary would have to determine at what “percentage” in the tail distribution policy liability requirements should be set, etc.), it does avoid period-to-period application of a stochastic model. Another alternative would set the policy liabilities as a percentage of the total balance sheet provision obtained by applying the factor-based approach as promulgated by OSFI.

The Task Force strongly believes that “single” scenario deterministic tests, such as those previously governing the prescribed minimum liability, are inappropriate and should not be considered acceptable actuarial practice in establishing the liabilities for segregated fund guarantees.

5. DYNAMIC CAPITAL ADEQUACY TESTING

The Task Force enthusiastically endorses the role of DCAT to determine the risks that segregated fund investment guarantees pose to the overall solvency of a company. Resiliency testing of catastrophic loss scenarios is a natural fit for a liability where the majority of the risk is catastrophic tail loss exposure. Indeed, deterministic equity correction tests naturally fit as DCAT scenarios.

Because of the general nature of the risk of loss associated with these guarantees (low incidence of loss, but potentially high severity of loss in scenarios where losses occur), it is unlikely that the policy liabilities and minimum capital held against these specific contingencies will cover the potential losses in truly catastrophic scenarios. A strongly focussed DCAT requirement that requires companies with material business volumes to annually assess their consolidated balance sheet exposure to a catastrophic equity market correction is the ultimate test of a company's capacity to prudently accept this risk.

DCAT testing is the most appropriate way to assess the ability of a company to absorb catastrophic loss exposure as:

- Loss events that can cause catastrophic losses, such as significant and prolonged equity market corrections, can be explicitly tested.
- The DCAT takes into account the overall resources available to the company (e.g., free surplus as well as minimum capital and policy liabilities).
- The DCAT takes into account the overall diffusion of risk across the company (e.g., product and territorial diffusion), and allows the company to use the overall strength of its balance sheet to absorb catastrophic losses.
- The DCAT looks at both current and potential future exposure as the volume of inforce business changes. This is a key concern, since for many companies, segregated fund guarantee risk is being added at a faster rate than other risks and, therefore, constitutes an increasing share of overall balance sheet risk.
- DCAT analysis can take into account that there may be a significant correlation between catastrophic losses on segregated fund guarantee and other risks on the balance sheet. As an example, for an equity correction scenario, there may be significant correlation of segregated fund guarantee exposure with the following exposures – loss of value on surplus equities, the need for additional policy liabilities on general account liabilities backed by equities, and the loss of fee income on segregated fund products (which may have a capitalized impact on insurance products and lead to a DAC write-off for deposit contracts).

The current DCAT Standard of Practice and supporting educational note do not explicitly address segregated fund investment guarantee risks, or the broader issue of equity market exposure. These risks are becoming much more material for a significant number of companies, not just due to segregated fund investment guarantees, but also due to the general increase in the volume of index-linked on-balance sheet business and off-balance sheet segregated fund business being written.

Given these dynamics, the Task Force believes that existing DCAT Standards of Practice/Educational Guidance should be expanded to provide more direction with respect to both segregated fund guarantee risks, and more generally, exposure to equity markets.

The ability to withstand a Japan-type equity correction scenario (i.e., a significant run up in the market, a market correction of 50% and then a sustained five-to-ten-year period of flat markets) would be an appropriate scenario for an actuary with material segregated fund investment guarantee exposure to analyse in DCAT.

Given OSFI's significant concern with respect to the risk exposure of segregated fund guarantees, the CIA should consider working with OSFI/IGIF to determine DCAT guidelines. For instance, the deterministic scenario proposed by OSFI for determination of minimum total balance sheet coverage in its December 23, 1999 memorandum could easily, with some adjustments, be developed into a DCAT scenario which could also consider the company's other correlated loss exposures.

6. EDUCATION AND AWARENESS

To support the introduction of stochastic techniques and encourage their widespread adoption in appropriate circumstances, the Task Force has undertaken a number of educational initiatives during its mandate. The Task Force recommends that the CIA continue with these initiatives and that the appropriate CIA committees, as mentioned in Section 1.5, give high priority to continuing with this educational effort.

We recommend that the CIA continue to offer technical educational sessions on these topics at future actuarial meetings and also continue to sponsor seminars that allow for more in-depth treatment of these complex topics. The CIA membership has made significant progress in understanding the areas of modelling and stochastic valuation during the mandate of the Task Force, but these are still developing techniques and the profession and industry would be well served if the current momentum was maintained.

The Task Force education efforts focused on these main topics:

- *Stochastic modelling of investment returns and segregated fund products.* We recommend that responsibility for this area be assigned to The Committee on Investment Practice.
- *Developing standards for stochastic valuation.* We recommend that this work be assigned to The Committee on Life Insurance Financial Reporting. The CLIFR Working Group on The Use of Stochastic Valuation Techniques has already produced a Research Paper “The Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP” (August 2001).
- *Developing standards for stochastic capital.* We recommend that this area be assigned to The Committee on Risk Management and Capital Requirements.

The educational efforts of the Task Force took the form of numerous presentations, papers and articles by members of the Task Force.

Of course, this report is the main formal publication produced by the Task Force. We hope that this will serve as a useful resource to the profession. Previous drafts of this report have already been considered for the SOA Course 8-Finance syllabus and included on the CIA Investment / Finance Practice Education Course (PEC).

In addition to their contributions to this report, Task Force members have produced a number of publications including articles for the North American Actuarial Journal, SOA Study Notes and even a book on Segregated Funds commissioned by the SOA.

Appendix A provides a list of these publications as well as a list of other papers and resources we have found useful our research.

Members of the Task Force have also spoken at various industry meetings and seminars to explain the topics discussed in this report. Presentations have also been made to regulators and to other industry and professional bodies, including a presentation to representatives from the American Academy of Actuaries.

The major educational events sponsored by the Task Force were the *2000 Segregated Fund Seminar* held on September 20, 2000 and the *2001 Symposium on Stochastic Modelling for Variable Annuity/Segregated Fund Investment Guarantees* held on September 5, 2001. These sessions built on the work started at the *1999 Symposium on Stochastic Modelling*. Links to the programs for each of these events, including a number of the presentations and papers, are also provided in Appendix A.

In addition to these seminars, Task Force members have or are scheduled to make presentations at a number of meetings including:

2000

- September CIA Appointed Actuary Seminar
- October Quebec Valuation Actuaries Meeting
- October CLHIA Segregated Fund Section Meeting
- November Philadelphia SOA Investment Seminar
- November CIA Investment Seminar
- November CIA General meeting
- Fields Institute Workshop on valuing options in financial products

2001

- June CIA Annual meeting
- June Joint CIA/SOA meeting
- June SOA Spring Meeting
- August ARC (Actuarial Research Conference)
- September AFIR Meeting
- November CIA Investment Seminar
- November CIA General meeting

2002

- March IAA International Congress

APPENDIX A – Recommended Reading and Resources

1. CIA Reports and Papers

Report of the CIA Task Force on Segregated Fund Investment Guarantees (August 1, 2000). This version was posted on Tuesday January 23, 2001:

<http://www.actuaries.ca/publications/2000/20020e.pdf>

November 1998 CIA Research Paper: *Financial Considerations of Segregated Fund Investment Guarantees*

<http://www.actuaries.ca/publications/1998/9858e.pdf>

CIA Working Group Report "The Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP" (August 2001): <http://www.actuaries.ca/publications/2001/20169e.pdf>

2. CIA Speciality Seminars

These links include presentations and papers from the following seminars:

2001 Symposium on Stochastic Modelling for Variable Annuity/Segregated Fund Investment Guarantees:

http://www.actuaries.ca/meetings/archive_segfund2001_e.html

2000 Segregated Fund Seminar:

http://www.actuaries.ca/meetings/archive_segfund2000_e.html

1999 Symposium on Stochastic Modelling for Segregated Fund/Variable Annuity Investment Guarantees:

http://www.actuaries.ca/meetings/archive_segfund1999_e.html

3. Links to Other CIA Meeting Presentations

November 2000 CIA General Meeting: PD-21 Implementation of MCCSR Capital Requirements for Segregated Fund Guarantees:

http://www.actuaries.ca/meetings/general/2000/m_taylor.pdf

September 2000 Seminar for the Appointed Actuary: PD-11 CLIFR Topics Segregated Funds:

http://www.actuaries.ca/meetings/aa/2000/Geoff_Hancock_PD11.htm

June 2000 CIA Annual meeting: PD-34 Report from the CIA Task Force on Segregated Fund Guarantees:

http://www.actuaries.ca/meetings/annual/2000/Curtis_Hancock.pdf

4. MCCR Guidelines for Segregated Funds

The OSFI website (www.osfi-bsif.gc.ca) includes documentation of the MCCR requirements for Segregated Funds.

The IGIF website includes documentation of the Capital Adequacy Requirements (CAR) for Segregated Funds, including guidance on capital offset for Segregated Fund hedging programs: <http://www.igif.gouv.qc.ca> (see the guidelines section).”

5. Other Reports and Papers

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Brender Allan (2001) “The Use of Internal Models for Determining Liabilities and Capital Requirements”. *Presented to the 2001 Bowles Symposium Georgia State University Atlanta, GA*

Brennan M.J. and E.S. Schwartz (1976) “The Pricing of the Equity-Linked Life Insurance Policies with an Asset Value Guarantee”, *Journal of Financial Economics*, Vol. 3, pp.195-213

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Hardy M. R. (2001), “A regime switching model of long term stock returns”, *North American Actuarial Journal*, 5.2:41-53.

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APPENDIX B – TSE300 Total Return Index Jan 1956 to Dec 1999

1956.01	246.77	1961.01	321.39	1966.01	604.39	1971.01	768.13	1976.01	987.24
1956.02	256.42	1961.02	332.57	1966.02	590.76	1971.02	768.66	1976.02	1022.24
1956.03	278.04	1961.03	340.11	1966.03	585.10	1971.03	802.39	1976.03	1006.70
1956.04	276.03	1961.04	356.51	1966.04	592.00	1971.04	794.09	1976.04	1031.13
1956.05	265.26	1961.05	364.01	1966.05	573.16	1971.05	778.74	1976.05	1033.91
1956.06	273.65	1961.06	367.96	1966.06	572.38	1971.06	795.96	1976.06	1018.99
1956.07	294.41	1961.07	375.49	1966.07	564.55	1971.07	787.11	1976.07	1019.02
1956.08	288.20	1961.08	381.77	1966.08	523.94	1971.08	783.56	1976.08	1019.27
1956.09	271.73	1961.09	376.89	1966.09	513.34	1971.09	758.44	1976.09	1003.73
1956.10	266.09	1961.10	379.96	1966.10	524.97	1971.10	710.90	1976.10	970.65
1956.11	252.16	1961.11	391.82	1966.11	522.32	1971.11	731.75	1976.11	904.56
1956.12	273.42	1961.12	402.71	1966.12	537.86	1971.12	803.48	1976.12	1000.00
1957.01	275.72	1962.01	393.28	1967.01	576.48	1972.01	877.00	1977.01	992.88
1957.02	265.21	1962.02	396.21	1967.02	583.22	1972.02	909.65	1977.02	1009.55
1957.03	275.35	1962.03	397.06	1967.03	605.26	1972.03	888.65	1977.03	1028.14
1957.04	288.42	1962.04	387.27	1967.04	619.42	1972.04	900.10	1977.04	1002.94
1957.05	297.03	1962.05	355.65	1967.05	597.15	1972.05	917.17	1977.05	993.16
1957.06	291.66	1962.06	334.16	1967.06	615.23	1972.06	908.47	1977.06	1050.85
1957.07	284.78	1962.07	343.14	1967.07	636.82	1972.07	932.88	1977.07	1055.55
1957.08	257.03	1962.08	352.16	1967.08	630.53	1972.08	978.63	1977.08	1028.45
1957.09	239.95	1962.09	336.76	1967.09	645.60	1972.09	967.21	1977.09	1030.73
1957.10	222.99	1962.10	342.95	1967.10	609.70	1972.10	933.31	1977.10	1002.32
1957.11	228.43	1962.11	370.21	1967.11	626.10	1972.11	995.88	1977.11	1055.48
1957.12	217.14	1962.12	374.14	1967.12	635.15	1972.12	1023.50	1977.12	1107.06
1958.01	226.37	1963.01	393.65	1968.01	619.06	1973.01	1044.42	1978.01	1044.55
1958.02	224.46	1963.02	381.51	1968.02	593.28	1973.02	1029.03	1978.02	1056.54
1958.03	232.31	1963.03	394.45	1968.03	581.37	1973.03	1040.96	1978.03	1124.21
1958.04	231.22	1963.04	414.56	1968.04	634.95	1973.04	1000.60	1978.04	1145.62
1958.05	241.05	1963.05	421.77	1968.05	629.91	1973.05	956.91	1978.05	1200.09
1958.06	248.08	1963.06	407.89	1968.06	665.81	1973.06	972.25	1978.06	1205.44
1958.07	260.92	1963.07	398.11	1968.07	663.87	1973.07	1034.76	1978.07	1280.39
1958.08	264.79	1963.08	401.47	1968.08	682.47	1973.08	1017.08	1978.08	1325.42
1958.09	274.55	1963.09	415.11	1968.09	717.69	1973.09	1065.89	1978.09	1388.49
1958.10	278.12	1963.10	419.49	1968.10	730.39	1973.10	1135.71	1978.10	1316.75
1958.11	279.70	1963.11	415.83	1968.11	758.78	1973.11	1011.86	1978.11	1382.25
1958.12	284.99	1963.12	432.51	1968.12	777.71	1973.12	1026.30	1978.12	1436.08
1959.01	293.48	1964.01	445.28	1969.01	799.95	1974.01	1052.72	1979.01	1488.17
1959.02	296.18	1964.02	442.34	1969.02	763.47	1974.02	1084.59	1979.02	1526.03
1959.03	295.19	1964.03	461.23	1969.03	790.57	1974.03	1053.51	1979.03	1623.65
1959.04	298.60	1964.04	479.38	1969.04	814.65	1974.04	959.39	1979.04	1646.32
1959.05	299.41	1964.05	496.53	1969.05	836.87	1974.05	899.00	1979.05	1692.56
1959.06	302.81	1964.06	497.63	1969.06	749.48	1974.06	884.15	1979.06	1813.17
1959.07	317.12	1964.07	510.63	1969.07	708.94	1974.07	901.59	1979.07	1748.91
1959.08	301.47	1964.08	508.06	1969.08	745.52	1974.08	811.60	1979.08	1911.69
1959.09	288.11	1964.09	531.69	1969.09	743.29	1974.09	738.66	1979.09	1983.15
1959.10	288.78	1964.10	534.86	1969.10	756.17	1974.10	811.65	1979.10	1791.16
1959.11	287.53	1964.11	537.63	1969.11	776.43	1974.11	759.12	1979.11	1937.24
1959.12	298.06	1964.12	542.51	1969.12	771.42	1974.12	760.21	1979.12	2079.01
1960.01	285.31	1965.01	574.34	1970.01	743.83	1975.01	885.96	1980.01	2327.62
1960.02	276.46	1965.02	572.96	1970.02	764.70	1975.02	912.71	1980.02	2524.47
1960.03	277.52	1965.03	570.05	1970.03	767.41	1975.03	901.76	1980.03	2079.19
1960.04	273.38	1965.04	585.26	1970.04	701.78	1975.04	922.57	1980.04	2167.01
1960.05	281.77	1965.05	584.59	1970.05	634.44	1975.05	945.03	1980.05	2294.76
1960.06	276.03	1965.06	546.83	1970.06	624.46	1975.06	972.61	1980.06	2411.25
1960.07	270.85	1965.07	542.64	1970.07	655.40	1975.07	974.11	1980.07	2577.58
1960.08	289.21	1965.08	557.04	1970.08	673.63	1975.08	964.09	1980.08	2609.33
1960.09	276.00	1965.09	568.53	1970.09	706.91	1975.09	910.29	1980.09	2668.43
1960.10	276.66	1965.10	580.04	1970.10	688.98	1975.10	872.15	1980.10	2648.72
1960.11	287.62	1965.11	564.46	1970.11	720.17	1975.11	920.68	1980.11	2850.93
1960.12	303.37	1965.12	578.76	1970.12	743.91	1975.12	900.72	1980.12	2705.49

APPENDIX B – Calibration Data Series: TSE300 Total Return Index Jan 1956 to Dec 1999

1981.01	2658.85	1986.01	4159.78	1991.01	5654.56	1996.01	9913.64
1981.02	2611.52	1986.02	4186.74	1991.02	5999.86	1996.02	9861.55
1981.03	2807.32	1986.03	4486.28	1991.03	6083.50	1996.03	9963.35
1981.04	2778.42	1986.04	4539.68	1991.04	6050.81	1996.04	10323.89
1981.05	2866.99	1986.05	4612.00	1991.05	6213.23	1996.05	10543.37
1981.06	2869.25	1986.06	4577.12	1991.06	6102.37	1996.06	10167.15
1981.07	2745.31	1986.07	4361.33	1991.07	6242.16	1996.07	9944.05
1981.08	2659.84	1986.08	4508.28	1991.08	6220.42	1996.08	10391.76
1981.09	2311.46	1986.09	4451.70	1991.09	6014.44	1996.09	10716.42
1981.10	2265.88	1986.10	4546.86	1991.10	6249.65	1996.10	11350.62
1981.11	2486.19	1986.11	4568.24	1991.11	6146.27	1996.11	12217.32
1981.12	2428.29	1986.12	4618.32	1991.12	6291.90	1996.12	12061.95
1982.01	2224.23	1987.01	5049.63	1992.01	6452.51	1997.01	12444.13
1982.02	2087.86	1987.02	5283.82	1992.02	6442.57	1997.02	12560.15
1982.03	1998.61	1987.03	5666.89	1992.03	6162.76	1997.03	11961.20
1982.04	1952.15	1987.04	5638.25	1992.04	6069.05	1997.04	12227.41
1982.05	1931.99	1987.05	5602.05	1992.05	6143.26	1997.05	13079.21
1982.06	1745.70	1987.06	5705.34	1992.06	6170.05	1997.06	13222.76
1982.07	1808.33	1987.07	6156.23	1992.07	6279.34	1997.07	14135.23
1982.08	2074.04	1987.08	6109.49	1992.08	6221.23	1997.08	13605.16
1982.09	2071.64	1987.09	5987.36	1992.09	6054.57	1997.09	14513.63
1982.10	2297.60	1987.10	4638.82	1992.10	6131.98	1997.10	14116.30
1982.11	2365.03	1987.11	4589.86	1992.11	6052.93	1997.11	13454.71
1982.12	2528.74	1987.12	4889.82	1992.12	6201.72	1997.12	13868.54
1983.01	2627.11	1988.01	4737.55	1993.01	6124.83	1998.01	13881.55
1983.02	2708.26	1988.02	4976.47	1993.02	6406.98	1998.02	14711.40
1983.03	2802.36	1988.03	5166.26	1993.03	6714.88	1998.03	15706.32
1983.04	3046.42	1988.04	5213.41	1993.04	7071.07	1998.04	15938.75
1983.05	3215.38	1988.05	5088.04	1993.05	7271.67	1998.05	15799.89
1983.06	3263.28	1988.06	5410.06	1993.06	7455.35	1998.06	15367.27
1983.07	3309.45	1988.07	5317.08	1993.07	7463.91	1998.07	14469.33
1983.08	3325.25	1988.08	5196.58	1993.08	7798.17	1998.08	11560.18
1983.09	3359.74	1988.09	5212.87	1993.09	7544.63	1998.09	11761.87
1983.10	3179.01	1988.10	5399.30	1993.10	8052.76	1998.10	13017.91
1983.11	3442.98	1988.11	5255.58	1993.11	7926.13	1998.11	13319.00
1983.12	3472.33	1988.12	5431.68	1993.12	8220.23	1998.12	13648.84
1984.01	3364.18	1989.01	5803.90	1994.01	8670.34	1999.01	14169.33
1984.02	3306.70	1989.02	5745.48	1994.02	8436.07	1999.02	13306.03
1984.03	3269.40	1989.03	5782.84	1994.03	8283.08	1999.03	13937.91
1984.04	3194.12	1989.04	5871.01	1994.04	8170.41	1999.04	14828.82
1984.05	3075.06	1989.05	6025.56	1994.05	8301.33	1999.05	14481.01
1984.06	3078.32	1989.06	6138.42	1994.06	7748.33	1999.06	14864.79
1984.07	2973.02	1989.07	6492.54	1994.07	8051.04	1999.07	15028.68
1984.08	3328.25	1989.08	6574.01	1994.08	8393.82	1999.08	14811.04
1984.09	3346.41	1989.09	6489.76	1994.09	8426.82	1999.09	14811.58
1984.10	3296.98	1989.10	6463.05	1994.10	8312.82	1999.10	15458.63
1984.11	3328.00	1989.11	6520.92	1994.11	7945.24	1999.11	16044.51
1984.12	3389.25	1989.12	6592.58	1994.12	8205.73	1999.12	17977.46
1985.01	3669.57	1990.01	6162.06	1995.01	7830.41		
1985.02	3677.69	1990.02	6146.24	1995.02	8053.21		
1985.03	3720.81	1990.03	6093.70	1995.03	8451.13		
1985.04	3759.58	1990.04	5602.46	1995.04	8391.00		
1985.05	3917.24	1990.05	6027.09	1995.05	8741.88		
1985.06	3899.51	1990.06	5990.42	1995.06	8923.67		
1985.07	4001.53	1990.07	6031.03	1995.07	9104.07		
1985.08	4071.54	1990.08	5686.33	1995.08	8926.86		
1985.09	3814.15	1990.09	5390.49	1995.09	8977.74		
1985.10	3882.56	1990.10	5268.72	1995.10	8845.80		
1985.11	4157.03	1990.11	5406.51	1995.11	9265.66		
1985.12	4238.78	1990.12	5617.01	1995.12	9397.97		

APPENDIX C – Model Calibration of Fixed Volatility Lognormal Model

Using the standard statistical definition, if the ratio of the stock prices is lognormal then:

$$\log_e \left(\frac{S_T}{S_t} \right) \sim N(\mu(T-t), \sigma^2(T-t)) \quad (1)$$

For $T > t$.

If Y is the ratio of the one period stock prices ($Y = S_{t+1} \div S_t$) then

$$E[Y] = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

However, a number of finance texts use a different definition where the assumption that stock returns follow a lognormal distribution is denoted as:

$$\log_e \left(\frac{S_T}{S_t} \right) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)(T-t), \sigma^2(T-t)\right) \quad (2)$$

In this case, $E[Y] = \exp(\mu)$.

Clearly, in order to correctly calibrate your model, you should be sure which of these definitions applies. We will use the second definition provided by (2) in this appendix.

