Economic Scenarios

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Changes to the C-3 Phase I Interest Rate Model

- Model Parameters Updated
- Log volatility process time step made consistent with time step for other processes
- Log volatility process seed updated

Difference Equations that describe the processes

$$\ln(r_{t+1}) = \ln(r_t) - \beta \ln\left(\frac{r_t}{\bar{r}}\right) + \psi \times \left(d_t + \bar{d}\right) + z_1 \sigma_t^r$$

$$\sigma_t^r = e^{\upsilon_t / 2} = \sqrt{\omega_t}$$

$$d_{t+1} = d_t - \gamma \times (dt + \bar{d}) - \varphi \times \ln\left(\frac{r_t}{\bar{r}}\right) + \sigma^d \times \left(\rho \times z_1 + \sqrt{1-\rho^2} \times z_2\right)$$

$$\upsilon_{t+1} = \upsilon_t + \lambda \times \left(\bar{\upsilon} - \upsilon_t\right) + \sigma^{\upsilon} \times z_3$$

Evolution of Model Parameters

Parameter	Current	SS4	SS5	SS6	SS7	SS8	SS8 (a)	SS8 (b)
$\bar{r}$	6.55%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%
LRSMR	.0048	.00072	.00401	.00265	.00265	.00265	.00265	.00265
$\bar{z}$	-6.92	-7.345	-7.345	-7.345	-7.345	-7.525	-7.525	-7.525
zSMR	.347	.02808	.02808	.02808	.02808	.02808	.02808	.02808
zVOL	.59	.22854	.22854	.22854	.22854	.22854	.22854	.22854
$\bar{d}$	.0105	.01155	.0155	.01155	.01155	.01155	.01271	.01271
DSMR	.042	.02777	.02777	.02777	.02777	.02777	.02777	.02777
v	.003809	.00322	.00322	.00322	.00322	.00322	.00322	.00322
SADJF	.21	.21375	.21375	.21375	.21375	.21375	.21375	.21375
YLF	.00024	.0002	.0002	.0002	.0002	.0002	.0002	.0002
Rho	.16	.12296	.12296	.12296	.12296	.12296	.12296	.12296

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Data

- Monthly average interest rates from US Federal Reserve Bank (“FRED”) for period April 1953 – April 2006
- Simple methodology for filling in the gaps in the time series of the 20 year interest rates
- Data in the Report

Information used to determine Model Parameters SS4 (MLE with Long Rate MRP = 5.4%

				2.183668595
Outputs	**r12**	**r180**	**r240**	**r360**
Simulation	**1**	**1**	**1**	**1**
Statistics / Cell	**D17**	**D185**	**D245**	**D365**
Minimum	0.036460672	0.005987468	0.003361304	0.003892351
Maximum	0.082191862	0.593731701	0.596752226	0.643189907
Mean	0.052568997	0.061312277	0.062489115	0.064465133
Standard Deviation	0.005040019	0.032024332	0.035923612	0.042214481
Variance	2.54018E-05	0.001025558	0.001290506	0.001782062
Skewness	0.344282866	3.538013344	2.8755806	3.08092358
Kurtosis	3.691484487	39.7532442	23.27939132	24.58186767
Number of Errors	0	0	0	0
Mode	0.05183192	0.047141355	0.041993382	0.038530903
5.0%	0.044951271	0.025886957	0.024036279	0.021835055
10.0%	0.046519887	0.031107729	0.028984325	0.026736639
15.0%	0.047523126	0.034820322	0.032923128	0.030616449
20.0%	0.048397433	0.038233582	0.036391348	0.034133442
25.0%	0.04920746	0.041088607	0.039752923	0.037685443
30.0%	0.04988008	0.044099409	0.042888347	0.040710963
35.0%	0.050517142	0.046907101	0.04576876	0.043823853
40.0%	0.051128022	0.04951233	0.04870199	0.04727485
45.0%	0.05175028	0.052001603	0.051838275	0.050684109
50.0%	0.052285172	0.054947298	0.054944176	0.054064665
55.0%	0.05289863	0.058228392	0.058348134	0.058262467
60.0%	0.053524755	0.061642185	0.06156737	0.062822208
65.0%	0.054203257	0.064981528	0.065378197	0.067497171
70.0%	0.054878376	0.069183879	0.069933176	0.073094398
75.0%	0.055652235	0.073944651	0.074795127	0.079647809
80.0%	0.056556724	0.078880817	0.081804201	0.087246209
85.0%	0.057651926	0.086480632	0.089965776	0.097056247
90.0%	0.059083126	0.097117379	0.10317225	0.112013154
95.0%	0.061099075	0.115528673	0.126728237	0.139894366

Information used to determine Model Parameters SS8 (b) (Recommended Parameterization)