The following points use the step numbers from the calibration process discussed in Section 2.1.2 *Calibration of Investment Return Models*.

Step 1. Fit the model to the existing data.

First we compute μ and σ from the monthly TSE 300 total return data from January 1956 to December 1999 inclusive (month end closing values as provided in Appendix B) as follows:

Since this is monthly data, define $T - t = 1/12$.

Using the monthly closing price at the end of each month, calculate

$$r_i = \log_e \left(\frac{S_{i+1}}{S_i} \right) \quad \text{where } r_i \text{ is the one-month return on the index, it is not annualized.}$$

Compute the sample mean and the sample standard deviation of the r_i

$$\hat{r} = 0.81374\%$$

$$\hat{\sigma}_r = 4.51133\%$$

Convert these to the instantaneous return μ and volatility σ

$$\sigma = \hat{\sigma}_r \sqrt{12} = 15.6277\%$$

$$\mu = \hat{r}(12) + \frac{1}{2}\sigma^2 = 10.9860\%$$

Note: the expected one-year accumulation factor (Y) is:

$$E[Y] = \exp(\mu) = 1.116122$$

Steps 2 and 3. Test the accumulation factors generated by the model.

Note that there are nine calibration points expressed as percentiles included in Table 1. These points are a test of the skewness or fatness of the tail of the distribution at the end of one, five and ten years. There is also a requirement that the expected value of the one-year accumulation factor should lie in the range 1.10 to 1.12 and that the standard deviation of the annual accumulation factor should be at least 0.175. So we have at least 11 possible constraints to use in calibrating our model.

However, the simple lognormal model has only two parameters – representing drift and volatility. To satisfy the percentile constraints, we will have to adjust the volatility of the distribution. So if we find the most severe of the nine percentile constraints we can use it to calibrate the model and the others will also be satisfied. For the lognormal model and the calibration criteria from Table 1, the one-year 2.5th percentile condition turns out to be the most important constraint.

If Y_p is the expected 100 p percentile of the Δt period accumulation factor and given (2)

$$Y_p \text{ must satisfy } \Phi\left(\frac{\log_e(Y_p) - (\mu - \frac{1}{2}\sigma^2)\Delta t}{\sigma\sqrt{\Delta t}}\right) = p \quad (3)$$

Here, $\Phi(\cdot)$ is the standard N(0,1) cumulative distribution function (c.d.f.)

To test the one-year 2.5th percentile calibration point, we set $p = 0.025$ and $\Delta t = 1$ and solve equation (3) for Y_p .

This gives $Y_p = \exp(\sigma\Phi^{-1}(p) + (\mu - \sigma^2/2)) = .812$. Since 0.812 is greater than the required value of 0.76, the model does not satisfy this calibration criteria (because the tail is not fat enough) and the parameters must be adjusted or another model selected.

Step 4. Adjust the model to fit the criteria.

We can rewrite equation (3) as:

$$\frac{1}{2}\sigma^2\Delta t - \sigma\sqrt{\Delta t}\Phi^{-1}(p) + \log_e(Y_p) - \mu\Delta t = 0$$

Using the one-year, 2.5th percentile value, $Y_p = .76$, and assuming that μ will not change, the equation to solve becomes:

$$\frac{1}{2}\sigma^2 - \sigma\Phi^{-1}(.025) + \log_e(.76) - 10.9860\% = 0 \Rightarrow \sigma = 18.7140\%$$

So $\mu = 10.9860\%$ and $\sigma = 18.714\%$ will meet the one-year 2.5th percentile calibration criteria.

You can redo this calculation for each of the other nine percentile calibration points to confirm that they produce lower values for volatility with the same expected return. This confirms that for the lognormal model calibrated to the prescribed criteria, the one-year 2.5th percentile condition is the most important constraint and the one that will drive the final parameters that should be used.

Step 5. Adjustment if you are using different data.

This does not apply in this case since we are using the TSE data directly. However, if you are using other data to calibrate a lognormal model you should look at the adjustment calculated in step 3 ($3.0863 = 18.714 - 15.6277$) and use the same adjustment to increase the volatility calculated from your new data set.

Step 6. Each of the maximum return criteria must be met.

This was done and discussed in steps 2 and 4.

Step 7. Simulated values should reproduce analytically calculated results.

In the steps above, we have calibrated the model analytically rather than by simulation. This step is a reminder that the simulation could still not meet the calibration criteria if there are problems with the random number generator used or if not enough simulations are run. This could be tested by running a number of simulations and comparing the results to the expected values.

Step 8. Check mean and standard deviation of the one-year accumulation factor.

As previously noted above $E[Y] = \exp(\mu) = 1.116122$ which is within the required range of 1.10 to 1.12.

We can also calculate the standard deviation of Y as 21.1% using the equation:

$$\text{Var}[Y] = \exp(2\mu) [\exp(\sigma^2) - 1]$$

This is greater than the minimum required value of 17.5%.

The lognormal distribution with $\mu = 10.9860\%$ and $\sigma = 18.714\%$ has now been shown to satisfy all the calibration criteria and this model can now be used to generate a set of scenarios for TSE returns to be used in policy liability calculations.

APPENDIX D – Calibration of Other Stochastic Investment Return Models

A more general procedure for calibrating models is as follows.

Step 1. Fit the model to the existing data

The model should be fitted to the TSE total return data from January 1956 to December 1999 using maximum likelihood estimation or a similar statistical procedure.

Maximum likelihood estimation:

The question which is answered with maximum likelihood estimation is: *Given a particular set of observed data, what set of parameters gives the highest probability of observing the data?* Maximum likelihood estimates in the lognormal case are simply given by the mean and the standard deviation of the observed log-returns.

For models other than the lognormal model, maximum likelihood parameters cannot in general be simply extracted from statistical moments of the data. You have to construct the likelihood function and find the parameters which give the maximum value.

The likelihood function is proportional to the probability of actually observing the data, given the assumed model and a set of parameters. So, maximizing the likelihood function is equivalent to maximizing the probability of observing the data. Technically, the likelihood function is the joint probability density function of the observed data for the given set of parameters.

Suppose your model has m parameters to be estimated. The set of parameters is represented by the vector Θ . For example, a first order autoregressive model has three parameters, the long-term mean, μ , say; the conditional standard deviation, σ and the autoregression parameter a . In this case $\Theta = (\mu, a, \sigma)$.

The likelihood function, $L(\Theta)$, is the joint density function of the observed data given the parameters in Θ . Let x_t denote the t -th value of the total return data, or, more commonly, of the log-return data.

If the returns in successive periods are assumed to be independent, then this density is the product of all the individual density functions. This is true, for example, using the lognormal model or the stable distribution model. For a sample dataset of size n , the likelihood function is:

$$L(\Theta) = \prod_{i=1}^n f(x_i | \Theta)$$

For many models the returns are not assumed to be independent. For example, for auto regressive models, regime switching models, stochastic volatility models, and the Wilkie model, the values for successive periods depend on the previous values in some way. In this case, the likelihood function can be constructed iteratively as:

$$L(\Theta) = \prod_{i=1}^n f(x_i | x_1, \dots, x_{i-1}, \Theta)$$

It is usually more convenient to work with the log-likelihood,

$$l(\Theta) = \log L(\Theta) = \sum_{r=1}^n \log f(x_r | x_1, x_2, \dots, x_{r-1}, \Theta)$$

This is simple to calculate for AR, ARCH and GARCH type models, since the conditional density of the log-return at t is known, usually assumed normal, with mean and variance dependent on the previous values for the log-return. This means that, for example, for an autoregressive model of order 1, each value for x_t depends only on the previous value, x_{t-1} and the parameters, Θ . For regime switching models, the calculation is slightly more complicated. Details for the general regime switching family are given in Hamilton and Susmel (1994) and for the regime switching lognormal model, in Hardy(1999).

Maximization:

Once the log-likelihood function is evaluated, the maximum likelihood estimator for the parameters is found by maximizing over the parameter vector. This means finding the set of parameters that gives the largest value for $l(\Theta)$. This can be done using spreadsheet functions (such as ‘Solver’) or with statistical software. These parameters are the maximum likelihood parameters for use in the next section. In maximizing the likelihood function, care should be taken to have a good maximization algorithm: in some instances, a maximization sub-routine can get stuck on a local maxima which is not the global maxima.

If you are using the Wilkie model, the procedure described in this step can be followed separately for each series, moving down the “cascade.” Start by calculating the maximum likelihood parameters for the inflation model. Then find the parameters for the next series in the cascade which is the dividend yield, followed by the dividend index and the fixed interest models.

Step 2. Test the accumulation factors generated by the model

Use the model to calculate the 2.5th, 5th and 10th percentiles of the one-year, five-year and ten-year accumulation functions. For some models it will be possible to calculate the percentiles analytically; if not, generate the percentiles through simulation – go to step 2A. If it is possible, analytic calculation is preferable as it removes the effect of sampling error.

Use neutral starting values. This can be achieved for example by using long-term, unconditional mean values for past observations required to generate future values. If any of the percentile values calculated are less than the values in Table 1, it will be necessary to adjust the parameters.

It may be simpler in some cases to calculate the distribution functions for the one-year, five-year and ten-year accumulation factors; that is, if the random one-year accumulation factor is A_1 , the distribution function is:

$$F_{A_1}(y) = \Pr[A_1 \leq y]$$

Evaluate the appropriate distribution function for each of the values in Table 1. If the value of the distribution function is greater than the percentile being tested, the model passes the calibration test. If it is less, then it will be necessary to adjust the parameters.

For example, the 2.5th percentile of the one-year accumulation factor must be less than the table value of 0.76. Calculate $F_{A_1}(0.76)$. If this is bigger than 0.025, then the 2.5th percentile of the model distribution must be less than 0.76 as required. If $F_{A_1}(0.76)$ is less than 0.025, then the tail of the distribution is too thin and the parameters need adjustment to meet the calibration requirements.

Step 2A. Test the accumulation factors using simulation

Using simulation it is important to ensure that the calibration test is not too susceptible to sampling variability. The following instructions will ensure that the calibration criteria are met with 95% probability.

Simulate n values for each of the three accumulation factors in Table 1 (one-year, five-year and ten-year).

For each column in Table 1, take the calibration point value and count how many of the simulated values for the accumulation factor fall below the appropriate Table value; let this number be N . For example, the first calibration point for the one-year accumulation factor is 0.76, so N represents the number of simulated values of the one-year accumulation factor which are less than 0.76.

Then $\hat{p} = \frac{N}{n}$ is an estimate of p , the true underlying probability that the accumulation factor is less than the calibration value. That is according to your model, 0.76 lies approximately at the $100\hat{p}$ percentile of the distribution of the accumulation factor. By using the Normal approximation to the binomial distribution, it is approximately 95% certain that:

$$p > \hat{p} - 1.645 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

So, for example, to test the probability that the one-year accumulation factor, $A_1 \leq 0.76$, generate, say, 10,000 simulated values of A_1 . Suppose that the number of these simulations falling below 0.76 is $N=280$. Then $\hat{p}=0.0280$, and we have that, with 95% certainty, $p > 0.0253$. So, we are 95% sure that more than 2.5% of the distribution lies below the critical value of 0.76, so the calibration criterion is satisfied.

Step 3. Adjust the model to fit the calibration criteria

For models with many parameters there will not, in general, be a ‘right’ way to adjust the parameters. Keep in mind the objective, which is to ensure an adequate probability in the left tail of the return distribution, whilst maintaining overall mean returns consistent with the data. Let θ' denote the adjusted parameter vector.

Step 4. Check mean and standard deviation of one-year accumulation factor

Using the adjusted parameter vector θ' , calculate analytically, if possible, or by simulation if necessary, the mean and standard deviation of A_1 . If the mean value does not lie in the range 1.10 to 1.12, or the standard deviation is less than 17.5%, then return to Step 3.

Step 5. Adjustment if you are using different data

If you estimate the stock return model using other market data, the adjustments arrived at in Step 3 should be applied. That is, if the best fit parameter vector for the new data is Φ , then the parameter vector used should be $\Phi' = \Phi + (\theta' - \theta)$.

References:

Hamilton J D (1994) *Time Series Analysis*, Princeton University Press.

Hamilton J D and Susmel R(1994) Autoregressive conditional heteroscedasticity and changes in regime. *Journal of Econometrics* **64** 307-333

Hardy M R (1999) Stock return models for segregated fund guarantees. *IIPR research report 99-14*. University of Waterloo.

APPENDIX E – Development of Calibration Points

The 1956-1999 monthly TSE total return data series allows for 527 non-overlapping end-of-month return observations; 43 non-overlapping observations of annual returns, eight non-overlapping observations of five-year returns and only four non-overlapping observations of ten-year returns. Since it is possible to choose different starting points for determining the non-overlapping returns there are several different non-overlapping series to choose from (e.g., for the annual returns, there would be twelve sets of non-overlapping returns for annual periods corresponding to different monthly starting points of January to January, February to February, etc). These series are not independent, but provide slightly different empirical estimates of the underlying distributions.

The table below summarizes the left tail returns based on these empirical observations. To interpret the table, the 2.27% percentile for the one-year return is based on the worst result of 43 independent observation periods of annual returns, where $2.27\% = 1 / (n + 1) = 1 / 44$. The 4.55% result is based on the second worst result (i.e., $2 / 44$). Because there are twelve possible starting points for the 43 years of non-overlapping returns, corresponding to starting points of Jan 56, Feb 56...Dec 56, the “empirical” range shows the maximum and minimum of the results from the looking at each of the 12 possible non-overlapping series.

TABLE E1 – Non-overlapping time period returns

Accumulation Period	Percentile	Empirical Range
one-year	2.27	(0.61, 0.82)
	4.55	(0.76, 0.85)
	9.09	(0.85, 0.92)
five-year	11.11	(0.98, 1.41)
ten-year	20.00	(1.60, 2.59)

The data are too sparse to provide credible empirical calibration criteria for the annual, five-year and ten-year accumulation factors. It is, therefore, necessary to extrapolate from these data. The extrapolation is performed by fitting an appropriate model of stock returns using the monthly data and using this model to generate distributions for the one-year, five-year and ten-year accumulation factors.

In choosing a model for the extrapolation, the main consideration is the goodness of fit to the monthly data. Stochastic volatility models can be shown to fit the monthly data significantly better than fixed volatility models. The models used to produce possible calibration figures were the following: a regime switching lognormal (RSLN) model, with two regimes, a stochastic volatility lognormal model (SVLN), and a Stable Distribution Model.

The tables below show the values produced by these models for the 1956 – 1999 TSE total return data when fitted to the observable data.

TABLE E2: RSLN Quantile calibration criteria for accumulation factors

Accumulation Period	First Percentile	2.5th Percentile	Fifth Percentile	Tenth Percentile
one-year	0.6541	0.7379	0.8128	0.8940
five-year		0.6920	0.8182	0.9805
ten-year		0.8172	1.0165	1.2925

TABLE E3: SVLN Quantile calibration criteria for accumulation factors

Accumulation Period	First Percentile	2.5th Percentile	Fifth Percentile	Tenth Percentile
one-year	0.6975	0.7632	0.8164	0.8797
five-year		0.7278	0.8308	0.9666
ten-year		0.8344	0.9746	1.2461

TABLE E4: Stable Model Quantile calibration criteria for accumulation factors

Accumulation Period	First Percentile	2.5th Percentile	Fifth Percentile	Tenth Percentile
one-year	0.6848	0.7828	0.8405	0.9044
five-year		0.7156	0.8506	1.0127
ten-year		0.8379	1.0513	1.3352

In Table –E5 below, the quantile estimates for the observed data are compared with the corresponding quantiles of the above. The table shows that the models fit well with the observed accumulation factors.

TABLE –E5

Accumulation Period	Percentile	Empirical Range	RSLN	SVLN	Stable
one-year	2.27	(0.61, 0.82)	0.74	0.76	0.77
	4.55	(0.76, 0.85)	0.82	0.81	0.83
	9.09	(0.85, 0.92)	0.89	0.87	0.90
five-year	11.11	(0.98, 1.41)	1.05	0.99	1.04
ten-year	20.00	(1.60, 2.59)	1.88	1.60	1.73

Based on this fit, a set of calibration points were selected that overall represented the results of the three fitted models. These points are shown below in Table E6, along with the “fitted” annualized μ and σ that they imply for the simple fixed volatility lognormal model.

TABLE –E6: Proposed quantile calibration criteria for accumulation factors and annualized (μ, σ) for fixed parameter lognormal model.

Note: μ and σ are defined as described in Appendix C, equation (1).

That is, $E[Y] = \exp(\mu + \frac{1}{2} \cdot \sigma^2)$

Accumulation Period		2.5th Percentile	Fifth Percentile	Tenth Percentile
one-year		0.76 (0.0923, 0.1871)	0.82 (0.0940, 0.1778)	0.90 (0.0974, 0.1582)
five-year		0.75 (0.0948, 0.1738)	0.85 (0.0949, 0.1732)	1.05 (0.098, 0.1540)
ten-year		0.85 (0.0940, 0.1779)	1.05 (0.0949, 0.1730)	1.35 (0.0964, 0.1639)

For fixed volatility models, since only one mean and volatility assumption can be chosen, the parameters selected must be conservative enough to encompass all the left-tail points in the above table. The one-year 2.5% results achieve this.

APPENDIX F – Dec 2001 OSFI MCCR Guideline (Sec. 9)

Segregated Fund Guarantee Risk

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Segregated Fund Guarantee Risk

This component is for the risk associated with investment or performance-related guarantees on segregated funds or other similar products.

Documentation and Reporting

Given the complexity of this calculation, for auditing purposes the Actuary should keep supporting schedules of all the calculations for each step building up to the final numbers detailed in the MCCSR form. Also, the Appointed Actuary is required to detail the calculation in the segregated fund section of the Appointed Actuary's Report.

Form Details

The columns of the reporting form on page 90.010 are filled in as follows:

Column 01 - Guaranteed Value

This is the amount guaranteed in all segregated funds. If the funds are subject to guarantees of different amounts, for example 100% on death and 75% on maturity, report the larger amount here.

Column 02 - Market Value

This is the market value of these segregated funds.

Column 03 - Total Gross Calculated Requirements

The calculation is detailed below.

Column 04 - Actuarial Liabilities Ceded

Report amounts for policy liabilities ceded in column 04. Note that policy liabilities ceded to unregistered reinsurers per Guideline B-3, must be deducted from Total Capital on page 20.030, line 085 of the OSFI 87 form by Canadian companies, and on page 25.010, line 050 of the OSFI 86 form by foreign branches.

Column 05 - Net Requirements

This is determined as:

(Total Gross Calculated Requirements – Actuarial Liabilities Ceded)

Column 06 –Credit for OSFI Approved Hedging Programs

This is the dollar equivalent of the maximum allowable reduction. It is determined as:

(Maximum allowable percentage reduction * Net Requirements)

Where the maximum percentage reduction is limited to 50% of the percentage reduction shown by the models. See OSFI Guidance Note “*Capital Offset for Segregated Fund Hedging Programs (MCCSR)*”.

Column 07 - Net Actuarial Liabilities Held

This is the total net actuarial liability held on the balance sheet for segregated fund guarantee risks.

Column 08 - Net Required Component

This is determined as:

[Net Requirements (column 5) – Credit for OSFI Approved Hedging Programs – Net Actuarial Liabilities Held]

Line 099 must not be less than zero in total.

Foreign Business Capital Requirements (Refer to Section 9-10 Modelling and Calibration)

For those institutions that have foreign business (policies issued outside of Canada) in segregated funds, the reporting form on page 90.015 should also be completed, and the information on page 90.015 should be incorporated into page 90.010. The columns of the reporting form on page 90.015 should be filled in as follows:

Column 01 – Factor Requirements for Foreign Business

This is the gross calculated requirement based on the OSFI approved foreign factors.

Column 02 - Internal Model Requirements (OSFI approved models only)

For OSFI approved models, this is the gross calculated requirement based on company specific internal models.

Column 03 - Total Gross Calculated Requirements

For OSFI approved models, transition rules apply:

In the first year of approval, Total Gross Calculated Requirements = 50% of the Foreign Factor Requirements + 50% of the Internal Model Requirements.

Thereafter, Total Gross Calculated Requirements = 100% of the Internal Model Requirements.

Otherwise, Total Gross Calculated Requirements = 100% of the Foreign Factor Requirements.

Column 04 - Actuarial Liabilities Ceded

Report amounts ceded for policy liabilities in column 04. Note that policy liabilities ceded to unregistered reinsurers with respect to foreign business per Guideline B-3, must be deducted from Total Capital on page 20.030, line 085 of the OSFI 87 form by Canadian companies, and on page 25.010, line 050 of the OSFI 86 form by foreign branches.

Column 05 - Net Requirements

This is determined as:

(Total Gross Calculated Requirements – Actuarial Liabilities Ceded)

Column 06 – Credit for OSFI Approved Hedging Programs

This is the dollar equivalent of the maximum allowable reduction. It is determined as:

(Maximum allowable percentage reduction * Net Requirements)

Where the maximum percentage reduction is limited to 50% of the percentage reduction shown by the models. See OSFI Guidance Note “*Capital Offset for Segregated Fund Hedging Programs (MCCSR)*”.

Column 07 - Net Actuarial Liabilities Held

This is the total net actuarial liability held on the balance sheet for foreign business segregated fund guarantee risks.

Column 08 - Net Required Component

This is determined as:

[Net Requirements (column 5) – Credit for OSFI Approved Hedging Programs – Net Actuarial Liabilities Held]

Total Gross Calculated Requirements

Factors applied to the market value of segregated fund and similar risk assets will be used to set Total Gross Calculated Requirements. This is a five-step factor process that can be expressed as follows:

$$\text{Total Gross Calculated Requirement} = \text{Market Value} \times [(\text{A1} \times \text{A2} \times \text{B}) + \text{C} + \text{D}]$$

where the factors are:

1. Basic Factor (**A1**) – This factor is based on the product type and investment fund class.
2. Time Diversification Adjustment (**A2**) – An adjustment factor applicable if the maturities are sufficiently spread out over time.
3. MV/GV and Time to Maturity Adjustment (**B**) – An adjustment factor for the relationship between market value and guaranteed value, and for the time left to maturity.
4. MER Adjustment (**C**) – An adjustment to account for the differences between actual management expense ratios (MER) and those assumed in the development of the factors. This reflects the deduction of MERs from investment funds.
5. Margin Offset Adjustment (**D**) – An adjustment factor to reflect the margins available to cover the cost of the guarantee.

Step 1 – Basic Factor (A1)

Determine the appropriate guarantee and investment types(s) and apply factors from the Basic Factor Table (Table 1a) to set A1.

For each guarantee, the company will select a factor from Table 1a based on the product characteristics and the types of investments held.

Factors are determined separately for death, maturity and income benefit guarantees and the results added together.

The guaranteed amounts are to include all implicit or explicit investment performance-related guarantees on segregated funds and other similar contracts (e.g., mutual funds, variable annuities, etc.) in force, including both Canadian and non-Canadian business.

In determining the appropriate factors for a particular benefit form, the descriptions that follow in the tables on the next pages should be used as a guide to select a row that most closely corresponds to the product form being evaluated. The “Attained age ~ Maturity age” and “Years to Next Maturity” values were used in developing the basic factors. It is expected that the factor tables will be used for most products/benefit forms. Should the company be evaluating a product type that is materially different from those in the tables, refer to *Modelling and Calibration* found in section 9-10.

| **Benefit/Product Descriptions: Guaranteed Minimum Death Benefits**

	Attained Age ~ Maturity Age	Years to Next Maturity	
GMDB: Minimum Death Benefits			
No resets or ratchets; to contract maturity (age 70)	50 ~ 70	20	Level guaranteed minimum death benefit to age 70.
No resets or ratchets; to contract maturity (age 90)	60 ~ 90	30	Level guaranteed minimum death benefit to age 90.
Annual ratchet; to contract maturity (age 90)	60 ~ 90	30	Automatic annual reset at each contract anniversary (no limit), irrespective of fund performance. No resets beyond attained age 85.
5% annual roll-up; to contract maturity (age 90)	60 ~ 90	30	Guaranteed benefit increases 5% per annum at each contract anniversary (no limit), irrespective of fund performance. No resets beyond attained age 85.
No resets or ratchets; 75% 10-year (1)	50 ~ 78	8	Guaranteed benefit rolls over at 75% of market value at next renewal (maturity) date.
No resets or ratchets; 100% 10-year (1)	50 ~ 78	8	Guaranteed benefit rolls over at 100% of market value at next renewal (maturity) date.
With resets/ratchets; 75% 10-year (1), (2)	50 ~ 78	8	GMDB can be reset (75% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final contract maturity.
With resets/ratchets; 100% 10-year (1), (2)	50 ~ 78	8	GMDB can be reset (100% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final contract maturity.

| Benefit/Product Descriptions: Guaranteed Minimum Maturity & Surrender Benefits

	Attained Age ~ Maturity Age	Years to Next Maturity	
GMMB: Minimum Maturity & Surrender Benefits			
No resets or ratchets; To contract maturity (age 70)	50 ~ 70	20	Level guaranteed minimum maturity benefit to age 70.
No resets or ratchets; 75% 10-year (1)	50 ~ 78	8	Guaranteed benefit rolls over at 75% of market value at next renewal (maturity) date.
No resets or ratchets; 100% 10-year (1)	50 ~ 78	8	Guaranteed benefit rolls over at 100% of market value at next renewal (maturity) date.
No resets or ratchets; 10-year CSV guarantee	50 ~ 78	8	Level guaranteed minimum surrender benefit to age 78; applies 10 years after contract issue.
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	50 ~ 78	8	Level guaranteed minimum surrender benefit to age 78; applies 10 years after contract issue. Policyholders assumed to surrender if MV/GV < 0.85 ("in-the-money termination").
With resets/ratchets; 75% to contract maturity (2)	50 ~ 70	20	GMMB can be reset (75% of market gains since last exercise); fixed maturity date. No resets permitted in the 10 years prior to final contract maturity.
With resets/ratchets; 100% to contract maturity (2)	50 ~ 70	20	GMMB can be reset (100% of market gains since last exercise); fixed maturity date. No resets permitted in the 10 years prior to final contract maturity.
With resets/ratchets; 75% 10-year (1), (2)	50 ~ 78	8	GMMB can be reset (75% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final contract maturity.
With resets/ratchets; 100% 10-year (1), (2)	50 ~ 78	8	GMMB can be reset (100% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final contract maturity.
With resets/ratchets; 75% 10-year CSV (2)	50 ~ 78	8	Guaranteed minimum surrender benefit can be reset (75% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final maturity.
With resets/ratchets; 100% 10-year CSV (2)	50 ~ 78	8	Guaranteed minimum surrender benefit can be reset (100% of market gains since last exercise); term-to-maturity resets to 10 years. No resets permitted in the 10 years prior to final maturity.
With resets/ratchets; 75% 10-year CSV ("in-the-money" surrender) (2)	50 ~ 78	8	Same as "With resets/ratchets; 75% 10-year CSV", except policyholders assumed to surrender if MV/GV < 0.85 ("in-the-money termination").
With resets/ratchets; 100% 10-year CSV ("in-the-money" surrender) (2)	50 ~ 78	8	Same as "With resets/ratchets; 100% 10-year CSV", except policyholders assumed to surrender if MV/GV < 0.85 ("in-the-money termination").
GMIB: Minimum Income Benefits			
Company must calibrate and model separately			

Notes:

- (1) Contract rolls over (renews) at the end of 10-year term for another 10-year term. Guarantee resets to X% of market value (after payment of any top-up maturity benefit for in-the-money guarantees).
- (2) 100% utilization of the elective reset option whenever MV/GV > 1.15; maximum 2 resets per year.

Table 1a – Basic Factor Table

	Money Market	Fixed Income	Balanced	Low Volatility Diversified Equity	Broad-Based Diversified Equity	Intermediate Risk Equity	Exotic or Aggressive Equity
Basic Total Management Expense Ratios (basis points, per annum):	110	200	250	265	265	280	295
Minimum Death Benefits							
No resets or ratchets; level guarantee to contract maturity (age 70)	0.0002	0.0007	0.0037	0.0076	0.0092	0.0114	0.0143
No resets or ratchets; level guarantee to contract maturity (age 90)	0.0005	0.0016	0.0084	0.0171	0.0201	0.0248	0.0309
Annual ratchet; to contract maturity (age 90)	0.0016	0.0042	0.0207	0.0425	0.0515	0.0856	0.1688
5% annual roll-up; to contract maturity (age 90)	0.0270	0.0321	0.0453	0.0576	0.0640	0.0678	0.0753
No resets or ratchets; 75% 10-year	0.0002	0.0006	0.0029	0.0066	0.0076	0.0125	0.0232
No resets or ratchets; 100% 10-year	0.0003	0.0012	0.0065	0.0148	0.0175	0.0302	0.0543
With resets/ratchets; 75% 10-year	0.0002	0.0006	0.0033	0.0094	0.0113	0.0207	0.0425
With resets/ratchets; 100% 10-year	0.0005	0.0016	0.0104	0.0233	0.0283	0.0503	0.0956
Minimum Maturity & Surrender Benefits							
No resets or ratchets; level guarantee to contract maturity (age 70)	0.0001	0.0001	0.0010	0.0044	0.0088	0.0125	0.0195
No resets or ratchets; 75% 10-year	0.0002	0.0025	0.0434	0.1038	0.1154	0.1414	0.1772
No resets or ratchets; 100% 10-year	0.0002	0.0030	0.0482	0.1082	0.1202	0.1504	0.2012
No resets or ratchets; 10-year CSV	0.0001	0.0005	0.0111	0.0309	0.0412	0.0523	0.0685
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	0.0001	0.0013	0.0446	0.1027	0.1142	0.1379	0.1671
With resets/ratchets; 75% to contract maturity	0.0001	0.0001	0.0028	0.0142	0.0259	0.0411	0.0761
With resets/ratchets; 100% to contract maturity	0.0001	0.0001	0.0141	0.0381	0.0544	0.0811	0.1365
With resets/ratchets; 75% 10-year	0.0002	0.0024	0.0438	0.1042	0.1158	0.1456	0.1983
With resets/ratchets; 100% 10-year	0.0002	0.0036	0.0567	0.1208	0.1342	0.1787	0.2695
With resets/ratchets; 75% 10-year CSV guarantee	0.0001	0.0004	0.0117	0.0334	0.0445	0.0607	0.0950
With resets/ratchets; 100% 10-year CSV guarantee	0.0001	0.0007	0.0178	0.0439	0.0585	0.0840	0.1427
With resets/ratchets; 75% 10-yr CSV ("in-the-money" surrender)	0.0001	0.0013	0.0460	0.1027	0.1141	0.1431	0.1947
With resets/ratchets; 100% 10-yr CSV ("in-the-money" surrender)	0.0001	0.0019	0.0575	0.1180	0.1312	0.1749	0.2644
Minimum Income Benefits							
Guaranteed Minimum Income Benefit	Company must calibrate and model separately						

The “with resets” rows have assumed a 100% rate of utilization (the proportion of policyholders expected to exercise the elective reset option given favourable investment performance). If the product offers an elective reset feature and the company has reliable and pertinent experience regarding the rate of utilization, the company should interpolate between the corresponding “no resets” and “with resets” rows according to the proportion of business that exercises the reset option. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

For products which offer a reset option (elective or automatic), but the contract is outside the period in which resets are permitted (e.g. beyond some duration or attained age limit), the exposure should be valued as if the reset option were not present.

The “CSV guarantee” product type pays a “top-up” maturity benefit only upon surrender of the fund units to which the guarantee applies. The costs associated with this benefit form are very sensitive to the underlying surrender (lapse) assumption and the assumed policyholder behaviour in adverse investment performance scenarios. The “in-the-money surrender” rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is “in-the-money” by more than 15% (i.e., MV/GV ratio is less than 0.85). If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and “in-the-money surrender” rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

In determining the appropriate factors for a particular fund, the criteria below are used to select a column. Where stated, the volatility of the long-term annualized total return for the fund (or an appropriate benchmark) should conform to the limits presented. For this purpose, “long-term” is defined as twice the average projection period that would be applied to test the product in a stochastic model. Where data for the fund or benchmark is too sparse or unreliable, the fund should be moved to the next higher class than otherwise indicated. In reviewing the fund classifications, care should be taken to reflect any additional volatility of returns added by the presence of currency risk, liquidity (bid-ask) effects, short selling and speculative positions.

All fund types must be categorized as one of the seven classes identified below.

Money Market/Short-Term

The fund is invested in money market instruments with an average remaining term-to-maturity of less than 365 days.

Fixed Income

The fund is invested primarily in investment grade (rated better than BBB) fixed income securities. Up to 25% of the fund within this class may be invested in equities. The volatility of the fund returns will be lower than the Balanced fund class.

Balanced

This class is a combination of fixed income securities with a larger equity component. The fixed income component should exceed 25% of the portfolio. Additionally, the aggressive equity component of the equity holdings should not exceed 15% of the total equities. Should the fund violate either of these constraints, it should be categorized as an equity fund. These funds usually have a long-term volatility in the range of 8% – 13%.

Low Volatility Diversified Equity

This fund is comparable to the Broad-Based Diversified Equity with the additional attributes noted below. Only funds that otherwise would be classified as Broad-Based Diversified Equity are candidates for this fund classification. For foreign funds, volatility should take into account the impact of currency fluctuations.

If the expected volatility of the fund is less than 15.5% (annualized) and the aggressive/exotic equity component of the equity holdings is less than 20% of the total equities by market value, and the overall asset holdings satisfy at least one of the following conditions:

- The fund permanently maintains relatively large cash or fixed income balances (greater than 10% of the market value of assets) as part of its investment strategy;
- The fund is "income" oriented and contains a significant (greater than 5% of the market value of assets) proportion of stocks paying material (at least a 2% yield) and regular dividends that are automatically reinvested in the fund.

If the above conditions are met, then the fund can be classified as Low Volatility Diversified Equity.

Note: For Tables 1b, 2, 3, and 4, the factors for Broad-Based Diversified Equity apply.

Broad-Based Diversified Equity

The fund is invested in a well-diversified (broad based) mix of Canadian, U.S. or global equities. Funds in this category would exhibit long-term volatility comparable to that of the TSE 300. These funds should usually have a long-term volatility in the range of 13% – 19%.

Intermediate Risk Equity

The fund has a mix of characteristics from both the Broad-Based Equity and Aggressive Equity Classes. These funds have a long-term volatility in the range of 19% – 25%.

Exotic or Aggressive Equity

This class comprises riskier funds where risk can arise from: (a) underdeveloped markets, (b) uncertain markets, (c) high volatility of returns, (d) narrow focus (e.g., specific market sector), etc. The fund either does not have sufficient history (or the market does not have sufficient experience) to allow for the calculation of a long-term volatility, or the volatility is very high. In those cases where there is no determinable long-term volatility and/or the long-term volatility exceeds 25%, the fund would be grouped in this class.

Selecting Appropriate Investment Categories

The selection of an appropriate investment type should be done at the level for which the guarantee applies. For guarantees applying on a deposit-by-deposit basis, the fund selection is straightforward. However, where the guarantee applies across deposits or for an entire contract, the approach can be more complicated. In such instances, the approach is to identify, for each client, where the “grouped holdings” fit within the categories listed and to categorize the associated assets on this basis. A seriatim process is used to identify the “grouped” fund holdings, to assess the risk profile of the current fund holdings (possibly calculating the volatility of the funds held), and to classify the current fund holdings into one of the six choices.

For example, suppose three funds (fixed income, broad-based diversified equity and aggressive equity) are offered to clients on a product with a contract level guarantee. That is, the guarantee applies across all funds held by the client.

The current fund holdings (in dollars) classification for five sample contracts is:

Client	1	2	3	4	5
MV Fund X (Fixed Income):	5,000	4,000	8,000	-	5,000
MV Fund Y (Broad Based):	9,000	8,000	2,000	6,000	-
MV Fund Z (Aggressive):	1,000	3,000	-	4,000	5,000
Total Market Value:	15,000	15,000	10,000	10,000	10,000
Total Equity Market Value:	10,000	11,000	2,000	10,000	5,000
Fixed Income % (a):	33%	27%	80%	0%	50%
Fixed Income Test (a>75%):	Fail	Fail	Pass	Fail	Fail
Aggressive % of Equity (b):	10%	27%		40%	100%
Balanced Test (a>25% & b<15%):	Pass	Fail		Fail	Fail
Volatility of Current Fund Holdings:		12.50%		19.50%	16.75%
Current Fund Holdings Classification:	Balanced	Broad Based Equity*	Fixed Income	Intermediate Risk Equity	Broad Based Equity

** Although the volatility suggests “Balanced Fund”, the Balanced Fund criteria were not met. Therefore, this holding is moved up one class to Broad-Based Diversified Equity. For those funds classified as Broad Based Diversified Equity, additional analysis would be required to assess whether they can be reclassified as Low Volatility Diversified Equity. In the example above, none qualify.*

If the guarantee applies separately for each deposit year within a client’s contract, then the above process would be applied separately for the exposure of each deposit year.

Step 2 – Time Diversification Adjustment (A2)

Determine A2, an adjustment for companies having guarantee maturities that are sufficiently spread out over time.

This adjustment only applies to the maturity benefit factors, not the death benefit factors.

This factor is determined in two steps. First, the portfolios must satisfy a time diversification test. Second, a factor is determined based on the results of the test.

To perform the test, the in-force maturity dates for each product/maturity guarantee form (“row” in Table 1a) are grouped by “quarter-to-maturity” (i.e., 1, 2, ..., N). For limited-term contracts that offer the client the opportunity to renew (“rollover”), the next maturity date should be used (not final contract maturity). Using current market value (at the calculation date), the current market value in each future 3-month time period is determined.

If the current market value in any given quarter exceeds 10% of the total, then the portfolio fails the test and A2 equals 1. If the current market value in each quarter is less than or equal to 10% of the total, the portfolio passes the test, and the factor in Table 1c for that product form is applied.

As for Table 1a, the “with resets” rows assume 100% utilization. If the product offers an elective reset feature and the company has reliable and pertinent experience regarding the rate of utilization, the company should interpolate between the corresponding “no resets” and “with resets” rows according to the proportion of business that exercises the reset option. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* in section 9-9.

For products which offer a reset option (elective or automatic), but the contract is outside the period in which resets are permitted (e.g. beyond some duration or attained age limit), the exposure should be valued as if the reset option were not present.

Furthermore, the “in-the-money surrender” rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is “in-the-money” by more than 15% (i.e., MV/GV ratio is less than 0.85). If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and “in-the-money surrender” rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

Table 1b – Time Diversification Adjustment Factor

	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity & Surrender Benefits						
No resets or ratchets; Level guarantee to contract maturity (age 70)	1.000	1.000	1.000	1.000	1.000	0.985
No resets or ratchets; 75% 10-year	1.000	1.000	0.960	0.935	0.935	0.940
No resets or ratchets; 100% 10-year	1.000	1.000	0.935	0.940	0.935	0.925
No resets or ratchets; 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	1.000	1.000	1.000	0.965	0.955	0.960
With resets/ratchets; 75% to contract maturity	1.000	1.000	0.965	0.920	0.905	0.885
With resets/ratchets; 100% to contract maturity	1.000	1.000	0.940	0.905	0.890	0.895
With resets/ratchets; 75% 10-year	1.000	1.000	0.965	0.965	0.970	0.985
With resets/ratchets; 100% 10-year	1.000	1.000	0.985	0.990	1.000	1.000
With resets/ratchets; 75% 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
With resets/ratchets; 100% 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
With resets/ratchets; 75% 10-year CSV ("in-the-money" surrender)	1.000	1.000	1.000	0.975	0.975	0.990
With resets/ratchets; 100% 10-year CSV ("in-the-money" surrender)	1.000	1.000	0.990	0.990	1.000	1.000
Minimum Income Benefits						
Guaranteed Minimum Income Benefit	Company must calibrate and model separately					

* For funds classified as "Low Volatility Diversified Equity", the factors for "Broad-Based Diversified Equity" apply.

Step 3 - MV/GV and Time to Maturity Adjustment (B)

Determine *B*, an adjustment for the relationship of current market value (*MV*) to guaranteed value (*GV*) and for the time to maturity/expiry.

MV/GV is the Market Value divided by the Guaranteed Value as of the valuation date.

For this factor, the ratio of current market value to guarantee value is calculated. Using this result, the type of fund, and the product categories, the appropriate factor is chosen from Table 2.

For each row of Table 2, the representative number can be found in the second column. It is at the low end of the range. For $MV/GV < 0.50$, the value for 0.25 is used. Linear interpolation between these representative numbers is used to determine more refined factors based on the actual MV/GV value. Linear extrapolation should be used for MV/GV ratios above 2.00 or below 0.25.

The “with resets” rows have assumed a 100% rate of utilization. If the product offers an elective reset feature and the company has reliable and pertinent experience regarding the rate of utilization, the company should interpolate between the corresponding “no resets” and “with resets” rows according to the proportion of business that exercises the reset option. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

For products which offer a reset option (elective or automatic), but the contract is outside the period in which resets are permitted (e.g. beyond some duration or attained age limit), the exposure should be valued as if the reset option were not present.

The “in-the-money surrender” rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is “in-the-money” by more than 15% (i.e., MV/GV ratio is less than 0.85). If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and “in-the-money surrender” rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

For product types where term-to-maturity is specified in Table 2, the company may interpolate between corresponding rows in the relevant sections based on the actual remaining term-to-next-maturity of the exposure being valued. The assumed term-to-next maturity “T” is shown for each product type as “T=n” above the MV/GV ratios in each section. Linear extrapolation beyond the assumed terms-to-maturity is not permitted.