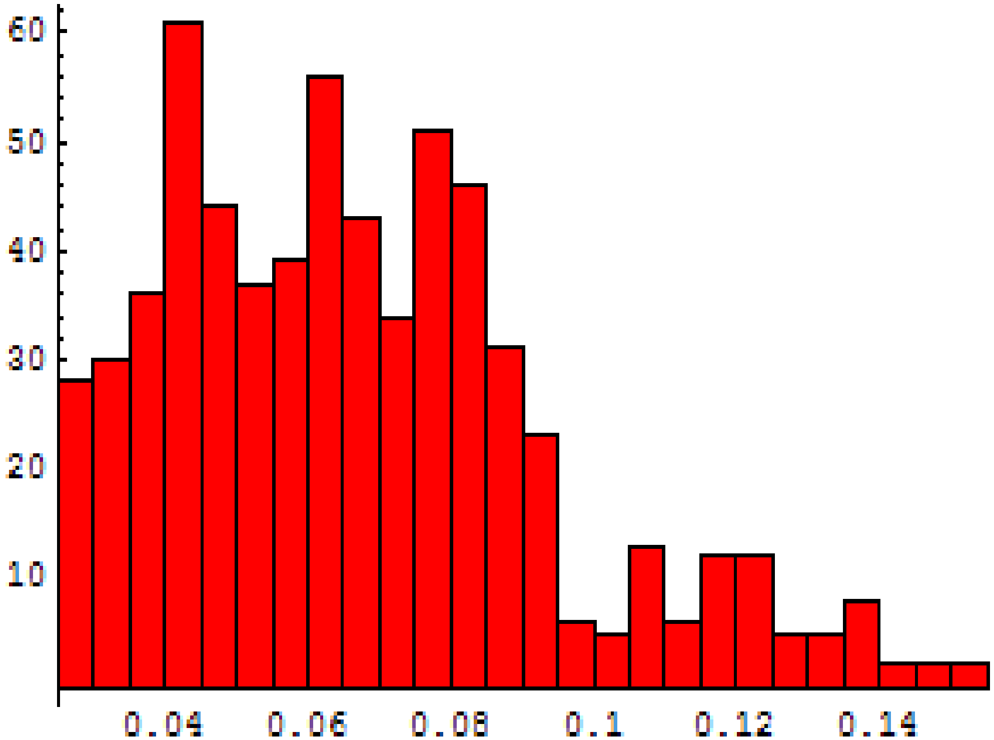
Outputs	r12	r60	r120	r180	r240	r360	Count
Simulation	1	1	1	1	1	1	1
Statistics / Cell	D17	D65	D125	D185	D245	D365	N367
Minimum	0.026485126	0.010382106	0.011189969	0.007703606	0.005582088	0.007666491	0
Maximum	0.101632059	0.185539126	0.174468577	0.172911748	0.180015147	0.181762934	253
Mean	0.052997576	0.057192974	0.058697889	0.05917558	0.059192833	0.059203824	68.421
Standard Deviation	0.008196121	0.017764168	0.021977848	0.023776574	0.024843843	0.025505028	41.28941802
Variance	6.71764E-05	0.000315566	0.000483026	0.000565325	0.000617217	0.000650506	1704.816041
Skewness	0.516978318	1.326361707	1.229317484	1.190048834	1.209275793	1.157387438	0.722115823
Kurtosis	3.90504046	6.965510716	5.563685243	5.205673465	5.064366788	4.740555907	3.35235544
Number of Errors	0	0	0	0	0	0	0
Mode	0.051400064	0.053000818	0.051317261	0.051034753	0.049912112	0.041845451	63
5.0%	0.040815629	0.03352952	0.03020799	0.02884252	0.028066471	0.026985127	12
10.0%	0.043290675	0.03774225	0.034852181	0.033528574	0.032391597	0.03195126	19
15.0%	0.044852998	0.040673137	0.038219776	0.036620196	0.035759185	0.035230398	26
20.0%	0.046224069	0.043108728	0.040964168	0.039712824	0.038631245	0.038294669	31
25.0%	0.047442112	0.045411274	0.043501318	0.042423014	0.041599073	0.041178424	37
30.0%	0.048479047	0.047366407	0.045876257	0.045128152	0.044425461	0.043575466	42
35.0%	0.049481932	0.04919615	0.048263155	0.04761954	0.046941776	0.046232253	47
40.0%	0.05053841	0.051070135	0.05067813	0.049871169	0.049685862	0.048917305	53
45.0%	0.051482335	0.0528448	0.052804835	0.05231116	0.052165318	0.05135832	58
50.0%	0.052398406	0.054801747	0.055153985	0.054978024	0.054622553	0.05421621	63
55.0%	0.05338214	0.056616731	0.057543948	0.057708185	0.057652894	0.057119165	69
60.0%	0.054365512	0.058691304	0.060015816	0.060818225	0.060520988	0.060397942	74
65.0%	0.055490211	0.060845435	0.062860988	0.063784167	0.063529208	0.063826099	80
70.0%	0.056584373	0.063288428	0.065990306	0.067243181	0.066865027	0.067563243	87
75.0%	0.057811439	0.06597814	0.069573931	0.071414679	0.07111045	0.07202296	93
80.0%	0.059327692	0.069015324	0.07385537	0.075783707	0.076027036	0.077455193	102
85.0%	0.061279722	0.073134251	0.078888699	0.081424184	0.082482487	0.083566956	112
90.0%	0.063507497	0.078807905	0.086114869	0.089721903	0.091405511	0.092751876	125
95.0%	0.067179441	0.08862821	0.100029662	0.103937127	0.108151361	0.109547779	144
Filter Minimum							
Filter Maximum						1.522840711	