For example, suppose the exposure being valued is 4 years from the next maturity date for a 10-year product (no resets) which guarantees 100% return-of-premium at maturity. Assume the MV/GV ratio is 1.20 and the funds are classified as “Broad-Based Diversified Equity”. The Status Factor “B” would be determined in 3 steps as follows:

- (a) Determine the Status Factor for the “*No Elective Resets: 100% GMMB 10-yr Term, > 3 years to Maturity*” product type by interpolating based on MV/GV ratio:

$$\begin{aligned} &= 0.80 \times 0.65 + (1 - 0.80) \times 1.00 \\ &= 0.72 \end{aligned}$$

- (b) Determine the Status Factor for the “*No Elective Resets: 100% GMMB 10-yr Term, ≤ 3 years to Maturity*” product type by interpolating based on MV/GV ratio:

$$\begin{aligned} &= 0.80 \times 1.08 + (1 - 0.80) \times 2.08 \\ &= 1.28 \end{aligned}$$

- (c) Using the results of the previous two steps, determine the final Status Factor by interpolating based on years-to-next-maturity:

$$\begin{aligned} &= 0.3333 \times 0.72 + (1 - 0.3333) \times 1.28 \\ &= 1.094 \end{aligned}$$

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor)

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Death Benefit	T=20	No Elective Resets: Level GMDB to Age 70					
MV/GV ≥ 2.00	2.00	-	-	0.01	0.05	0.10	0.16
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.04	0.21	0.30	0.37
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.21	0.46	0.54	0.60
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	36.14	12.38	3.61	2.24	1.99	1.82
0.50 ≤ MV/GV < 0.75	0.50	168.92	48.97	10.40	5.05	4.25	3.63
MV/GV < 0.50	0.25	619.03	169.66	31.97	13.68	11.23	9.19
Minimum Death Benefit	T=30	No Elective Resets: Level GMDB to Age 90					
MV/GV ≥ 2.00	2.00	-	-	-	0.05	0.09	0.16
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.04	0.21	0.29	0.37
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.21	0.46	0.53	0.59
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	33.94	11.82	3.58	2.25	2.01	1.83
0.50 ≤ MV/GV < 0.75	0.50	146.31	44.09	10.08	5.10	4.31	3.69
MV/GV < 0.50	0.25	537.46	153.64	31.27	14.03	11.53	9.49
Minimum Death Benefit	T=30	No Elective Resets: GMDB: Annual Automatic Ratchet					
MV/GV ≥ 2.00	2.00	0.95	0.96	0.98	0.99	1.00	1.00
1.50 ≤ MV/GV < 2.00	1.50	0.95	0.96	0.98	0.99	1.00	1.00
1.25 ≤ MV/GV < 1.50	1.25	0.95	0.96	0.98	0.99	1.00	1.00
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	9.63	4.48	1.54	1.12	1.04	1.01
0.50 ≤ MV/GV < 0.75	0.50	41.12	16.40	3.98	1.95	1.36	1.09
MV/GV < 0.50	0.25	151.00	57.08	12.30	5.29	3.23	1.80
Minimum Death Benefit	T=30	No Elective Resets: GMDB: 5% Annual Roll-Up					
MV/GV ≥ 2.00	2.00	-	-	0.05	0.17	0.22	0.26
1.50 ≤ MV/GV < 2.00	1.50	0.03	0.08	0.23	0.39	0.43	0.48
1.25 ≤ MV/GV < 1.50	1.25	0.23	0.32	0.49	0.62	0.64	0.67
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	2.74	2.46	2.02	1.72	1.67	1.60
0.50 ≤ MV/GV < 0.75	0.50	6.24	5.40	4.11	3.19	3.06	2.85
MV/GV < 0.50	0.25	16.74	14.23	10.37	7.62	7.24	6.61

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Death Benefit	T=8	No Elective Resets: 75% GMDB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.29	0.76	0.91	0.98
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.31	0.77	0.91	0.98
1.25 ≤ MV/GV < 1.50	1.25	-	-	0.41	0.81	0.93	0.98
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	32.65	10.90	3.10	1.69	1.31	1.11
0.50 ≤ MV/GV < 0.75	0.50	110.96	33.26	7.67	3.36	2.25	1.53
MV/GV < 0.50	0.25	345.91	100.35	21.41	8.58	5.38	3.14
Minimum Death Benefit	T=2	No Elective Resets: 75% GMDB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.32	0.70	0.69	0.67
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.32	0.70	0.69	0.67
1.25 ≤ MV/GV < 1.50	1.25	-	-	0.32	0.70	0.69	0.67
1.00 ≤ MV/GV < 1.25	1.00	0.67	0.46	0.43	0.72	0.70	0.67
0.75 ≤ MV/GV < 1.00	0.75	11.72	3.63	1.01	0.88	0.79	0.71
0.50 ≤ MV/GV < 0.75	0.50	34.09	10.02	2.32	1.36	1.07	0.85
MV/GV < 0.50	0.25	101.20	29.18	6.24	2.86	1.98	1.34
Minimum Death Benefit	T=8	No Elective Resets: 100% GMDB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.79	0.82	0.94	0.99	0.99	1.00
1.50 ≤ MV/GV < 2.00	1.50	0.79	0.82	0.95	0.99	0.99	1.00
1.25 ≤ MV/GV < 1.50	1.25	0.79	0.82	0.95	0.99	0.99	1.00
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	16.77	5.66	1.64	1.14	1.05	1.02
0.50 ≤ MV/GV < 0.75	0.50	56.83	17.05	3.64	1.75	1.32	1.14
MV/GV < 0.50	0.25	177.03	51.25	9.86	4.01	2.54	1.74
Minimum Death Benefit	T=2	No Elective Resets: 100% GMDB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.82	0.82	0.91	0.82	0.71	0.64
1.50 ≤ MV/GV < 2.00	1.50	0.82	0.82	0.91	0.82	0.71	0.64
1.25 ≤ MV/GV < 1.50	1.25	0.82	0.82	0.91	0.82	0.71	0.64
1.00 ≤ MV/GV < 1.25	1.00	0.91	0.85	0.92	0.83	0.71	0.64
0.75 ≤ MV/GV < 1.00	0.75	6.25	2.26	1.12	0.89	0.74	0.65
0.50 ≤ MV/GV < 0.75	0.50	17.69	5.52	1.71	1.10	0.86	0.71
MV/GV < 0.50	0.25	52.02	15.29	3.49	1.76	1.24	0.92

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Death Benefit	T=8	With Resets: 75% GMDB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.67	0.97	0.99	1.00
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.66	0.97	0.99	1.00
1.25 ≤ MV/GV < 1.50	1.25	-	-	0.67	0.97	0.99	1.00
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	32.65	10.90	2.67	1.29	1.09	1.03
0.50 ≤ MV/GV < 0.75	0.50	110.96	33.26	6.55	2.30	1.53	1.17
MV/GV < 0.50	0.25	345.91	100.36	18.25	5.71	3.32	1.92
Minimum Death Benefit	T=2	With Resets: 75% GMDB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.52	0.74	0.66	0.58
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.51	0.73	0.66	0.58
1.25 ≤ MV/GV < 1.50	1.25	-	-	0.50	0.73	0.66	0.58
1.00 ≤ MV/GV < 1.25	1.00	0.67	0.46	0.54	0.73	0.66	0.58
0.75 ≤ MV/GV < 1.00	0.75	11.72	3.63	0.99	0.82	0.71	0.60
0.50 ≤ MV/GV < 0.75	0.50	34.09	10.02	2.10	1.14	0.87	0.67
MV/GV < 0.50	0.25	101.20	29.18	5.44	2.12	1.41	0.93
Minimum Death Benefit	T=8	With Resets: 100% GMDB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	1.01	1.00	1.00	1.00	1.00	1.00
1.50 ≤ MV/GV < 2.00	1.50	1.01	1.00	1.00	1.00	1.00	1.00
1.25 ≤ MV/GV < 1.50	1.25	1.01	1.00	1.00	1.00	1.00	1.00
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	10.91	4.05	1.23	1.05	1.02	1.01
0.50 ≤ MV/GV < 0.75	0.50	36.86	12.08	2.35	1.32	1.14	1.06
MV/GV < 0.50	0.25	114.73	36.18	6.16	2.63	1.79	1.35
Minimum Death Benefit	T=2	With Resets: 100% GMDB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.85	0.84	0.80	0.71	0.63	0.55
1.50 ≤ MV/GV < 2.00	1.50	0.85	0.84	0.80	0.71	0.63	0.55
1.25 ≤ MV/GV < 1.50	1.25	0.85	0.84	0.80	0.71	0.63	0.55
1.00 ≤ MV/GV < 1.25	1.00	0.81	0.82	0.80	0.71	0.63	0.55
0.75 ≤ MV/GV < 1.00	0.75	4.18	1.77	0.90	0.74	0.65	0.56
0.50 ≤ MV/GV < 0.75	0.50	11.59	4.06	1.27	0.87	0.71	0.59
MV/GV < 0.50	0.25	33.83	10.95	2.36	1.27	0.94	0.71

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=20	No Elective Resets: Level Maturity Benefit at Age 70					
MV/GV \geq 2.00	2.00	-	-	0.02	0.06	0.11	0.14
1.50 \leq MV/GV < 2.00	1.50	-	-	0.10	0.20	0.28	0.33
1.25 \leq MV/GV < 1.50	1.25	-	-	0.28	0.42	0.51	0.56
1.00 \leq MV/GV < 1.25	1.00	-	0.12	1.00	1.00	1.00	1.00
0.75 \leq MV/GV < 1.00	0.75	-	2.90	5.43	2.66	2.20	1.79
0.50 \leq MV/GV < 0.75	0.50	110.13	173.83	34.26	6.17	4.69	3.38
MV/GV < 0.50	0.25	1,041.20	1,104.90	131.20	16.69	12.16	8.15
Minimum Maturity Benefit	T=4	No Elective Resets: Level GMMB at Age 70, \leq 7 years to Maturity					
MV/GV \geq 2.00	2.00	-	-	0.24	0.64	1.30	1.64
1.50 \leq MV/GV < 2.00	1.50	-	-	3.20	4.11	5.48	5.59
1.25 \leq MV/GV < 1.50	1.25	-	0.58	20.57	11.31	11.27	9.31
1.00 \leq MV/GV < 1.25	1.00	28.13	197.26	118.01	23.61	20.00	14.88
0.75 \leq MV/GV < 1.00	0.75	1,678.04	2,000.48	306.89	44.12	34.55	24.17
0.50 \leq MV/GV < 0.75	0.50	5,306.02	5,628.45	684.64	85.15	63.66	42.74
MV/GV < 0.50	0.25	16,189.95	16,512.38	1,817.89	208.21	150.98	98.46
Minimum Maturity Benefit	T=8	No Elective Resets: 75% GMMB 10-yr Term, > 3 year to Maturity					
MV/GV \geq 2.00	2.00	-	-	0.05	0.23	0.32	0.46
1.50 \leq MV/GV < 2.00	1.50	-	-	0.09	0.34	0.44	0.57
1.25 \leq MV/GV < 1.50	1.25	-	0.01	0.20	0.54	0.62	0.72
1.00 \leq MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 \leq MV/GV < 1.00	0.75	347.17	31.95	3.30	1.87	1.70	1.54
0.50 \leq MV/GV < 0.75	0.50	1,646.71	113.31	7.92	3.61	3.13	2.68
MV/GV < 0.50	0.25	5,545.33	357.40	21.81	8.85	7.42	6.13
Minimum Maturity Benefit	T=2	No Elective Resets: 75% GMMB 10-yr Term, \leq 3 year to Maturity					
MV/GV \geq 2.00	2.00	-	-	0.09	0.34	0.44	0.58
1.50 \leq MV/GV < 2.00	1.50	-	-	0.13	0.50	0.66	0.80
1.25 \leq MV/GV < 1.50	1.25	-	0.02	0.52	0.94	1.08	1.17
1.00 \leq MV/GV < 1.25	1.00	91.09	19.03	3.33	2.12	2.06	1.95
0.75 \leq MV/GV < 1.00	0.75	1,660.28	117.35	8.93	4.24	3.79	3.33
0.50 \leq MV/GV < 0.75	0.50	4,801.16	314.00	20.11	8.46	7.25	6.11
MV/GV < 0.50	0.25	14,223.80	903.93	53.69	21.18	17.69	14.53

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=8	No Elective Resets: 100% GMMB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	0.18	0.34	0.47	0.56	0.70
1.50 ≤ MV/GV < 2.00	1.50	-	0.18	0.37	0.53	0.62	0.75
1.25 ≤ MV/GV < 1.50	1.25	-	0.19	0.45	0.65	0.73	0.82
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	347.17	26.30	2.96	1.80	1.62	1.41
0.50 ≤ MV/GV < 0.75	0.50	1,646.71	93.28	7.11	3.47	2.96	2.39
MV/GV < 0.50	0.25	5,545.33	294.23	19.57	8.51	7.03	5.47
Minimum Maturity Benefit	T=2	No Elective Resets: 100% GMMB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.09	0.28	0.59	0.71	0.76	0.83
1.50 ≤ MV/GV < 2.00	1.50	0.09	0.28	0.61	0.80	0.88	0.94
1.25 ≤ MV/GV < 1.50	1.25	0.09	0.29	0.90	1.08	1.17	1.18
1.00 ≤ MV/GV < 1.25	1.00	91.17	15.70	3.01	2.08	1.98	1.78
0.75 ≤ MV/GV < 1.00	0.75	1,660.28	96.63	8.02	4.11	3.61	2.99
0.50 ≤ MV/GV < 0.75	0.50	4,801.16	258.53	18.07	8.19	6.90	5.48
MV/GV < 0.50	0.25	14,223.80	744.27	48.28	20.59	16.97	13.18
Minimum Maturity Benefit	T=8	No Elective Resets: CSV Guarantee, 10-year Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.01	0.06	0.11	0.18
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.06	0.22	0.31	0.40
1.25 ≤ MV/GV < 1.50	1.25	-	0.03	0.21	0.47	0.55	0.62
1.00 ≤ MV/GV < 1.25	1.00	0.06	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	87.16	38.10	4.05	2.11	1.89	1.71
0.50 ≤ MV/GV < 0.75	0.50	903.49	229.54	12.65	4.63	3.87	3.24
MV/GV < 0.50	0.25	4,163.78	952.37	42.22	12.69	10.18	8.07
Minimum Maturity Benefit	T=2	No Elective Resets: CSV Guarantee, 10-year Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.02	0.13	0.23	0.37
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.18	0.56	0.75	0.88
1.25 ≤ MV/GV < 1.50	1.25	-	0.12	0.89	1.22	1.36	1.42
1.00 ≤ MV/GV < 1.25	1.00	15.71	17.95	4.22	2.61	2.52	2.37
0.75 ≤ MV/GV < 1.00	0.75	735.98	207.54	13.71	5.54	4.81	4.17
0.50 ≤ MV/GV < 0.75	0.50	3,162.30	756.92	36.97	12.08	9.91	8.09
MV/GV < 0.50	0.25	11,599.45	2,620.66	113.00	32.57	25.98	20.37

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.]

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=8	No Elective Resets: CSV Guar. 10-yr Term, > 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	-	-	0.01	0.07	0.12	0.22
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.05	0.27	0.37	0.44
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.18	0.55	0.60	0.65
1.00 ≤ MV/GV < 1.25	1.00	0.06	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	265.42	61.09	3.19	1.87	1.72	1.60
0.50 ≤ MV/GV < 0.75	0.50	2,533.53	214.04	7.68	3.62	3.17	2.79
MV/GV < 0.50	0.25	8,531.71	675.11	21.14	8.88	7.52	6.38
Minimum Maturity Benefit	T=2	No Elective Resets: CSV Guar. 10-yr Term, ≤ 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	-	-	0.01	0.14	0.27	0.40
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.13	0.65	0.77	0.83
1.25 ≤ MV/GV < 1.50	1.25	-	0.04	0.82	1.18	1.23	1.26
1.00 ≤ MV/GV < 1.25	1.00	17.13	12.33	3.53	2.16	2.11	2.04
0.75 ≤ MV/GV < 1.00	0.75	2,554.41	221.67	8.65	4.24	3.82	3.47
0.50 ≤ MV/GV < 0.75	0.50	7,386.78	593.12	19.49	8.47	7.33	6.36
MV/GV < 0.50	0.25	21,883.88	1,707.49	52.03	21.17	17.84	15.03
Minimum Maturity Benefit	T=20	With Resets: Level Maturity Benefit at Age 70 (75% ROP)					
MV/GV ≥ 2.00	2.00	-	0.01	0.83	0.95	0.98	1.00
1.50 ≤ MV/GV < 2.00	1.50	-	0.01	0.82	0.95	0.98	1.00
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.82	0.95	0.98	1.00
1.00 ≤ MV/GV < 1.25	1.00	-	0.11	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	-	2.76	2.23	1.22	1.10	1.02
0.50 ≤ MV/GV < 0.75	0.50	104.47	164.89	11.13	2.08	1.53	1.17
MV/GV < 0.50	0.25	987.69	1,048.12	42.54	5.41	3.53	2.05
Minimum Maturity Benefit	T=20	With Resets: Level Maturity Benefit at Age 70 (100% ROP)					
MV/GV ≥ 2.00	2.00	-	0.89	1.01	1.00	1.00	1.00
1.50 ≤ MV/GV < 2.00	1.50	-	0.89	1.01	1.00	1.00	1.00
1.25 ≤ MV/GV < 1.50	1.25	-	0.89	1.01	1.00	1.00	1.00
1.00 ≤ MV/GV < 1.25	1.00	-	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	-	2.27	1.15	1.01	1.00	1.00
0.50 ≤ MV/GV < 0.75	0.50	105.38	111.81	2.38	1.20	1.07	1.01
MV/GV < 0.50	0.25	996.28	710.70	8.55	2.61	1.85	1.30

* For funds classified as "Low Volatility Diversified Equity", the factors for "Broad-Based Diversified Equity" apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=8	With Resets: 75% GMMB 10-yr Term, > 3 year to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.22	0.56	0.66	0.81
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.23	0.57	0.68	0.82
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.32	0.67	0.75	0.86
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	347.17	31.95	3.22	1.84	1.64	1.40
0.50 ≤ MV/GV < 0.75	0.50	1,646.71	113.31	7.74	3.56	3.00	2.38
MV/GV < 0.50	0.25	5,545.33	357.40	21.29	8.72	7.13	5.45
Minimum Maturity Benefit	T=2	With Resets: 75% GMMB 10-yr Term, ≤ 3 year to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.20	0.55	0.64	0.76
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.21	0.62	0.75	0.86
1.25 ≤ MV/GV < 1.50	1.25	-	0.02	0.60	0.98	1.10	1.14
1.00 ≤ MV/GV < 1.25	1.00	91.09	19.03	3.26	2.11	2.00	1.78
0.75 ≤ MV/GV < 1.00	0.75	1,660.28	117.35	8.72	4.18	3.66	3.00
0.50 ≤ MV/GV < 0.75	0.50	4,801.16	314.00	19.65	8.35	7.00	5.49
MV/GV < 0.50	0.25	14,223.80	903.93	52.44	20.93	17.12	13.14
Minimum Maturity Benefit	T=8	With Resets: 100% GMMB 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.04	0.53	0.82	0.90	0.94	0.98
1.50 ≤ MV/GV < 2.00	1.50	0.04	0.53	0.82	0.90	0.94	0.98
1.25 ≤ MV/GV < 1.50	1.25	0.04	0.53	0.83	0.91	0.94	0.98
1.00 ≤ MV/GV < 1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	328.72	21.66	2.47	1.59	1.38	1.17
0.50 ≤ MV/GV < 0.75	0.50	1,559.18	76.81	5.96	3.09	2.48	1.82
MV/GV < 0.50	0.25	5,250.57	242.26	16.40	7.59	5.90	4.12
Minimum Maturity Benefit	T=2	With Resets: 100% GMMB 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.01	0.48	0.82	0.89	0.90	0.91
1.50 ≤ MV/GV < 2.00	1.50	0.01	0.48	0.82	0.89	0.91	0.92
1.25 ≤ MV/GV < 1.50	1.25	0.01	0.49	0.87	0.98	1.01	0.98
1.00 ≤ MV/GV < 1.25	1.00	86.26	12.94	2.53	1.84	1.66	1.40
0.75 ≤ MV/GV < 1.00	0.75	1,572.03	79.57	6.76	3.68	3.06	2.31
0.50 ≤ MV/GV < 0.75	0.50	4,545.96	212.87	15.20	7.34	5.85	4.23
MV/GV < 0.50	0.25	13,467.74	612.82	40.64	18.52	14.48	10.28

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.]

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=8	With Resets: 75% CSV Guar. 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.29	0.62	0.73	0.86
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.30	0.64	0.74	0.86
1.25 ≤ MV/GV < 1.50	1.25	-	0.03	0.38	0.71	0.80	0.89
1.00 ≤ MV/GV < 1.25	1.00	0.06	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	85.38	38.10	3.74	1.92	1.62	1.33
0.50 ≤ MV/GV < 0.75	0.50	885.06	229.54	11.67	4.19	3.27	2.32
MV/GV < 0.50	0.25	4,078.82	952.37	38.97	11.48	8.60	5.70
Minimum Maturity Benefit	T=2	With Resets: 75% CSV Guar. 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	-	-	0.33	0.80	0.91	1.04
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.42	0.89	1.01	1.10
1.25 ≤ MV/GV < 1.50	1.25	-	0.12	1.00	1.30	1.34	1.31
1.00 ≤ MV/GV < 1.25	1.00	15.39	17.95	3.92	2.40	2.17	1.82
0.75 ≤ MV/GV < 1.00	0.75	720.96	207.54	12.65	5.01	4.07	2.98
0.50 ≤ MV/GV < 0.75	0.50	3,097.78	756.92	34.12	10.93	8.36	5.71
MV/GV < 0.50	0.25	11,362.79	2,620.66	104.30	29.48	21.94	14.39
Minimum Maturity Benefit	T=8	With Resets: 100% CSV Guar. 10-yr Term, > 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.01	0.50	0.87	0.93	0.96	0.99
1.50 ≤ MV/GV < 2.00	1.50	0.01	0.50	0.87	0.93	0.96	0.99
1.25 ≤ MV/GV < 1.50	1.25	0.01	0.50	0.87	0.94	0.96	0.99
1.00 ≤ MV/GV < 1.25	1.00	0.07	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	84.52	24.51	2.42	1.48	1.27	1.11
0.50 ≤ MV/GV < 0.75	0.50	876.17	147.66	7.61	3.16	2.34	1.60
MV/GV < 0.50	0.25	4,037.83	612.63	25.41	8.65	6.14	3.77
Minimum Maturity Benefit	T=2	With Resets: 100% CSV Guar. 10-yr Term, ≤ 3 years to Maturity					
MV/GV ≥ 2.00	2.00	0.01	0.62	1.16	1.21	1.20	1.18
1.50 ≤ MV/GV < 2.00	1.50	0.01	0.62	1.16	1.22	1.20	1.18
1.25 ≤ MV/GV < 1.50	1.25	0.01	0.62	1.21	1.26	1.25	1.20
1.00 ≤ MV/GV < 1.25	1.00	15.19	11.71	2.61	1.83	1.64	1.38
0.75 ≤ MV/GV < 1.00	0.75	713.72	133.51	8.24	3.78	2.92	2.04
0.50 ≤ MV/GV < 0.75	0.50	3,066.65	486.90	22.25	8.23	5.98	3.78
MV/GV < 0.50	0.25	11,248.60	1,685.79	68.02	22.20	15.67	9.49

* For funds classified as “Low Volatility Diversified Equity”, the factors for “Broad-Based Diversified Equity” apply.

Table 2 – MV/GV and Time to Maturity Adjustments (Status Factor) – continued

	MV/GV Ratio	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity Benefit	T=8	With Resets: 75% CSV Guar. 10-yr Term, > 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	-	-	0.22	0.57	0.67	0.82
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.23	0.58	0.68	0.82
1.25 ≤ MV/GV < 1.50	1.25	-	0.01	0.32	0.67	0.75	0.86
1.00 ≤ MV/GV < 1.25	1.00	0.06	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	260.49	61.09	3.04	1.83	1.64	1.39
0.50 ≤ MV/GV < 0.75	0.50	2,486.45	214.04	7.30	3.56	3.00	2.36
MV/GV < 0.50	0.25	8,373.19	675.11	20.09	8.71	7.11	5.38
Minimum Maturity Benefit	T=2	With Resets: 75% CSV Guar. 10-yr Term, ≤ 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	-	-	0.20	0.56	0.65	0.77
1.50 ≤ MV/GV < 2.00	1.50	-	-	0.26	0.69	0.79	0.87
1.25 ≤ MV/GV < 1.50	1.25	-	0.04	0.88	1.15	1.17	1.14
1.00 ≤ MV/GV < 1.25	1.00	16.81	12.33	3.36	2.12	2.00	1.74
0.75 ≤ MV/GV < 1.00	0.75	2,506.95	221.67	8.22	4.16	3.62	2.92
0.50 ≤ MV/GV < 0.75	0.50	7,249.53	593.12	18.53	8.32	6.93	5.36
MV/GV < 0.50	0.25	21,477.26	1,707.49	49.44	20.79	16.87	12.66
Minimum Maturity Benefit	T=8	With Resets: 100% CSV Guar. 10-yr Term, > 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	0.01	0.50	0.82	0.91	0.94	0.98
1.50 ≤ MV/GV < 2.00	1.50	0.01	0.50	0.82	0.91	0.94	0.98
1.25 ≤ MV/GV < 1.50	1.25	0.01	0.50	0.83	0.91	0.94	0.98
1.00 ≤ MV/GV < 1.25	1.00	0.07	1.00	1.00	1.00	1.00	1.00
0.75 ≤ MV/GV < 1.00	0.75	259.70	41.72	2.41	1.59	1.37	1.16
0.50 ≤ MV/GV < 0.75	0.50	2,478.91	146.20	5.83	3.08	2.45	1.77
MV/GV < 0.50	0.25	8,347.80	461.12	16.04	7.56	5.80	3.95
Minimum Maturity Benefit	T=2	With Resets: 100% CSV Guar. 10-yr Term, ≤ 3 years to Maturity ("in-the-money" surrender)					
MV/GV ≥ 2.00	2.00	0.01	0.52	0.82	0.90	0.92	0.92
1.50 ≤ MV/GV < 2.00	1.50	0.01	0.52	0.82	0.91	0.93	0.93
1.25 ≤ MV/GV < 1.50	1.25	0.01	0.52	0.89	0.99	1.01	0.98
1.00 ≤ MV/GV < 1.25	1.00	16.71	8.64	2.64	1.81	1.61	1.33
0.75 ≤ MV/GV < 1.00	0.75	2,499.35	151.41	6.56	3.61	2.95	2.16
0.50 ≤ MV/GV < 0.75	0.50	7,227.55	405.12	14.79	7.22	5.65	3.93
MV/GV < 0.50	0.25	21,412.14	1,166.27	39.48	18.03	13.77	9.30

* For funds classified as "Low Volatility Diversified Equity", the factors for "Broad-Based Diversified Equity" apply.

Step 4 - MER Adjustment (C)

Determine C, an adjustment reflecting differences in the actual MER (Management Expense Ratio) from those assumed in developing the factor tables (optional where the difference is less than 0.25% per annum).

This adjustment factor is calculated as follows:

$$C = g(MV/GV) \times (\text{Actual MER} - \text{Assumed MER})$$

Where: **$g(MV/GV)$** is determined by formula from Table 3.

Actual MER is the MER (management expense ratio) charged by the company for the product on a “per annum” basis, expressed in decimal form.

Assumed MER is the MER (expressed in decimal form) assumed in the development of the factors and is shown in Table 3 (in basis points per annum).

The function $g(MV/GV)$ is evaluated by applying the following formula:

$$g\left(\frac{MV}{GV}\right) = \left(a - b \times \frac{MV}{GV}\right)^3$$

In the above, MV/GV is the actual MV/GV ratio for the exposure being valued, subject to a maximum of 2.00. The parameters “ a ” and “ b ” vary by product type and fund class according to Table 3.

Should refined MER data not be available, an adjustment should be made based on a conservative estimation of the average MERs (i.e., higher MERs are conservative).

Where applicable, any additional taxes (e.g., GST and/or HST) should be reflected in the MER before determining the difference between actual and assumed MERs. For example, if the actual MER for a Diversified Equity fund is 3% + GST, the difference would be 3.21% – 2.65% = 0.56% where 3.21% is derived as 3% + 7% GST.

Table 3 – Formula Parameters for MER Adjustment

	Money Market		Fixed Income		Balanced		Broad-Based Diversified Equity *		Intermediate Risk Equity		Exotic or Aggressive Equity	
Basic Total Management Expense Ratios (basis points, per annum):	110		200		250		265		280		295	
	a	b	a	b	a	b	a	b	a	b	a	b
Minimum Death Benefits												
No resets or ratchets; level guarantee to contract maturity (age 70)	0.964	0.545	0.938	0.489	0.863	0.357	0.756	0.211	0.726	0.190	0.681	0.152
No resets or ratchets; level guarantee to contract maturity (age 90)	1.300	0.782	1.266	0.709	1.165	0.526	1.022	0.323	0.983	0.294	0.929	0.247
Annual ratchet; to contract maturity (age 90)	1.285	0.744	1.239	0.641	1.164	0.565	1.149	0.863	-0.515	0.220	-1.058	0.112
5% annual roll-up; to contract maturity (age 90)	1.229	0.254	1.200	0.227	1.123	0.177	0.994	0.111	0.980	0.122	0.937	0.110
No resets or ratchets; 75% 10-year	0.546	0.228	0.532	0.202	0.473	0.104	0.446	0.154	0.423	0.384	-0.441	0.061
No resets or ratchets; 100% 10-year	0.535	0.193	0.502	0.116	0.485	0.122	-0.267	0.099	-0.514	0.052	-0.748	0.028
With resets/ratchets; 75% 10-year	0.546	0.228	0.532	0.202	0.466	0.095	0.490	0.380	-0.338	0.079	-0.644	0.037
With resets/ratchets; 100% 10-year	0.525	0.170	0.487	0.077	0.516	0.257	-0.455	0.058	-0.688	0.031	-0.949	0.015
Minimum Maturity & Surrender Benefits												
No resets or ratchets; level guarantee to contract maturity (age 70)	1.476	1.009	1.450	0.966	1.307	0.682	1.078	0.300	1.026	0.252	0.920	0.150
No resets or ratchets; 75% 10-year	1.749	0.767	1.699	0.705	1.516	0.395	1.309	0.207	1.272	0.215	1.203	0.232
No resets or ratchets; 100% 10-year	1.749	0.766	1.692	0.680	1.492	0.339	1.392	0.314	1.360	0.340	1.352	0.500
No resets or ratchets; 10-year CSV	1.957	1.262	1.915	1.163	1.739	0.826	1.495	0.469	1.430	0.414	1.335	0.336
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	1.791	0.686	1.734	0.669	1.519	0.383	1.283	0.188	1.205	0.141	1.108	0.094
With resets/ratchets; 75% to contract maturity	1.476	1.009	1.450	0.966	1.272	0.587	0.925	0.086	0.853	0.046	0.735	0.012
With resets/ratchets; 100% to contract maturity	1.476	1.008	1.442	0.940	1.077	0.142	0.874	0.109	0.798	0.025	0.581	0.008
With resets/ratchets; 75% 10-year	1.749	0.767	1.699	0.705	1.478	0.328	1.323	0.230	1.288	0.268	1.318	0.488
With resets/ratchets; 100% 10-year	1.748	0.763	1.659	0.595	1.442	0.237	1.421	0.336	1.434	0.474	1.593	1.114
With resets/ratchets; 75% 10-year CSV	1.957	1.262	1.915	1.163	1.724	0.790	1.488	0.476	1.451	0.504	1.395	0.575
With resets/ratchets; 100% 10-year CSV	1.957	1.261	1.905	1.135	1.678	0.690	1.489	0.515	1.486	0.641	1.482	0.977
With resets/ratchets; 75% 10-year CSV ("in-the-money" surrender)	1.791	0.686	1.734	0.669	1.465	0.287	1.281	0.215	1.231	0.238	1.195	0.405
With resets/ratchets; 100% 10-year CSV ("in-the-money" surrender)	1.791	0.685	1.716	0.623	1.403	0.219	1.274	0.241	1.247	0.339	1.328	1.004
Minimum Income Benefits												
Guaranteed Minimum Income Benefit	Company must calibrate and model separately											

* For funds classified as "Low Volatility Diversified Equity", the factors for "Broad-Based Diversified Equity" apply.

Step 5 - Margin Offset Adjustment (D)

Determine D, a reduction to adjust for available margins.

The requirement to this point is based on a present value of costs without reduction for the existence of margins available to be drawn upon to make guarantee payments. The following table provides factors that are used to reflect the existence of margins to offset the costs. The table reflects the percentage of market value reduction for each 1% of available margins. The factors vary by fund type and product type. Factor D can be expressed as:

$$\mathbf{D = Available Margin \times Table 4 Factor}$$

Available margins should be consistent with the unitized margins/charges for these features that can reasonably be expected to be available as revenue to offset these costs in those scenarios which give rise to benefit payments in the “worst” 5% of cases (“worst” is defined as those scenarios which give rise to the “highest” net benefit costs on a present value basis at the valuation date). In other words, this offset should be the sustainable margin that is expected to be available based on company valuation practices, and should not double count any revenue that would support the base product valuation or other non-valuation costs in a similar adverse investment performance scenario.

For example, if the anticipated company practice is that unitized margins/charges would be re-allocated to the base product valuation in an adverse investment performance scenario (e.g., to ensure acquisition cost recoverability), then the available margins that are assumed for the offset calculation should be net of the projected re-allocation that would occur. In such circumstances, the true margin available as an offset could be materially lower than the amount that a simple analysis reflecting only the current environment would imply.

Table 4 – Margin Offset Factors (per 1% of margin offset)

	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Death Benefits						
No resets or ratchets; level guarantee to contract maturity (age 70)	-0.0735	-0.0714	-0.0635	-0.0562	-0.0548	-0.0568
No resets or ratchets; level guarantee to contract maturity (age 90)	-0.0631	-0.0614	-0.0549	-0.0465	-0.0449	-0.0417
Annual ratchet; to contract maturity (age 90)	-0.0654	-0.0655	-0.0766	-0.1073	-0.1723	-0.2758
5% annual roll-up; to contract maturity (age 90)	-0.0621	-0.0603	-0.0550	-0.0474	-0.0461	-0.0428
No resets or ratchets; 75% 10-year	-0.0778	-0.0751	-0.0765	-0.0778	-0.0856	-0.1103
No resets or ratchets; 100% 10-year	-0.0787	-0.0771	-0.0835	-0.0897	-0.1137	-0.1708
With resets/ratchets; 75% 10-year	-0.0768	-0.0742	-0.0779	-0.0829	-0.1010	-0.1558
With resets/ratchets; 100% 10-year	-0.0787	-0.0772	-0.0888	-0.1051	-0.1455	-0.2329
Minimum Maturity & Surrender Benefits						
No resets or ratchets; level guarantee to contract maturity (age 70)	-0.0735	-0.0714	-0.0635	-0.0562	-0.0548	-0.0568
No resets or ratchets; 75% 10-year	-0.0778	-0.0751	-0.0765	-0.0778	-0.0856	-0.1103
No resets or ratchets; 100% 10-year	-0.0787	-0.0771	-0.0835	-0.0897	-0.1137	-0.1708
No resets or ratchets; 10-year CSV	-0.0774	-0.0747	-0.0697	-0.0605	-0.0587	-0.0556
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	-0.0774	-0.0742	-0.0550	-0.0418	-0.0405	-0.0389
With resets/ratchets; 75% to contract maturity	-0.0728	-0.0701	-0.0672	-0.0903	-0.1051	-0.1403
With resets/ratchets; 100% to contract maturity	-0.0728	-0.0703	-0.0865	-0.1066	-0.1315	-0.1678
With resets/ratchets; 75% 10-year	-0.0768	-0.0742	-0.0779	-0.0829	-0.1010	-0.1558
With resets/ratchets; 100% 10-year	-0.0787	-0.0772	-0.0888	-0.1051	-0.1455	-0.2329
With resets/ratchets; 75% 10-year CSV guarantee	-0.0774	-0.0747	-0.0749	-0.0784	-0.0966	-0.1398
With resets/ratchets; 100% 10-year CSV guarantee	-0.0774	-0.0755	-0.0886	-0.1040	-0.1359	-0.2109
With resets/ratchets; 75% 10-yr CSV ("in-the-money" surrender)	-0.0774	-0.0742	-0.0633	-0.0520	-0.0641	-0.1081
With resets/ratchets; 100% 10-yr CSV ("in-the-money" surrender)	-0.0774	-0.0750	-0.0692	-0.0749	-0.1014	-0.1838
Minimum Income Benefits						
Guaranteed Minimum Income Benefit	Company must calibrate and model separately					

- For funds classified as "Low Volatility Diversified Equity", the factors for "Broad-Based Diversified Equity" apply.

The factor from the table, scaled by actual available margins, is added to the factor already calculated in steps 1-5.

Should refined margin offset data not be available, an adjustment should be made based on a conservative estimate of the offsets available (i.e., low offsets are conservative).

Where more than one feature is present in a product, unless the company has a justifiable alternative for allocating the total available margin between the benefit types, the split should be based on the proportionate gross costs developed in steps 1 to 5.

With respect to Table 1a, the “with resets” rows have assumed 100% utilization. If the product offers an elective reset feature and the company has reliable and pertinent experience regarding the rate of utilization, the company should interpolate between the corresponding “no resets” and “with resets” rows according to the proportion of business that exercises the reset option. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* in section 9-9.

For products which offer a reset option (elective or automatic), but the contract is outside the period in which resets are permitted (e.g. beyond some duration or attained age limit), the exposure should be valued as if the reset option were not present.

The “in-the-money surrender” rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is “in-the-money” by more than 15% (i.e., MV/GV ratio is less than 0.85). If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and “in-the-money surrender” rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

Credit for Reinsurance Ceded or Capital Markets Hedging

This is the reduction in the risk due to risk mitigation strategies. Risk mitigation includes reinsurance and hedging.

For reinsurance of the actuarial liabilities that can be expressed directly in terms of the factors, credit can be taken on that basis.

For more complex reinsurance arrangements that cannot be expressed using the factors, the impact will need to be modelled (refer to *Modelling and Calibration* in section 9-10) and brought to OSFI for approval. For example, a reinsurance treaty that has the ceding company retain losses to a predetermined level (a “deductible”), with the reinsurer assuming losses above this level, but with a cap on the reinsurance claims (e.g., a maximum annual payment cap under the treaty).

Policy liabilities ceded to unregistered reinsurers per Guideline B-3, must be reported on page 20.030 line 085 by Canadian companies, and on page 25.010 line 050 by foreign branches.

Guideline B-3 *Unregistered Reinsurance* will apply to segregated funds. Deposits held for unregistered reinsurance per Guideline B-3, for the period not less than the fund guarantee term remaining which are in excess of the actuarial liabilities for the risk reinsured, may reduce the net required segregated fund risk component requirement on the reinsured policies to a minimum of zero (report this amount in column 04 on page 90.010). For Canadian business, the deposits must be held in Canada, and OSFI must have given the company permission to reduce its reserves by the deposits held corresponding to the reserves. The reduction is limited to that available had the business been ceded to a reinsurer subject to these requirements.

Institutions seeking to obtain credit for segregated funds for hedging programs should refer to OSFI Guidance Note “*Capital Offset for Segregated Fund Hedging Programs (MCCSR)*” for instructions and requirements.

Interpolation/Extrapolation in the Factor Tables

To keep the table and adjustment factor structure fairly simple, it is necessary in some cases for the company to interpolate or extrapolate values. As a general rule, when interpolating between values, a linear interpolation should be used. When a factor lies outside the ranges, a linear extrapolation should be used. However, if a company's own modelling shows an extrapolation based on the above approach to be inadequate, higher appropriate values should be used. In all cases, the low range values would be constrained so that they are positive. Details of all interpolations should be maintained by the actuary and detailed in the Appointed Actuary's Report.

Modelling and Calibration

Should the company be evaluating a product type that is materially different from those presented in the tables, or where a company needs to evaluate a complex reinsurance or hedging arrangement, it will be necessary to use stochastic modelling to calculate factors for their particular product or treaty.

Any use of modelling to calculate factors specific to a product will need to be approved by the Actuarial Division of OSFI. First, it will need to be determined whether modelling is appropriate to the situation; then the process to generate the factors will need to be reviewed.

To set the appropriate factors, the company will need to:

1. Calibrate its models/assumptions on a stochastic basis to particular product type in Table 1a. This calibration will be performed using the investment model calibration process and assumption described in the CIA Report of the Task Force on Stochastic Techniques for Segregated Fund Guarantees on a CTE 95 basis. Calculate the ratio between the table factors and the calculated factors
2. Recalculate factors using this calibrated model, but with the product parameters adjusted to fit the product or arrangement in question.
3. Adjust the factors in 2 by the ratio calculated in 1.
4. Use these factors to calculate the Total Gross Calculated Requirements.

The details of the factor generated must be documented and a report must be produced for OSFI approval.

Foreign Business Capital Requirements

OSFI will permit, on a limited basis, and under strict criteria, the use of internal models for the development of segregated fund capital requirements for foreign business (policies outside of Canada) only. Institutions seeking to use their internal models must consult with OSFI regarding specific requirements. Internal model usage requires explicit written prior OSFI approval. The requirements also include transitional rules: in the first year of approval only 50% credit is permitted (i.e., the Total Gross Calculated Requirement is equal to 50% of the value calculated under the approved internal model plus 50% of the value calculated using the factor requirements). However in subsequent years the requirement is based 100% upon the value determined by the approved internal model.

APPENDIX G – Dec 2001 OSFI MCCSR Guideline (Sec. 9): Documentation

Executive Summary

The Office of the Superintendent of Financial Institutions Canada (“OSFI”) engaged a financial consulting firm (the “Firm”) to assist in the development of the factor-based methodology for the segregated fund investment guarantee component of the regulatory Minimum Continuing Capital and Surplus Requirement (“MCCSR”). This component is for the risks associated with investment or performance-related guarantees on segregated funds and other similar investment/insurance products.

The MCCSR component for segregated fund guarantee risk is determined by applying a series of factors to the market value of segregated/separate account assets for each unit of exposure. The factor tables were developed from the results of extensive stochastic testing and reflect the primary risk drivers for a range of standardized product forms covering guaranteed minimum death (GMDDB) and maturity benefits (GMMB). The risk drivers include: market value-to-guaranteed value (MV/GV) ratio, investment class, time-to-maturity, management fees/charges and margins available to offset the costs of the guaranteed benefits.

The mandate for the assignment required the Firm to build projection models and conduct testing and analysis on behalf of OSFI in order to construct the factor tables for the total gross calculated requirement (“TGCR”). The Firm also reviewed the December 2001 MCCSR Guideline – Chapter 9 and suggested revisions where appropriate.

The future performance of the standardized products was simulated under a set of stochastic investment return scenarios from a correlated regime-switching lognormal process (with two regimes). The model generated 10,000 scenarios for each asset class using Monte Carlo simulation. All other projection assumptions (e.g., persistency, discount rates, etc.) were deterministic.

The factor-based methodology remains substantially unchanged from December 2000 and is founded upon the pioneering work by the Canadian Institute of Actuaries Task Force on Segregated Fund Investment Guarantees (August 2000). However, some factor tables have been expanded and/or revised to capture more appropriately the risks on the standardized products. For reference, a summary of changes is included later in this report. All table entries are based on the results of extensive testing and calculated at a confidence level consistent with a 95th percentile conditional right-tail expectation (“CTE⁺”).

This report documents the assumptions and models, offering key results where appropriate. The final factor tables are presented in the MCCSR Guideline (see the Appendices). The complete set of model output, including all supporting calculations, was submitted by the Firm to OSFI in August 2001 via Microsoft[®] Excel 97 workbooks.

Factor Basis

The MCCR component for segregated fund guarantee risk extends to all investment performance related guarantees on variable insurance and annuity products, including both Canadian and non-Canadian business. Factors applied to the market value of segregated fund and similar risk assets are used to set Total Gross Calculated Requirements for MCCR in respect of the investment performance guarantees. The TGCR establishes a total balance sheet provision, whereas minimum regulatory capital (net required component) is equal to the difference between the TGCR and net actuarial liabilities (after any OSFI approved credit for hedging and reinsurance).

The TGCR calculation basis is fully documented in OSFI's MCCR Guideline – Section 9 along with the published factor tables. This is a five-step factor process that can be expressed as follows:

$$\text{Total Gross Calculated Requirement} = \text{Market Value} \times [(\mathbf{A1} \times \mathbf{A2} \times \mathbf{B}) + \mathbf{C} + \mathbf{D}]$$

where the factors are:

1. *Basic Factor (A1)* – This factor is based on the product type and investment fund class.
2. *Time Diversification Adjustment (A2)* – An adjustment factor applicable if the maturities are sufficiently spread out over time.
3. *MV/GV and Time to Maturity Adjustment (B)* – An adjustment factor for the relationship between market value and guaranteed value, and for the time left to maturity.
4. *MER Adjustment (C)* – An adjustment to account for the differences between actual management expense ratios (MER) and those assumed in the development of the factors. This reflects the deduction of MERs from investment funds.
5. *Margin Offset Adjustment (D)* – An adjustment factor to reflect the margins available to cover the cost of the guarantee.

On a limited basis and under strict criteria in 2001, OSFI is permitting companies to use the results of internal (company-specific) models for the development of segregated fund capital requirements in respect of investment performance guarantees on foreign business. Prior written approval from OSFI is required. Application for approval and the specific requirements imposed by OSFI are outside the scope of this assignment.

Summary of Changes for December 2001

1. Actuarial Liabilities Ceded:

Guideline B-3 *Unregistered Reinsurance* will apply to segregated funds. Deposits held for unregistered reinsurance per Guideline B-3, for the period not less than the fund guarantee term remaining, which are in excess of the actuarial liabilities for the risk reinsured, may reduce the net required segregated fund risk component requirement on the reinsured policies to a minimum of zero.

2. Transition:

The full Net Required Component is required at year-end 2001 and must not be less than zero in total.

3. Total Gross Calculated Requirement (TGCR):

The factor methodology to determine the total gross calculated requirement is a 5-step process expressed as:

$$\text{TGCR} = \text{Market Value} \times [(A1 \times A2 \times B) + C + D]$$

where the factors are described in the previous section "*Factor Basis*".

4. Total Management Expense Ratios (MERs):

The total management expense ratio (MER) assumed in the development of the factor tables varies only by investment fund class, not by product type. The Basic MERs (in basis points per annum) are: Money Market – 110, Bond – 200, Balanced – 250, Broad-Based Diversified Equity – 265, Intermediate Risk Equity – 280, Exotic/Aggressive Equity – 295.

5. Intermediate Risk Equity Class:

The "Intermediate Risk Equity" investment class is now explicitly modelled. In December 2000, "Intermediate Risk Equity" factors were calculated as a 53/47 weighted-average of the respective factors for "Diversified Equity" and "Aggressive/Exotic Equity"

6. Step 1 – Basic Factor (A1):

Detailed benefit descriptions are provided for all product types.

The Basic Factor Table 1a includes a new type of maturity guarantee which pays a "top-up" maturity benefit only upon surrender of the fund units to which the guarantee applies. The costs associated with this benefit form are very sensitive to the underlying surrender (lapse) assumption and the assumed policyholder behaviour in adverse investment performance scenarios. The "in-the-money surrender" rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is "in-the-money" by more than 15% (i.e., MV/GV ratio is less than 0.85). If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and "in-the-money surrender" rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable.

7. Table 1a – Low Volatility Diversified Equity:

A new investment class has been introduced to accommodate lower volatility diversified equity funds. Only funds that would otherwise be classified as Broad-Based Diversified Equity are candidates for inclusion provided certain conditions are met. The expected volatility of the fund must be less than 15.5% (annualized) and the aggressive/exotic equity component of the equity holdings must be less than 20% of the total equities by market value. Other specific criteria also apply (e.g., the fund permanently maintains a relatively large cash or fixed income position and/or the fund is income oriented and contains a significant proportion of stocks paying material and regular dividends that are automatically reinvested).