Impact of changes to the model
1 and 30 year time horizons

	After	Current
0.05	0.0411002	0.04331
0.1	0.0432634	0.045382
0.15	0.0448481	0.046687
0.2	0.0462448	0.047743
0.25	0.0474006	0.048692
0.3	0.0484817	0.049476
0.35	0.0494311	0.050203
0.4	0.0504096	0.050917
0.45	0.0513289	0.051595
0.5	0.0523162	0.052321
0.55	0.0532404	0.053001
0.6	0.0542841	0.053675
0.65	0.0553961	0.054377
0.7	0.0565219	0.055182
0.75	0.0577944	0.056043
0.8	0.0592077	0.057094
0.85	0.0609055	0.058355
0.9	0.0632433	0.06006
0.95	0.0675022	0.062772

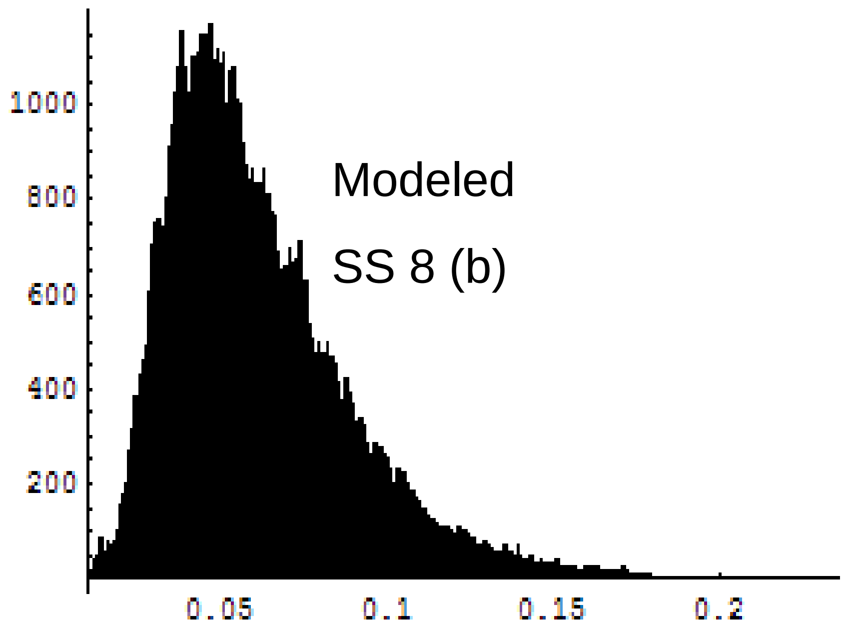
	After	Current
0.05	0.0291514	0.028387
0.1	0.0336288	0.033773
0.15	0.0370669	0.037348
0.2	0.0399654	0.040495
0.25	0.0427856	0.043819
0.3	0.0455697	0.046713
0.35	0.0480806	0.04944
0.4	0.0506282	0.052079
0.45	0.0531502	0.054846
0.5	0.0561833	0.058006
0.55	0.0589752	0.061225
0.6	0.062076	0.064665
0.65	0.0653381	0.06823
0.7	0.0693798	0.072911
0.75	0.0737366	0.078428
0.8	0.0790358	0.084431
0.85	0.0860194	0.0918
0.9	0.0959354	0.10173
0.95	0.111647	0.119777

Distribution of Monthly Average 20 YR UST Interest Rates
GS20 -- April 53 - April 2006



Historical GS20 -- April 53 - April 2006	
0.05	0.0306
0.1	0.0374
0.15	0.04
0.2	0.0419
0.25	0.0452
0.3	0.049
0.35	0.0536
0.4	0.0578
0.45	0.0604
0.5	0.0636
0.55	0.0674
0.6	0.0714
0.65	0.0757
0.7	0.0787
0.75	0.0814
0.8	0.0856
0.85	0.0908
0.9	0.1063
0.95	0.1216

Distribution of Modeled 20 YR UST Interest Rates SS 8 (b)



	Modeled SS 8 (b)
0.05	0.0272282
0.1	0.0317501
0.15	0.0354157
0.2	0.0382433
0.25	0.0411524
0.3	0.0439318
0.35	0.0465703
0.4	0.0492575
0.45	0.0521246
0.5	0.0549978
0.55	0.0583557
0.6	0.0619866
0.65	0.0657482
0.7	0.0702388
0.75	0.0747485
0.8	0.0805931
0.85	0.0876208
0.9	0.0973099
0.95	0.112576

Next steps

- Automatic updating of the mean reversion point of the long rate process.
- Develop calibration criteria.
- Coordinate equity scenarios with interest rate scenarios.
- Change interest rate model?