All funds in this asset class use the “Broad-Based Diversified Equity” factors in all subsequent calculations involving Tables 1b, 2, 3 and 4.

8. Step 2 – Time Diversification Adjustment (A2):

The adjustment factors for time diversification (Table 1b) include the guaranteed surrender benefit product type. See above under item 6. for more details.

9. Step 3 – MV/GV and Time-to-Maturity Adjustment: Status Factor (B):

A grid of Status Factors is provided in Table 2 for each product type referenced in the Basic Factor table.

For product types where term-to-maturity is specified in Table 2, the company may interpolate between corresponding rows in the relevant sections based on the actual remaining term-to-next-maturity of the exposure being valued, subject to the guidance provided in section 9-9: *Interpolation/Extrapolation in the Factor Tables*. Linear extrapolation beyond the assumed terms-to-maturity is not permitted. An example is provided to demonstrate how the interpolation is to be applied.

10. Step 4 – MER Adjustment (C):

The MER Adjustment “C” is now an additive component, not multiplicative. The adjustment is calculated as:

$$C = g(MV/GV) \times (\text{Actual MER} - \text{Assumed MER})$$

where the function $g(MV/GV)$ is determined by the following formula:

$$g\left(\frac{MV}{GV}\right) = \left(a - b \times \frac{MV}{GV}\right)^3$$

In the above, MV/GV is the actual MV/GV ratio for the exposure being valued, subject to a maximum of 2.00. The parameters “a” and “b” vary by product type and fund class according to Table 3.

11. Step 5 – Margin Offset Adjustment (D):

The adjustments for margin offset (Table 4) include the guaranteed surrender benefit product type. See above under item 6. for more details.

Standardized Products

The models included a variety of standardized product forms for guaranteed minimum death and maturity benefits commonly offered on variable insurance and annuity products in Canada and the United States. The tables in the MCCR Guideline (see the Appendices) provide descriptions for the benefit types that comprise the test policies in the factor development.

Three generic GMDB forms are presented that typify benefits commonly offered on variable annuities in the United States: level return-of-premium (to contract maturity at age 90), annual ratchet and 5% annual roll-up (collectively, the “U.S. GMDBs”). While many variations exist of these basic benefit forms, the factors presented for these contracts may be used as benchmarks for internal model calibration or by companies with immaterial blocks of business.

Assumptions

The Firm used its proprietary software to simulate the future performance of the standardized products under a set of stochastic investment return scenarios from a correlated regime-switching lognormal process (with two regimes). The model generated 10,000 scenarios for each asset class using Monte Carlo simulation. All other projection assumptions (e.g., persistency, discount rates, etc.) were deterministic.

All tabular entries are calculated at a confidence level consistent with a 95th percentile conditional right-tail expectation (“CTE⁺”). By definition, the sample CTE⁺(x%) is the weighted-average of the highest (1-x)% of results when ranked (ordered) by the net present value of guaranteed benefit costs (“PVGB”). Under Monte Carlo simulation where we consider all 10,000 scenarios to be equally likely, CTE⁺(95%) is the arithmetic average of the highest 500 net present values.

The cashflow models assume a three month time increment and apply decrements uniformly over each policy year. Cashflows are discounted at a level 6% annual effective rate along each scenario (i.e., the discount rates do not vary with the investment performance of the funds). Surrender charges and taxes are ignored.

Persistency, Transfers & Withdrawals

Policy movement (persistency) is calculated assuming a uniform distribution of decrements (UDD) in the associated single decrement table. Mortality is assumed to follow the CIA 1986-1992 Age Last Birthday Aggregate Ultimate insurance table. The valuation mortality rates underlying the factor development are a 60/40 blend of the respective male and female decrements and are shown in the table on the following page (per 1000). All sample contracts assume the life is attained age 50 (age last) at the valuation date except for the U.S. GMDB products which assume age 60.

Lapse rates are a constant 8% per annum at all policy durations irrespective of investment performance except for the U.S. GMDB (to age 90) product forms where the lapse rate is 10% per annum. All terminations other than by death are treated as voluntary surrenders. All policies are assumed to terminate by maturity at the end of the end of the contract period.

The model does not permit partial withdrawals nor future fund transfers between investment options. Only amounts currently at risk are considered (i.e., future deposits are ignored). The guaranteed benefit amounts are adjusted on a pro-rata basis (not dollar-for-dollar) to account for survivorship. Both mortality and lapse decrements were used in valuing all sample contracts (e.g., mortality was included in the survivorship calculations for the GMMBs).

CIA 1986-1992 Insurance Mortality Table (Age Last Birthday, Aggregate, Ultimate):

Attained Age (Last Birthday)	CIA86-92 Male	CIA86-92 Female	Valuation Rate ×1000	Attained Age (Last Birthday)	CIA86-92 Male	CIA86-92 Female	Valuation Rate ×1000
50	3.85	2.74	3.406	70	30.07	17.47	25.030
51	4.29	2.98	3.766	71	33.11	19.36	27.610
52	4.78	3.23	4.160	72	36.43	21.46	30.442
53	5.31	3.51	4.590	73	40.07	23.80	33.562
54	5.91	3.82	5.074	74	44.05	26.41	36.994
55	6.58	4.17	5.616	75	48.39	29.32	40.762
56	7.31	4.55	6.206	76	53.14	32.56	44.908
57	8.12	4.97	6.860	77	58.31	36.16	49.450
58	9.02	5.44	7.588	78	63.96	40.17	54.444
59	10.00	5.96	8.384	79	70.11	44.64	59.922
60	11.09	6.54	9.270	80	76.81	49.60	65.926
61	12.29	7.18	10.246	81	84.10	55.12	72.508
62	13.61	7.90	11.326	82	92.03	61.25	79.718
63	15.06	8.70	12.516	83	100.64	68.05	87.604
64	16.66	9.58	13.828	84	109.99	75.60	96.234
65	18.41	10.57	15.274	85	120.12	83.97	105.660
66	20.33	11.67	16.866	86	131.10	93.24	115.956
67	22.44	12.89	18.620	87	142.97	103.49	127.178
68	24.75	14.26	20.554	88	155.80	114.81	139.404
69	27.29	15.78	22.686	89	169.64	127.31	152.708
				90	184.54	141.07	167.152

Management Fees & Risk Margin Offsets

Administration and management fees (MERs) are charged to the funds at the end of each three month period. The quarterly equivalent total management fee used for calculation is:

$$1 - (1 - MER)^{\frac{1}{4}}$$

The fees vary only by fund class as indicated in the following table. Rates are shown as annualized charges, expressed in basis points, after all applicable taxes. For calculation purposes, a constant 10 basis points per annum is assumed available as a risk margin to offset the costs of the guaranteed benefits.

Investment Fund Class	Total MER
Money Market	110
Fixed Income	200
Balanced	250
Low Volatility Diversified Equity	265
Broad-Based Diversified Equity	265
Intermediate Risk Equity	280
Exotic/Aggressive Equity	295

Policyholder Behaviour

The “with resets” rows have assumed a 100% rate of utilization of the elective reset option (the proportion of policyholders expected to exercise the reset option given favourable investment performance). In developing the factor tables, “favourable” investment performance (prompting option exercise) is deemed to have occurred whenever the market value-to-guaranteed value (MV/GV) ratio equals or exceeds 115%. A maximum of two resets per policy year is permitted.

If the product offers an elective reset feature and the company has reliable and pertinent experience regarding the rate of utilization, the company should interpolate between the corresponding “no resets” and “with resets” rows according to the proportion of business that exercises the reset option. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

For products which offer a reset option (elective or automatic), but the contract is outside the period in which resets are permitted (e.g. beyond some duration or attained age limit), the exposure should be valued as if the reset option were not present. The testing did not permit resets of the guarantee benefits in the ten years prior to final contract maturity.

The “CSV guarantee” product pays a “top-up” maturity benefit only upon surrender of the fund units to which the guarantee applies. The costs associated with this benefit type are very sensitive to the underlying surrender (lapse) assumption and the assumed policyholder behaviour in adverse investment performance scenarios. The “in-the-money surrender” rows have assumed a 100% rate of utilization of the surrender option whenever the guarantee is “in-the-money” by more than 15% (i.e., MV/GV ratio is less than 0.85).

If the product offers a guaranteed surrender benefit, the company should interpolate between the corresponding base assumption and “in-the-money surrender” rows according to the proportion of business that is expected to exercise the surrender option through voluntary termination in those adverse scenarios in which a guaranteed benefit is payable. The proportion used should include an appropriate margin for adverse deviation. The same interpolation factor must be used for all the factor tables. Refer to *Interpolation and Extrapolation in the Factor Tables* found in section 9-9.

Investment Return Models

Regime-Switching Lognormal Process

Investment returns for the constituent asset classes are assumed to follow a monthly regime-switching lognormal process with two regimes (“RSLN2”). The RSLN2 model assumes that the asset return process switches randomly at each monthly time step between two lognormal processes. That is, the log returns are normally distributed with each regime. The process describing which regime is operative at any time is assumed to be Markov; that is, the probability of switching regimes depends only on the current regime and not on the history of the process. The RSLN model maintains some of the attractive simplicity and tractability of the independent lognormal (“ILN”) model, but more accurately captures the more extreme behaviour observed in historical data. It is one of the easiest ways to introduce a form of stochastic volatility into the model.

Regime switching models for investment returns have been well-documented in the academic literature. For a particularly salient treatment of regime-switching lognormal models in the context of valuing embedded options on long-term variable annuity contracts, please refer to “A Regime-Switching Lognormal Model of Long-Term Stock Returns” by Mary R. Hardy (*North American Actuarial Journal*, Volume 5, Number 2, April 2001).

Calibration Data Series

The monthly TSE300 Total Return data series (1956.01 to 2000.12 inclusive) is shown [in Appendix B to the CIA Task Force report (December 2001)]. This dataset is recommended by the CIA Task Force on Segregated Fund Investment Guarantees for calibration of stochastic equity return models (August 2000). The historic period for investment model calibration is 1956.01 – 1999.12 inclusive (528 index values, 527 log return observations).

Calibration

For the monthly TSE300 Total Return test data series (1956.01 to 1999.12 inclusive), we obtain the following monthly parameters for the independent lognormal model and regime-switching lognormal models (with 2 regimes). Note that μ_k and σ_k in each regime are for the associated normal distributions for the log returns. π_1 and π_2 are respectively the long-run unconditional (invariant) probabilities of being in regime 1 or 2 at any given epoch without knowledge of the prior state of the process. For the “uncalibrated ILN” and “RSLN2” models, the parameter estimates were obtained using maximum likelihood techniques.

The annualized mean return is the expected long-term total return on the asset expressed as an annual effective rate. The annualized volatility is the standard deviation of the log total returns. The skewness of the historical log returns is -0.912 (a skewness co-efficient of zero indicates perfect symmetry about the mean of the distribution).

Monthly ILN Model Parameters for TSE300 TR Performance (1956.01-1999.12):

	μ	σ	Annualized Mean	Annualized Volatility	Skewness (γ)
Uncalibrated ILN	0.00814	0.04507	0.1161	0.1561	0
Calibrated ILN *	0.00769	0.05401	0.1161	0.1871	0

* According to the criteria outlined in the Aug 2000 Report of the CIA Task Force on Segregated Fund Investment Guarantees.

Monthly RSLN2 Model Parameters for TSE300 TR Performance (1956.01-1999.12):

	μ_1	σ_1	$\rho_{1 \rightarrow 2}$	μ_2	σ_2	$\rho_{2 \rightarrow 1}$	π_1	π_2	γ
RSLN2 (Cons. Firm)	0.0124	0.0347	0.0375	-0.0157	0.0777	0.2108	0.8491	0.1509	-0.591
RSLN2 (M.R. Hardy) *	0.0123	0.0347	0.0371	-0.0157	0.0778	0.2101	0.8499	0.1501	-0.589

* The parameter estimates provided by Mary R. Hardy (NAAJ, April 2001) are not statistically different from those obtained by the Consulting Firm and could be due to small discrepancies in the historical total return data series.

The expected annualized mean total return (annual effective) and volatility for the RSLN2 model are 0.1159 and 0.1563 respectively.

Sample statistics for the three models are provided in the following table assuming a 30-year projection horizon. The CIA Task Force Report (August 2000) suggests that the *Mean of the 1-year Accumulation Factors* should lie in the range 1.10 to 1.12 and that the *Standard Deviation of the 1-year Accumulation Factor* should equal or exceed 0.175 (annualized).

Sample TSE300TR Investment Return Statistics (10,000 scenarios):

	Annualized Mean	Annualized Volatility	Mean 1-year Accumulation Factor	StDev of 1-year Accumulation Factor
ILN	11.63%	15.65%	1.1163	0.1759
Calibrated ILN	11.63%	18.76%	1.1163	0.2114
RSLN2	11.77%	15.65%	1.1177	0.1826

The sample left-tail TSE300TR percentiles (at the 2.5%, 5% and 10% confidence levels) for the stochastic accumulation factors over various holding periods are shown below. The CIA Task Force calibration statistics are labelled as “TARGET”. As expected, both the “Calibrated ILN” and “RSLN2” models satisfy the calibration criteria set forth in the August 1, 2000 Report of the CIA Task Force (within statistical fluctuation).

Sample Left-Tail TSE300TR Percentiles for Accumulation Factors (10,000 scenarios):

	1-year			5-years			10-years		
	2.5%	5%	10%	2.5%	5%	10%	2.5%	5%	10%
TARGET	0.76	0.82	0.90	0.75	0.85	1.05	0.85	1.05	1.35
ILN	0.81	0.85	0.90	0.82	0.92	1.04	1.01	1.18	1.41
Calibrated ILN	0.76	0.81	0.86	0.70	0.80	0.93	0.79	0.95	1.18
RSLN2	0.74	0.81	0.89	0.69	0.81	0.98	0.80	1.00	1.28

Parameters

The model parameters for each asset class were estimated by constrained maximum likelihood techniques (using the “Solver” Add-In in Microsoft® Excel 97) applied to monthly historic total return data up to and including December 2000. The estimation process constrained the state transition probabilities so that all funds would always be in the same regime (i.e., would “switch regimes” at the same time). This strengthens the correlation between asset classes.

The accuracy of the “Solver” Add-In has been confirmed through sensitivity testing of the initial estimates. That is, the routines do indeed solve for the parameters that globally maximize the log likelihood function for most reasonable initial values given the constraints.

Judgement was applied in setting the final parameter values for each benchmark (numbered 1 through 7) to maintain consistency in the market price of risk¹ for each asset class. The following starting dates for the historic monthly data series were used in estimating model parameters.

Data Series Start Dates:

DATA SERIES	DESCRIPTION	OSFI FUND CLASS	START DATE
S&P500 TR	U.S. EQUITY	LOW VOLATILITY DIVERSIFIED EQUITY	1956.01
TSE300 TR	CANADIAN EQUITY	BROAD-BASED DIVERSIFIED EQUITY	1956.01
MSCI-EAFE TR	INTERNATIONAL EQUITY	LOW VOLATILITY DIVERSIFIED EQUITY	1969.12
SBBI U.S. Small Co TR	U.S. SMALL CAP EQUITY	INTERMEDIATE RISK EQUITY	1956.01
AGGRESSIVE EQUITY TR	HIGH RISK EQUITY	AGGRESSIVE EQUITY	1980.12 (1)
BOND TR	BONDS	FIXED INCOME	1960.12 (2)
MONEY MARKET TR	MONEY MARKET	MONEY MARKET	1960.12 (3)
BALANCED TR	CANADIAN BALANCED	BALANCED	60% TSE300, 40% BOND
NASDAQ	U.S. TECHNOLOGY	AGGRESSIVE EQUITY (1)	1971.02
HANG SENG TR	FOREIGN / EMERGING	AGGRESSIVE EQUITY (1)	1980.12

TR = Total return (i.e., after reinvestment of dividends and coupons)

- (1) Based on judgement applied to MLE parameters for a hypothetical time series constructed from Hang Seng TR and NASDAQ data series (1980.12 – 2000.12) assuming an approximate 20/80% split between markets.
- (2) Based on judgement applied to MLE parameters for Scotia Capital Markets Bond Universe and the U.S. Variable Annuity bond markets (Morningstar data).
- (3) Based on judgement applied to MLE parameters for 3-month Treasury Bills and the U.S. Variable Annuity money markets (Morningstar data).

¹ For the K^{th} asset class, the “market price of risk” is defined by $\frac{E[R_K] - r_f}{\sigma_K}$, where $E[R_K]$ is the expected total

annualized return, r_f is the risk-free rate (assumed to be 5.5% p.a.) and σ_K is the annualized standard deviation of the log returns (volatility).

The investment returns for the various classes are assumed to be partially correlated. The investment models use 10,000 stochastic scenarios generated by Monte Carlo simulation using a highly robust random number generator and $N(0,1)$ mapping routines.

The stochastic generator produces gross total returns for each benchmark (i.e., before deduction of management fees and other charges) to which management expense ratios (“MERS”) are applied to obtain returns on the proxy funds. No other source of basis risk is considered (i.e., the proxy funds perfectly track the benchmark before fees).

The model assumes that funds remain fully invested at all times (no cash position) with bond coupons and stock dividends immediately reinvested. Currency exchange rates have been ignored to the extent that all historic return data (used to estimate the model parameters) are expressed in local dollars (i.e., current exchange rates are assumed to continue indefinitely).

Prior testing has confirmed that using an RSLN2 process to model the market total returns on fixed income investments (e.g. bond funds) is superior to the ILN model and generally leads to more conservative results.

Monthly RSLN2 model parameters are provided in the following table. The means (μ) and standard deviations (σ) are for the associated normal distribution for the log total returns in each regime. The invariant distribution is provided by π_k (the unconditional probability of being in regime “ k ” without knowing the prior state). The starting regime (at time zero) is randomized according to the unconditional distribution π . All benchmarks start in the same regime.

Monthly RSLN2 Model Parameters for Total Return Performance - Model:

		OSFI FUND CLASS	μ_1	σ_1	$\rho_{1 \rightarrow 2}$	μ_2	σ_2	$\rho_{2 \rightarrow 1}$	π_1	π_2
S&P500	1	LOW VOLATILITY EQUITY	0.0121	0.0348	0.0410	-0.0092	0.0687	0.2323	0.85	0.15
TSE300	2	DIVERSIFIED EQUITY	0.0128	0.0348	0.0410	-0.0169	0.0766	0.2323	0.85	0.15
MSCI-EAFE	3	LOW VOLATILITY EQUITY	0.0126	0.0321	0.0410	-0.0119	0.0691	0.2323	0.85	0.15
SMALL CO	4	INTERMEDIATE RISK EQUITY	0.0138	0.0429	0.0410	-0.0156	0.1016	0.2323	0.85	0.15
AGGRESSIVE	5	AGGRESSIVE EQUITY	0.0159	0.0515	0.0410	-0.0225	0.1200	0.2323	0.85	0.15
BOND	6	FIXED INCOME	0.0055	0.0147	0.0410	0.0090	0.0275	0.2323	0.85	0.15
MONEY MRKT	7	MONEY MARKET	0.0042	0.0086	0.0410	0.0085	0.0217	0.2323	0.85	0.15
NASDAQ		AGGRESSIVE EQUITY	0.0146	0.0437	0.0410	-0.0204	0.1128	0.2323	0.85	0.15
HANG SENG		AGGRESSIVE EQUITY	0.0244	0.0669	0.0410	-0.0309	0.1479	0.2323	0.85	0.15

The “Balanced” benchmark has been simulated by averaging the stochastically generated accumulation factors from the TSE300 and Bond funds (assuming a 60/40 split). The fund is assumed to be rebalanced quarterly to maintain the desired mix.

The following table shows the (unconstrained) monthly maximum likelihood estimates for several markets (in local currency) under the RSLN2 and ILN models.

Monthly Model Parameters for Total Return Performance – Maximum Likelihood:

	Dates		ILN		RSLN2							
	From	To	μ	σ	μ_1	σ_1	$\rho_{1 \rightarrow 2}$	μ_2	σ_2	$\rho_{2 \rightarrow 1}$	π_1	π_2
S&P500	1956.01	2000.12	0.0092	0.0417	0.0120	0.0357	0.0362	-0.0208	0.0758	0.3915	0.9153	0.0847
TSE300	1956.01	2000.12	0.0081	0.0454	0.0128	0.0347	0.0409	-0.0156	0.0758	0.2039	0.8330	0.1670
MSCI-EAFE	1969.12	2000.12	0.0085	0.0412	0.0141	0.0277	0.0924	-0.0097	0.0656	0.3017	0.7656	0.2344
U.S. SMALL CO.	1956.01	2000.12	0.0110	0.0599	0.0167	0.0410	0.0446	-0.0057	0.0935	0.1265	0.7392	0.2608
BOND	1960.12	2000.12	0.0056	0.0161	0.0044	0.0102	0.0097	0.0086	0.0253	0.0280	0.7433	0.2567
MONEY MRKT	1960.12	2000.12	0.0047	0.0023	0.0035	0.0015	0.0109	0.0076	0.0020	0.0326	0.7498	0.2502
NASDAQ	1971.02	2000.12	0.0089	0.0618	0.0145	0.0437	0.0369	-0.0184	0.1112	0.1755	0.8263	0.1737
HANG SENG	1980.12	2000.12	0.0128	0.0929	0.0275	0.0574	0.0540	-0.0079	0.1240	0.0750	0.5816	0.4184

The switching of regimes is assumed to be highly correlated for the equity markets. The monthly log returns for the modelled asset classes are imperfectly (but positively) correlated as indicated by the correlation coefficients in the following tables. The historic correlations are derived from the monthly log total return data from 1971.02 to 2000.12 inclusive.

At the beginning of each time step, a single $U^*=U(0,1)$ random variate is sampled to provide the “switching criterion”. The transition probability for each fund (appropriate to the current regime) is compared to U^* to determine if the process jumps to the other regime. Once all the current regimes have been established, a set of correlated $N(0,1)$ random variates is generated to produce the returns in the given month.

The overall correlation of the underlying asset returns comes from two sources: the switching of regimes (the “state transition” process) and the simulated normal random variates (the “investment return” process). Using the same transition probabilities for all equity classes forces the associated processes to “switch regimes” at the same time, thus maximizing the correlation from the state transition process.

The ordering of indices is important as the first asset in the list is designated the “lead benchmark” for purposes of establishing return correlations (here, the S&P500 is the “lead benchmark”). In each time period, the model looks to the state (regime) of the lead benchmark to determine which correlation matrix is applicable. The “Regime 1” correlation matrix applies if the lead benchmark is in regime 1, otherwise the “Regime 2” coefficients are used. In this way, the model more accurately captures the strong tendency of markets to “move together” in times of stress or adversity.

Correlation Coefficients (Lead Benchmark in Regime 1):

	S&P500	TSE300	MSCI-EAFE	SMALL CO.	AGGRESSIVE	BOND	MONEY
S&P500	1	0.56	0.35	0.58	0.73	0.56	0.09
TSE300		1	0.36	0.54	0.51	0.55	0.10
MSCI-EAFE			1	0.27	0.31	0.29	0.06
SMALL CO.				1	0.81	0.37	0.10
AGGRESSIVE					1	0.36	0.12
BOND						1	0.35
MONEY							1

Correlation Coefficients (Lead Benchmark in Regime 2):

	S&P500	TSE300	MSCI-EAFE	SMALL CO.	AGGRESSIVE	BOND	MONEY
S&P500	1	0.95	0.84	0.91	0.94	0.41	-0.08
TSE300		1	0.87	0.89	0.95	0.58	-0.03
MSCI-EAFE			1	0.85	0.82	0.44	-0.09
SMALL CO.				1	0.93	0.54	-0.06
AGGRESSIVE					1	0.66	-0.07
BOND						1	0.20
MONEY							1

Sample (overall) correlations from the investment return scenarios are shown in the table below. As can be seen, these closely approximate the historic market correlations.

Sample Correlations (Monthly Log Returns, 10,000 Scenarios):

	S&P500	TSE300	MSCI-EAFE	SMALL CO.	AGGRESSIVE	BOND	MONEY
S&P500	1	0.75	0.61	0.74	0.84	0.52	-0.02
TSE300		1	0.62	0.74	0.74	0.42	-0.06
MSCI-EAFE			1	0.58	0.59	0.29	-0.07
SMALL CO.				1	0.88	0.39	-0.03
AGGRESSIVE					1	0.43	-0.04
BOND						1	0.29
MONEY							1

Historic Correlations (Monthly Log Return Data 1971.02 – 2000.12):

	S&P500	TSE300	MSCI-EAFE	SMALL CO.	AGGRESSIVE	BOND	MONEY
S&P500	1	0.75	0.61	0.74	0.84	0.50	-0.08
TSE300		1	0.60	0.74	0.75	0.39	-0.08
MSCI-EAFE			1	0.53	0.57	0.28	-0.04
SMALL CO.				1	0.88	0.39	-0.04
AGGRESSIVE					1	0.43	-0.10
BOND						1	0.24
MONEY							1

Sample investment return statistics for the benchmarks and various equity markets are provided in the following table (simulated local currency returns). “Mean1” and “StDev1” are respectively the mean and standard deviation of the 1-year accumulation factor. The “Annual Mean” is the expected long-term total return on the asset expressed as an annual effective rate. The “Annual Vol.” is the annualized standard deviation of the sample monthly log returns.

Sample statistics for the RSLN2 models were calculated over a 30-year projection horizon. The correlated RSLN2 “constrained” model simultaneously projects the seven primary indices, while the independent model projects each index one at a time using the MLE parameters for that market/class. Within statistical fluctuation, all sample statistics agree with the calculated moments of the underlying distribution (given the parameters).

Sample Investment Return Statistics:

	Historic Data Assuming an ILN Model using MLEs				Correlated RSLN2 “Constrained”				Independent RSLN2 “MLE”	
	Annual Mean	Annual Vol.	Mean1	StDev1	Annual Mean	Annual Vol.	Mean1	StDev1	Annual Mean	Annual Vol.
S&P500	12.84%	14.45%	1.128	0.164	12.63%	15.48%	1.127	0.169	12.87%	14.44%
TSE300	11.58%	15.73%	1.116	0.177	12.15%	17.17%	1.121	0.182	11.53%	15.78%
MSCI-EAFE	11.87%	14.27%	1.119	0.160	12.67%	15.42%	1.126	0.165	11.90%	14.27%
SMALL CO	16.59%	20.75%	1.167	0.245	14.36%	20.75%	1.143	0.226	16.42%	20.84%
AGGRESSIVE					16.55%	25.22%	1.165	0.275		
BOND	7.12%	5.58%	1.072	0.060	7.70%	6.05%	1.077	0.065	6.95%	5.42%
MONEY MRKT	5.81%	0.80%	1.059	0.008	6.06%	4.15%	1.061	0.045	5.58%	0.84%
BALANCED					10.30%	11.20%	1.103	0.121		
NASDAQ	13.85%	21.41%	1.138	0.247					13.74%	21.58%
HANG SENG TR	22.80%	32.18%	1.228	0.406					22.65%	32.23%

Sample left-tail percentiles (at the 2.5%, 5% and 10% confidence levels) for the stochastic RSLN2 accumulation factors (local currency) over various holding periods are as follows:

RSLN2 Sample Percentiles for Accumulation Factors:

	1-year			5-years			10-years		
	2.5%	5%	10%	2.5%	5%	10%	2.5%	5%	10%
S&P500	0.79	0.85	0.92	0.83	0.94	1.08	1.07	1.27	1.55
TSE300	0.74	0.81	0.89	0.71	0.83	0.99	0.86	1.07	1.34
MSCI-EAFE	0.78	0.85	0.92	0.81	0.93	1.09	1.04	1.26	1.55
SMALL CO	0.70	0.78	0.87	0.64	0.77	0.96	0.78	1.00	1.31
AGGRESSIVE	0.63	0.73	0.83	0.55	0.68	0.87	0.64	0.87	1.21
BOND	0.96	0.98	1.00	1.11	1.15	1.21	1.43	1.51	1.62
MONEY MRKT	0.98	1.00	1.01	1.12	1.15	1.19	1.40	1.45	1.52
BALANCED	0.85	0.90	0.95	0.93	1.02	1.14	1.20	1.36	1.58

Model Fit & Selection

It has been well documented (see “A Regime-Switching Lognormal Model of Long-Term Stock Returns” by Mary R. Hardy, *North American Actuarial Journal*, Volume 5, Number 2, April 2001, 2001 for a discussion of TSE300 and S&P500 returns) that RSLN2 models generally provide a statistically better fit to the historic monthly time series than the simpler independent lognormal process. This can be demonstrated in several ways.

Adding more parameters to a model will always produce a higher value for the likelihood function (or, equivalently, the log-likelihood function) for a given time series (using a fixed number of data points). However, this does not necessarily imply a better “fit”. Any increase in the value of the likelihood function must be statistically significant for the model not to be rejected in favour of simpler embedded versions (in this case, the ILN process is embedded in the RSLN2 model). The Schwartz-Bayes Criterion (SBC) is a measure which takes into account the number of parameters and the sample size in order to “rank” models. Under the Schwartz-Bayes Criterion, the model which maximizes

$\left[l_j - \frac{1}{2} k_j \log n \right]$ is selected as “superior” in terms of fit (for the j^{th} model, l_j is the value of the log-likelihood function, k_j represents the number of parameters and n is the sample size).

The table below shows values for the log-likelihood function and SBC for various markets. In all cases, the RSLN2 model (with unconstrained MLE parameters) provides a significantly better “fit” to the data than the simpler ILN model. The likelihood ratio test provides another selection criteria to compare embedded models. In this case, we would favour the RSLN2 model (over the ILN model) for all funds on a local currency basis.

Selection Criteria using Maximum Likelihood Parameters (Local Currency):

	Number of Data Points	Independent Lognormal		Unconstrained RSLN2		Constrained RSLN2	
		Log Likelihood	SBC	Log Likelihood	SBC	Log Likelihood	SBC
S&P500	539	948.02	941.73	969.79	950.92	969.15	950.28
TSE300	539	901.38	895.09	938.11	919.24	938.02	919.15
MSCI-EAFE	372	658.69	652.77	690.17	672.41	688.59	670.84
U.S. SMALL CO.	539	752.59	746.30	800.99	782.12	799.34	780.47
BOND	480	1300.50	1294.32	1376.39	1357.87	1362.83	1344.31
MONEY MRKT	480	2232.54	2226.37	2461.38	2442.86	1806.53	1788.01
NASDAQ	358	488.61	482.73	526.57	508.92	526.33	508.69
HANG SENG	240	229.69	224.21	249.31	232.87	247.39	230.95

Factor Development

The development of the “factor tables” in respect of the investment guarantees on segregated funds followed from the stochastic modelling of sample (representative) contracts. Each test “cell” (combination of investment option and product/guarantee form) was composed of a single sample policy. Furthermore, the analysis of the stochastic results for each investment option (“fund”) was conducted in isolation. Hence, the tabular factors were developed independently for each fund category.

The above “simplified” approach neglects two issues which could have a material impact on the true risk profile of an actual portfolio of business. Namely, the simplified approach ignores the potential risk reduction effects (at the company level) of maturity profile (time diversification) and investment mix (fund diversification). These have been accommodated through additional adjustments, both implicit (fund diversification) and explicit (time diversification).

The remainder of this section documents the most significant issues in the factor development process.

Table 1a – Basic Factor Table (A1)

Table 1a in the MCCSR Guideline presents basic cost factors at the CTE⁺(95%) confidence level (expressed as a % of market value) for each combination of benefit form and investment fund class assuming MV/GV =1. The costs have been expressed in present value terms reflecting survivorship and interest. The Table 1a basic factors reflect a macro-level adjustment to account for fund diversification at the company level (see the following section for further details).

Product forms which offer a level guaranteed minimum benefit (i.e., return-of-premium on death, maturity or surrender) are shown without a “guaranteed percentage” since Table 1a provides factors when MV=GV. In this case, the actual level of the guarantee (e.g., 75% or 100% of net deposits) is irrelevant in determining future costs when expressed as a proportion of current market value.

Company Level Fund Diversification

A company's mix of investment options ("fund profile") can significantly reduce the potential costs of the guaranteed benefits to the extent that the various underlying asset classes are imperfectly correlated. This reduction is irrespective of the form of investment maturity guarantee offered to the retail client (i.e., "fund-by-fund" or "family-of-funds").

The TGCR methodology requires that a company slot its guaranteed benefit exposure by cell (benefit form & investment class combination) and apply the factor tables as appropriate depending on the characteristics of the contracts being valued (most notably, MV/GV ratio). In practical terms, this calculation is performed independently for each inforce contract, resulting in a "policy" TGCR at the CTE⁺(95%) confidence level. The sum of these independent values is the company's aggregate TGCR.

It should be clear that this process overstates the company's true exposure since it effectively assumes perfect correlation of the constituent asset classes that comprise the portfolio. A theoretically correct stochastic calculation would project guaranteed benefit costs for every contract consistent with the investment return scenarios and discount the aggregated "cost" stream to determine a balance sheet provision at the valuation date.

Under the "correct" approach, the stochastic scenarios would be internally consistent, reflecting the imperfect correlations between different markets. Though debt and equity returns are significantly positively correlated, even the most highly intertwined markets (e.g. Canada and the United States) exhibit correlations significantly less than 90% in all but the most adverse conditions. The bond and equity markets are even less highly correlated (around 40%), so it stands to reason that losses in one market or sector can be partially offset by gains in another.

To understand the impact of fund diversification, we compared the theoretical CTE⁺(95%) direct cost factor ("Total Company" value) versus that derived by application of the TGCR methodology ("Weighted-Average" value). Note that the "TGCR" result is simply the arithmetic average of the Table 1a Factors, weighted by the market value in each asset category.

The table entitled "Impact of Fund Diversification" expresses the "Total Company" result as a proportion of the "Weighted-Average" value from the TGCR methodology for hypothetical industry-average company-level fund profile (retail segregated funds in Canada as at June 30, 2001. Source: *Investor Economics*) shown in the table below.

Assume Industry Average Fund Profile:

Money Market	Fixed Income	Balanced	Diversified Equity	Intermediate Risk Equity	Aggressive Equity
4%	7%	28%	50%	4%	7%

The technically "correct" way of incorporating the impact of fund diversification into the factor-based TGCR methodology would be to allow individual companies to calculate a reduction factor based on its actual fund profile. However, this could be problematic in application, difficult to audit and subject to potential manipulation. Instead, a more practical approach was adopted that reduced the "unadjusted" Table 1a Basic Factors by the indicated amounts. The obvious drawback of this approach is that it gives the same reduction to every company regardless of its actual investment profile. This may not be a serious issue as most segregated fund/variable annuity writers offer an array of investment options.

Impact of Company-Level Fund Diversification (Reduction Factors) :

Minimum Death Benefits	
No resets or ratchets; level guarantee to contract maturity (age 70)	0.958
No resets or ratchets; level guarantee to contract maturity (age 90)	0.959
Annual ratchet; to contract maturity (age 90)	0.926
5% annual roll-up; to contract maturity (age 90)	0.948
No resets or ratchets; 75% 10-year	0.921
No resets or ratchets; 100% 10-year	0.935
With resets/ratchets; 75% 10-year	0.901
With resets/ratchets; 100% 10-year	0.919
Minimum Maturity & Surrender Benefits	
No resets or ratchets; level guarantee to contract maturity (age 70)	0.974
No resets or ratchets; 75% 10-year	0.953
No resets or ratchets; 100% 10-year	0.946
No resets or ratchets; 10-year CSV	0.953
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	0.949
With resets/ratchets; 75% to contract maturity	0.924
With resets/ratchets; 100% to contract maturity	0.932
With resets/ratchets; 75% 10-year	0.939
With resets/ratchets; 100% 10-year	0.932
With resets/ratchets; 75% 10-year CSV guarantee	0.934
With resets/ratchets; 100% 10-year CSV guarantee	0.924
With resets/ratchets; 75% 10-yr CSV ("in-the-money" surrender)	0.931
With resets/ratchets; 100% 10-yr CSV ("in-the-money" surrender)	0.928

Low Volatility Diversified Equity

A new investment class has been introduced to accommodate lower volatility diversified equity (“LVDE”) funds. Only funds that would otherwise be classified as Broad-Based Diversified Equity are candidates for inclusion provided certain conditions are met. The expected volatility of the fund must be less than 15.5% (annualized) and the aggressive/exotic equity component of the equity holdings must be less than 20% of the total equities by market value. Other specific criteria also apply (e.g., the fund permanently maintains a relatively large cash or fixed income position and/or the fund is income oriented and contains a significant proportion of stocks paying material and regular dividends that are automatically reinvested).

All funds in this asset class use the “Broad-Based Diversified Equity” factors in all subsequent TGCR calculations involving Tables 1b, 2, 3 and 4.

Basic cost factors (present value of guarantee benefits, ignoring available margin offset) were developed for an LVDE fund based on the results from the U.S. Equity (S&P500TR) and International Equity (MSCI-EAFE) investment return scenarios. First, the LVDE results were expressed as a proportion of the “Diversified Equity” Table 1a basic factors at the CTE⁺(95%) confidence level for a range of MV/GV ratios (call these the “LVDE ratios”). The final “LVDE adjustment factors” vary by benefit type and were conservatively derived at a 75% MV/GV ratio using the higher of the U.S. Equity and International Equity LVDE ratios, rounded up to the next multiple of 0.025 (GMDB) or 0.05 (GMMB).

The LVDE ratios vary materially by MV/GV ratio, with less impact (i.e., higher factors) when the guarantees are deeper “in-the-money”. The differences in the LVDE ratios for the U.S. Equity and International Equity fund classes were not significant.

The Table 1a values for “Low Volatility Diversified Equity” fund class were determined by multiplying the “Broad-Based Diversified Equity” factors by the LVDE Adjustment Factors shown in the table.

Low Volatility Diversified Equity Ratios at CTE⁺ (95%)

	LVDE Adjustment Factor
Minimum Death Benefits	
No resets or ratchets; level guarantee to contract maturity (age 70)	0.825
No resets or ratchets; level guarantee to contract maturity (age 90)	0.850
Annual ratchet; to contract maturity (age 90)	0.825
5% annual roll-up; to contract maturity (age 90)	0.900
No resets or ratchets; 75% 10-year	0.875
No resets or ratchets; 100% 10-year	0.850
With resets/ratchets; 75% 10-year	0.825
With resets/ratchets; 100% 10-year	0.825
Minimum Maturity & Surrender Benefits	
No resets or ratchets; level guarantee to contract maturity (age 70)	0.500
No resets or ratchets; 75% 10-year	0.900
No resets or ratchets; 100% 10-year	0.900
No resets or ratchets; 10-year CSV	0.750
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	0.900
With resets/ratchets; 75% to contract maturity	0.550
With resets/ratchets; 100% to contract maturity	0.700
With resets/ratchets; 75% 10-year	0.900
With resets/ratchets; 100% 10-year	0.900
With resets/ratchets; 75% 10-year CSV guarantee	0.750
With resets/ratchets; 100% 10-year CSV guarantee	0.750
With resets/ratchets; 75% 10-yr CSV ("in-the-money" surrender)	0.900
With resets/ratchets; 100% 10-yr CSV ("in-the-money" surrender)	0.900

Table 1b – Time Diversification Adjustment (A2)

The factor development process tested one contract (with a single maturity date) for each cell. In the event that this “time-to-maturity” well represents an “average” time-to-maturity for an existing book of business, it could still fail to capture the time diversification (cost reduction) resulting from a disperse maturity profile. It stands to reason that for a given volume of exposure, a more disperse maturity profile (maturities spread through time) is less risky (in terms of potential payout on the guaranteed maturity benefits) than a more concentrated one. This section describes the analysis undertaken to quantify the degree of risk reduction associated with time diversification across the maturity date spectrum.

Longer-term time diversification results from the ability of a company to “ride out” market cycles across the maturity date spectrum. In recent experience, such adverse economic cycles can persist for several years.

Additional testing considered the difference in “cost” (present value of guaranteed maturity benefits at a $CTE^+(95\%)$ confidence level) for the base single (“bullet”) maturity cells versus a “basket” of uniformly distributed maturities two (2) years on either side of the bullet maturity date. That is, the dispersed model had the same average time-to-maturity as the corresponding base cells. All maturity/surrender guarantee and fund class combinations were reviewed.

As anticipated, the reduction in direct cost (i.e., ignoring the margin offset) for the “dispersed profile” varies by guarantee form and fund category. There is no reduction in cost for Money Market and Bond funds, but more substantial diversification for the other asset classes. Specifically, the table on the following page provides the inter-year time diversification by expressing the $CTE^+(95\%)$ present values of the guaranteed benefit costs (“PVGB”) for the dispersed maturity model as a percentage of the base “bullet maturity” results. The factors have been rounded to the nearest 0.005 and capped at 1.

Table 1b – Time Diversification Adjustment Factor:

	Money Market	Fixed Income	Balanced	Broad-Based Diversified Equity *	Intermediate Risk Equity	Exotic or Aggressive Equity
Minimum Maturity & Surrender Benefits						
No resets or ratchets; Level guarantee to contract maturity (age 70)	1.000	1.000	1.000	1.000	1.000	0.985
No resets or ratchets; 75% 10-year	1.000	1.000	0.960	0.935	0.935	0.940
No resets or ratchets; 100% 10-year	1.000	1.000	0.935	0.940	0.935	0.925
No resets or ratchets; 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
No resets or ratchets; 10-year CSV ("in-the-money" surrender)	1.000	1.000	1.000	0.965	0.955	0.960
With resets/ratchets; 75% to contract maturity	1.000	1.000	0.965	0.920	0.905	0.885
With resets/ratchets; 100% to contract maturity	1.000	1.000	0.940	0.905	0.890	0.895
With resets/ratchets; 75% 10-year	1.000	1.000	0.965	0.965	0.970	0.985
With resets/ratchets; 100% 10-year	1.000	1.000	0.985	0.990	1.000	1.000
With resets/ratchets; 75% 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
With resets/ratchets; 100% 10-year CSV	1.000	1.000	1.000	1.000	1.000	1.000
With resets/ratchets; 75% 10-year CSV ("in-the-money" surrender)	1.000	1.000	1.000	0.975	0.975	0.990
With resets/ratchets; 100% 10-year CSV ("in-the-money" surrender)	1.000	1.000	0.990	0.990	1.000	1.000
Minimum Income Benefits						
Guaranteed Minimum Income Benefit	Company must calibrate and model separately					

* The factors for "Broad-Based Diversified Equity" also apply to funds classified as "Low Volatility Diversified Equity".

Table 2 – MV/GV & Time to Maturity Adjustments (B)

Table 2 in the MCCSR Guideline presents adjustment factors to account for the relationship between market value (MV) and guaranteed value (GV) at the valuation date. The factors vary by fund class, benefit type and remaining time-to-next maturity. The relationship between the tested MV/GV ratios is continuous and stable, so linear interpolation is appropriate. For practical reasons, linear extrapolation beyond the table limits (i.e., below MV/GV=0.25 or above MV/GV=2.00) is permitted.

The Table 2 factors are expressed as the ratio of the PVGB for the indicated MV/GV relationship to the PVGB when MV/GV=1 (Table 1a before adjustments for time diversification) at the CTE⁺(95%) confidence level. We would expect this ratio to be 1.00 for MV=GV, but there are two circumstances when this is not the case. First, the unadjusted Table 1a factors have been floored at 0.01% (this only affects certain benefit types for the Money Market and Bond fund classes). Second, the Table 2 factors account for time-to-next maturity (≤ 3 years and > 3 years). Whenever the time-to-next maturity in Table 2 differs from Table 1a, the factor can substantially deviate from 1.00 due to obvious changes in the risk profile.

Table 3 – MER Adjustments (C)

Table 3 shows the formula parameters (“a” and “b”) for the Management Expense Ratio (“MER”) adjustment factor “C”. This factor is meant to account for differences in a company’s actual MER from those assumed in the development of the MCCSR tables. The formula is:

$$C = g\left(\frac{MV}{GV}\right) \times \Delta$$

where:

$$g\left(\frac{MV}{GV}\right) = \left(a - b \times \frac{MV}{GV}\right)^3$$

$$\Delta = [\text{Actual MER} - \text{Assumed MER}]$$

In the above, MV/GV is the actual MV/GV ratio for the exposure being valued, subject to a maximum of 2.00. The parameters “a” and “b” vary by product type and fund class. In applying the formula, Δ must be expressed as an annualized decimal equivalent (i.e., the MERs should be the total charges levied against the funds on a “per annum” basis, expressed in decimal form).

The formula (methodology and parameters) was developed from extensive testing at the CTE⁺(95%) confidence level for a wide array of values for Δ and MV/GV ratio. The MER differences ranged from Δ = –1% up to +1% (per annum) in increments of 25 basis points.

The primary measure in the review was the ratio of CTE⁺95%(MER+Δ) to CTE⁺95%(MER), where MER denotes the “base” MERs underlying the factor development. The analysis showed the ratio to be very nearly linear in Δ for a given value of MV/GV, but significantly non-linear by MV/GV for fixed Δ. Note that Δ may be positive or negative.

The function $g(\bullet)$ accounts for MV/GV ratio. Various candidates were proposed and the parameters fit by least-squares regression. To keep the formula simple, only two-parameter families were considered. In the following, X represents the MV/GV ratio:

$$\begin{aligned} \text{Linear} : \quad g(X) &= a + bX \\ \text{Cubic} : \quad g(X) &= (a - bX)^3 \\ \text{Exponential} : \quad g(X) &= a + e^{-bX} \\ \text{Linear ArcTangent} : \quad g(X) &= a + b \cdot \text{ArcTan}(X) \\ \text{Exponential ArcTangent} : \quad g(X) &= a + [\text{ArcTan}(X)]^b \end{aligned}$$

No single candidate formula was superior for all benefit types and fund class combinations, but the *Cubic* family gave the best overall fit. The formula can give absurd results if $MV/GV > 2.00$ or $MV/GV < 0.25$ since such values were not considered in the fitting process. As such, MV/GV must be capped at 2.00 and floored at 0.25 in application.

A careful review of Table 3 will show that $g(X)$ may be negative for certain benefit forms and MV/GV ratios ($0.25 \leq X \leq 2.00$). In particular, this can occur for several of the GMDB types for the riskier fund classes since the parameter “ a ” is negative. Upon first inspection, this may seem to produce counter-intuitive values whereby a negative adjustment to the risk component would result when a company’s actual MER exceeded those assumed in the base factor development (i.e., when $\Delta > 0$). That is, one might assume that by retarding fund growth, a higher MER should lead to larger projected benefit costs. While this is a generally valid assertion, it is not uniformly true when the guaranteed amount resets or ratchets based on fund performance.

To understand this complex relationship, it is important to remember that the standardized product testing considered a life aged 50 at the measurement date and that projected GMDB costs are much more significant for older lives given the assumed mortality table. Hence, by impeding fund growth, higher MERs can limit the guaranteed values for ratchet/reset products and thereby reduce projected costs at the advanced ages (as measured in present value terms).

Notwithstanding the foregoing, it must be stated that the relationships between projected costs and MV/GV ratio, as suggested by the parameters in Table 3 and the functional form for $g(\bullet)$, may not hold for values of Δ well outside the tested range (i.e., MER differences that are substantially more than 100 basis points per annum).

Table 4 – Margin Offset Factors

Table 4 in the MCCSR Guideline presents offset factors (expressed as a % of market value) per 1% of available margin for each combination of benefit form and investment fund class assuming MV/GV=1. The offsets have been calculated at the CTE⁺(95%) confidence level and are expressed in present value terms taking account of survivorship and interest.

As with Table 1a, product forms which offer a level guaranteed minimum benefit (i.e., return-of-premium on death, maturity or surrender) are shown without a “guaranteed percentage” since Table 1a provides factors when MV=GV. In this case, the actual level of the guarantee (e.g., 75% or 100% of net deposits) is irrelevant in determining future margins when expressed as a proportion of current market value.

The factor development process tested “product forms” as stand-alone benefit types. That is, any single test policy only had a single guaranteed benefit (either death, maturity or surrender). This was a sensible approach since the factor-based TGCR methodology must accommodate the wide variety of death/maturity/surrender benefit combinations available in the market. However, any factors developed for a single benefit type (i.e., either GMDB or GMMB) in isolation will not necessarily be “correct” when the guaranteed benefit costs for an entire product are measured (i.e., both GMDB and GMMB). This will be discussed further in the next section.

The process of developing Table 4 factors severally by benefit type becomes problematic since a straightforward calculation can lead to offsets that differ significantly for “corresponding” GMDB and GMMB forms (e.g., 100% return-of-premium over a 10-year term), especially for the riskier fund classes.

Consider a company using internal models to measure the risk. The total balance sheet provision would be established at a CTE⁺(95%) confidence level based on the net present value of total guaranteed benefit costs for the portfolio. Any available margin (to offset costs) would not be allocated between the death and maturity guarantees. However, in applying the MCCSR factor-based methodology, inappropriate results can be obtained if the Table 4 offset factors differ for corresponding guaranteed benefit forms. A company could manipulate its MCCSR by optimizing the available margin allocation between the death and maturity benefits.

To circumvent this problem, a single offset factor for “corresponding” GMDB and GMMB products was calculated as the weighted-average of the individually derived offsets, with the weights determined by the present value of costs (i.e., the Table 1a factors before adjustment for fund diversification) for each benefit form. As an example, consider the 10-year 100% GMDB and GMMB forms without resets/ratchets for the Broad-Based Diversified Equity fund class:

Broad-Based Diversified Equity (MV/GV=1)	GMDB Basic	GMMB Basic	Total PVGB
Present Value of Guaranteed Benefits (unadjusted):	0.0187	0.1271	0.1458
Percentage of Total PVGB:	12.81%	87.19%	
Margin Offset (Marginal):	-0.1248	-0.0846	
Table 4 Offset = weighted-average of marginal offsets:			-0.0897

Outstanding Issues

While the testing underlying the TGCR methodology was extensive, there are three issues which remain largely unexplored and could be a source of considerable bias in the real-world results obtained through application. These are discussed below.

Aggregation

It is anticipated that most companies will apply the factor-based methodology on a policy-by-policy basis and then summing the results. In effect, the TGCR methodology provides a $CTE^+(95\%)$ for each exposure. Since the sum of CTE^+ s is always greater than or equal to the CTE^+ calculated in aggregate, the total reported component could significantly exceed that obtained by internal models (all else being equal).

Measurement

The factor development process tested “product forms” as stand-alone benefit types. This was appropriate since the methodology must accommodate the wide variety of benefit combinations available in the market. However, any factors developed for a single benefit type in isolation will not necessarily be “correct” when the guaranteed benefit costs for an entire product are measured. This cause of this conservatism has been noted above; namely, the sum of CTE^+ s is always greater than or equal to the CTE^+ calculated at a more aggregate level.

When a company offers both GMDB and GMMB on its products, the sum of the $CTE^+(95\%)$ death and maturity factors will exceed what would otherwise be obtained for the $CTE^+(95\%)$ on the combined guaranteed benefit costs. This is understandable as the future experience scenarios which comprise the worst 5% of cases (as measure by PVGB at the valuation date) can differ between the death and maturity/surrender guarantees. In some sense, this is a form of “benefit diversification”. Some superficial analysis suggests that the overstatement in the TGCR factors is about 7.5% for the 100% 10-year guarantees and 3.5% for the 10-year 75% benefits.

Bond Funds

Total returns on all investment classes, including bond funds, have been modelled using a monthly RSLN2 process with two regimes. However appropriate this may be for equity funds, it is questionable whether certain underlying assumptions (specifically, the regime-switching process being Markov) hold for fixed income investments where returns are clearly mean-reverting due to the high reinvestment component. The RSLN2 model could be conservative or aggressive relative to more realistic alternatives (e.g., generating yield curves and pricing a portfolio of bonds, reflecting reinvestment and periodic rebalancing), especially for balanced funds. Further research in this area is needed before any definitive conclusions can be reached.

APPENDIX H – Aug 2001 OSFI Guidance: Capital Offset for Segregated Fund Hedging Programs

Purpose

This note is intended to provide direction to life insurers wishing to obtain an MCCSR capital offset for hedging programs used in managing segregated funds market and insurance risks. Because this is an evolving area within the life insurance industry, OSFI expects that, as experience is gained, this guidance note will be revised.

Background

The capital requirements for segregated fund risks are specified in the MCCSR guideline. Currently, capital offsets are not provided for hedging programs for segregated fund risks. This guidance note outlines key criteria that must be met in order to obtain OSFI's approval for such offsets.

A strong risk management culture is a critical element in approving any hedging program for the purposes of a capital offset. Several of the requirements discussed later require an effective risk management culture. Examples include board and senior management involvement and approvals, independence of the risk management function, effective controls and validation processes, reporting that includes relevant management action plans, and appropriate skills to identify, monitor and manage the risks.

Federally Regulated Financial Institutions (FRFIs) that wish to obtain a capital offset for segregated fund hedging programs must submit an application to their Relationship Managers (RMs). Each application should address the requirements outlined in this note. In particular, each application should include copies of pertinent board approvals, documentation to support board approvals (see item 2), evidence of meeting operational requirements, sample reporting and a copy of the independent review and validation report (see item 6). All relevant documents should be available for onsite review. This transaction is subject to user pay.

REQUIREMENTS

The following are minimum expectations for the approval of hedging programs to obtain an MCCSR capital offset. In reviewing each application, consideration will be given to the FRFI's business profile and control environment. The term "model" refers to a cash flow projection model that encompasses both investment returns and segregated fund liability characteristics. The model must be able to evaluate financial market options accurately within the context of real world-based investment return simulations pertinent to actuarial projections. An audit trail must be maintained for independent verification and control.

1. Board Approval of Hedging and Other Pertinent Policies

A strong risk management culture is a prerequisite for any hedging program approval. As such, the institution should have in place policies, processes, controls, management and board approvals that show that a strong risk management framework is in place and is working effectively.

The segregated fund risk mitigation program must be presented for review to both senior management and the board and must be explicitly approved by the board or an appropriate board committee.

As appropriate, the board or board committee must also approve policies that address derivatives, capacity limits, and operational limits.

2. Documentation

A comprehensive summary of all the principles, techniques and processes used to implement the model should be available for external audit, internal audit, operational and executive management and OSFI reviews. In addition, the documentation to support the board's initial approval and OSFI's review and approval should, at a minimum, include a description of the:

- rationale for use of hedging;
- hedging program, including any rebalancing criteria;
- products to which that program is to be applied;
- implementation plan for the program;
- derivative or risk mitigation products contemplated in the hedging program;
- measurement criteria for basis risk, liquidity risk, counter-party credit risk or any other material risks associated with the hedging program;
- model's ability to evaluate financial market options (Q-measure) within a real world experience based valuation (P-measure);
- methodology, models and their limitations;
- use of hedging models within the day-to-day risk management process;
- review and approval process for the adoption of new models or modifications to existing hedging models;
- hedging model validation criteria;
- process and criteria for analysing hedge effectiveness and relationship to model validation criteria;
- frequency and types of model review;
- risk limits (corporate and trigger levels);
- escalation procedures for limit exceptions;
- stress testing and frequency of such tests;
- appropriateness of the chosen stress tests;
- reporting and monitoring requirements;
- systems used to support the monitoring and reporting;
- controls to ensure the integrity of data and results, including the peer review process;
- skills and expertise required of personnel to execute and monitor the hedging program; and
- actual CVs of individuals who develop, execute and manage the hedging program(s).

3. Operational Requirements

The program must be fully documented, implemented and proven to be effective for at least three months before OSFI will consider granting approval. During the initial implementation phase, increased monitoring and reporting of the program may be warranted.

4. Reporting

The results of the program must be reported to senior management at least monthly, with summaries of the relevant items to the board at least quarterly. More frequent reporting may be appropriate in certain circumstances. The reporting must define measurement criteria, quantify key risk exposures, analyse the hedge effectiveness and any residual risks, discuss financial implications, and specify appropriate action plans. Reporting must also include evidence of compliance with pertinent policies and limits.

Periodically, an analysis of the actual results of the hedging program versus the modelled outcomes should be conducted and reported.

5. Technical Analysis

The analysis underpinning the reporting must include a description of the stress testing undertaken (both quantitative and qualitative in nature and incorporating market risk, the liquidity aspects of market disturbances and liability characteristics), critical assumptions, demonstration of hedge effectiveness under routine and stressed conditions, and analysis of residual risks. Stress testing should be meaningful and should identify risks applicable to the hedging program and liability or product characteristics. Stress testing should be conducted at least monthly, consistent with management reporting, and should incorporate deterministic adverse scenarios. It could also include stochastic scenarios. Stress testing scenarios must take into account all material risks relating to the hedging strategy. Examples include: illiquidity, where financial options required to rebalance the portfolio are not readily available, correlation changes between asset classes, and failure to execute the hedging program.

In addition, key model limitations should be reported. Circumstances under which the model does or does not work effectively should be documented. The modelling should follow the guidance outlined in the CIA Task Force on Segregated Funds Investment Guarantees August 2000 report. See section 2.3, “Modelling of Hedges”, of the report.

6. Independent Review and Validation

The hedging program and the models used to implement the strategy should be subject to an independent review by a qualified resource(s). Qualified individual(s) within the organization may conduct the review, provided that they were not involved with developing, implementing, or executing the hedging program or model. Qualified individuals are those that have the requisite analytical and business skills to understand and evaluate the hedging program. The hedging program, in this context, should include the models, assumptions, reporting, and overall risk management infrastructure. The review should, at a minimum, include evaluation of:

- data integrity and controls;
- model logic;
- existence and appropriateness of validation methodology for the models and assumptions;
- replication of modelling results;
- ability of models to accurately capture the hedging strategy;
- appropriateness of the stress testing program, including the use of stress testing results;
- sufficiency of documentation supporting the program (including models and assumptions); and
- robustness of the process for reviewing hedging results and relationship to model validation criteria.

The actuary of the company should also provide a supporting letter indicating the appropriateness of the models, hedging program and stress tests, and the reasonableness of the results including the appropriateness of capital offsets under stress scenarios. This letter is required at the time of application or re-application.

7. Modifications to Program

Any capital offset approvals are for the specific programs presented to OSFI. If the approved program is materially modified, the company must re-apply to OSFI and receive written approval in order to continue to receive a capital offset. If the program is discontinued, the company must provide written notification to OSFI; the capital offset previously approved will be revoked.

Examples of material modifications include, but are not limited to a:

- change in board approval;
- change in model;
- change in the program; and
- change in the hedge effectiveness.

Any modified programs will need to satisfy the requirements of this guidance note. It will be sufficient, however, to model the results of the modified program for at least three months without actually implementing the program.

8. Maximum Capital Offset

Hedging strategies with respect to segregated funds market and insurance risks are relatively new and evolving. To allow for operational and execution risks in implementing such strategies successfully, the maximum offset will be limited to 50% of the reduction shown by the models. As the industry and OSFI gain confidence in implementing the strategies, this limitation will be reviewed.

The approach described below is applicable to capital offset under the current factor-based requirements.

The capital offset is determined by the use of a model –discussed previously – that accurately captures the hedging program in place and is approved by the board. This same model must be used to calculate the actuarial liability for these products. To determine the percentage reduction due to hedging, costs determined at the conditional tail expectation at 95% (CTE (95)) should be calculated on two bases: (1) with no hedging program in place and (2) with hedging. The underlying assumptions and scenarios must be the same for both calculations. The maximum allowable reduction will then be one-half of the difference between the two values, expressed as a percentage of the costs set at CTE (95) determined without hedging. This reduction is to be applied to the Net Requirements (NR) determined using factor tables as discussed in Chapter 9 of the MCCSR guideline. Therefore, the maximum allowable percentage reduction = $0.50 * ((1)-(2))/(1)$ and the MCCSR NR with capital offset is equal to $NR * (1 - \text{maximum allowable percentage reduction})$.

- END -

APPENDIX I – Guidance for the 2001 Valuation of Life Policy Liabilities: CIA CLIFR**MEMORANDUM**

TO: All Life Insurance Practitioners

FROM: Simon Curtis, Chairperson
Committee on Life Insurance Financial Reporting

DATE: October 2001

SUBJECT: Guidance for the 2001 Valuation of Policy Liabilities of Life Insurers

Valuation of Segregated Fund Investment Guarantees

For valuation of the general account liability associated with segregated fund guarantees, CLIFR believes that establishing the liability for the guarantee element using stochastic techniques represents appropriate actuarial practice.

CLIFR believes that the Appointed Actuary applying stochastic techniques to value segregated fund guarantees should review the following two papers "Use of Stochastic Techniques to Value Liabilities under Canadian GAAP" (July 2001) and "Report: CIA Task Force on Segregated Fund Investment Guarantees" (August 2000). These papers have Research Paper status, and therefore do not represent Standards of Practice or Illustrations / Expansions of Standards of practice. CLIFR believe that these documents are nonetheless a useful reference in the application of stochastics techniques in a Canadian GAAP valuation environment.

CLIFR does have the following specific recommendations for appropriate practice in applying stochastic techniques to value segregated fund guarantees:

- (a) The investment return model used to generate the investment return paths should follow the criteria and methodology laid out in section 2.1 (Investment Return Models) of the above referenced August 2000 Task Force Report,
- (b) Any modelling of hedges or other risk mitigation strategies should follow section 2.3 (Modelling of Hedges) of the above mentioned August 2000 Task Force Report,
- (c) The methodology to establish the PFAD for investment return risk should follow the CTE (Conditional Tail Expectation) approach described in both the above referenced papers. The appropriate range for the result is CTE(60) to CTE(80),
- (d) In determining the amount of unitized fee income available as revenue to offset the benefit expenses in the stochastic projection, the criteria laid out in section 3.3.5 of the August 2000 Task Force Report should be followed,
- (e) Unless there is clear intent and commitment to change, future total unitized revenues (management expense ratios) and the insurer's risk management strategies (do nothing, reinsure, hedge) should remain the same as those applicable on the valuation date,
- (f) Future deposits should be included at a reasonable level whenever future deposits materially increase the risk (e.g. fixed maturity date contracts where subsequent deposits are fully guaranteed over a period of less than 10 years),

- (g) Unless the Appointed Actuary has reliable experience to indicate otherwise, where elective resets of the guaranteed amount are available, not less than 75% of the cohort of policyholders eligible to reset should be assumed to reset each year where such a reset would cause a material increase in the guaranteed amount. A material increase in the guaranteed amount would be 15% or greater,
- (h) For contracts where a higher termination assumption reduces the net cost of the guaranteed benefits (i.e. after reflecting available margins to offset costs), unless the Appointed Actuary has reliable experience (i.e., credible and pertinent, such as experience on products with similar guarantees) to indicate otherwise, surrenders / lapses / withdrawals should not exceed a maximum rate of 8% per year at any duration. This 8% excludes any regular income withdrawals under payout features explicitly incorporated into the products (e.g. RRIF contracts income payouts),
- (i) The projection period should extend to contract maturity, including the impact of renewals (automatic resets) and voluntary resets,
- (j) The analysis should be done on a seriatim basis or on a basis that groups policies into cohorts having similar profiles with respect to nature of guarantee, time to maturity / expiry of the guarantee, and relationship of the starting unit value to the guaranteed unit value.

Once a liability for the segregated fund guarantee component of a policy / block of policies has been determined, the result should then be integrated with the valuation performed for the other elements of the policy/block of policies.

CLIFR recognizes that there may be situations where the exposure to this risk is immaterial and a simpler approach is warranted. In such circumstances, CLIFR recommends determining the policy liability for this risk by taking percentages of the total balance sheet requirement (TBSR) resulting from the application of the OSFI TBSR requirements for MCCSR (i.e. actuarial liability = F x calculated TBSR requirement).

The factor F varies as follows by type of Benefit and Fund Category, and therefore requires this split of the TBSR. The result is intended to correspond to approximately a CTE(80) result assuming conservative MV/GV ratios for each Fund Category.

Guaranteed Minimum Death Benefits	F = 0.65 (all fund types)
Guaranteed Minimum Maturity Benefits	F = 0.60 (Money market)
	F = 0.25 (bond)
	F = 0.30 (balanced)
	F = 0.50 (diversified equity)
	F = 0.55 (intermediate equity)
	F = 0.60 (aggressive equity)

APPENDIX J – Letter to CIA with Concluding Remarks

December 1, 2001

Practice Standards Council
Canadian Institute of Actuaries
360 Albert Street, Suite 820
Ottawa, ON K1R 7X7

Attention: Geoffrey I. Guy

Re: CIA Task Force on Segregated Fund Investment Guarantees

The work of the Task Force is now complete. We are thankful to the CIA for allowing us to serve the profession by contributing to the development of “best practices” for valuing long-term investment guarantees. Our report is now finished and is being prepared by the CIA office for distribution.

Although the Task Force focused on the segregated fund guarantee risk, we were able to contribute to evolving techniques using stochastic methods. We believe that continued efforts to develop stochastic modelling should be an ongoing initiative for the CIA. This work will be useful for application to Canadian situations and will be helpful in our contribution to emerging global standards for risk measurement.

In our report, we identify three areas of responsibility that should have continued focus. There are three CIA Committees, which can naturally continue to manage these areas as we have identified below:

1. Committee on Risk Management and Capital Requirements (Chair: Doug Brooks)

The Task Force has developed criteria and guidance for determining required capital using internal company models in respect of the investment guarantees on segregated funds and other similar vehicles. In principle, these methods have been accepted by the regulatory authorities (OSFI and IGIF).

This Committee should continue to monitor the evolution of model-based approaches to required capital and consider ways to support both regulators and actuaries in further implementation.

This Committee should also work with the regulators to review appropriate use of the MCCR factor methodology and track any needed changes for capital determination of the segregated fund risk.

Although most issues on capital will be dealt with by this Committee, we would also advise that the Committee on the Appointed/Valuation Actuary monitor developments on the development of the AA role in the calculation of stochastically modelled capital values.

2. Committee on Investment Practice (Chair: Charles Gilbert)

The Task Force has spent significant energy on education of the profession, regulators, industry and other interested parties on stochastic modelling techniques and their application to investment guarantee risks.

This Committee is in the best position within the CIA to assume continuing educational responsibilities on stochastic methods and applications.

3. Committee on Life Insurance Financial Reporting (CLIFR) (Chair: Simon Curtis)

CLIFR has a well established practice of developing standards for policy liabilities and assisting Appointed Actuaries through their Fall Guidance Letter to deal with interim needs as new products, risks and methods emerge. For the last few years, CLIFR has dealt with the segregated fund guarantee risk in the fall letter with numerous references to the reports of the Task Force.

CLIFR will continue to be responsible for any research and standards development in this area, including the use of stochastic methods to assess risk.

By way of this letter being sent to the PSC, with copies to the chairs of the three relevant Committees, we feel our duties have been fully discharged and our mandate fulfilled. The final English text wording of the Task Force report is attached for your reference.

Please direct any questions about the report or this letter to the undersigned by December 31, 2001.

On behalf of the Task Force,

Murray J. Taylor
Chair, CIA Task Force on Segregated Fund Guarantees

MJT/em

cc: Doug Brooks (Committee on Risk Management and Capital Requirements, Chair)
Charles Gilbert (Committee on Investment Practice, Chair)
Simon Curtis (Committee on Life Insurance Financial Reporting, Chair)
Mike Lombardi (Committee on the Appointed/Valuation Actuary, Chair)
Nick Bauer (PSC, Representative)
CIA Task Force Members