



AMERICAN ACADEMY *of* ACTUARIES

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**Modeling Efficiency Bibliography for Practicing Actuaries**

**By the American Academy of Actuaries'  
Modeling Efficiency Work Group**

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The American Academy of Actuaries' mission is to serve the public on behalf of the U.S. actuarial profession. The Academy assists public policymakers on all levels by providing leadership, objective expertise, and actuarial advice on risk and financial security issues. The Academy also sets qualification, practice, and professionalism standards for actuaries in the United States.

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The Modeling Efficiency Work Group would also like to acknowledge Mark Tenney.

# Modeling Efficiency Bibliography for Practicing Actuaries

## Background

This bibliography on modeling efficiency has been developed by the Modeling Efficiency Work Group (Work Group), a subgroup of the American Academy of Actuaries' Life Financial Soundness/Risk Management Committee. The Committee is responsible for making proposals for the implementation of a principle-based approach to statutory reserves and capital in the U.S. life insurance industry.

The Life Financial Soundness/Risk Management Committee recognizes that some of the calculations envisioned by a principle-based approach to the determination of reserves and capital can be onerous. The purpose of the Modeling Efficiency Work Group is to examine ways to make these calculations more manageable.

This bibliography has been prepared to give valuation actuaries a useful resource to aid with the process of improving model efficiency. The focus is primarily on techniques that could be useful for modeling work in connection with a principle-based approach to statutory reserves and capital calculations, but there are wider applications that the practicing actuary may wish to consider, such as model building for economic capital or market consistent embedded value.

Depending on the purpose of the model, the actuary will need to consider the suitability of the model design and implementation in terms of the accuracy and reliability of the results of the model, in combination with the gains in efficiency produced by the techniques selected. These considerations will include all relevant professional and regulatory guidance, such as minimum calibration requirements for economic scenarios stipulated by the American Academy of Actuaries (Academy).

The Modeling Efficiency Work Group has included in this Bibliography various papers that have come to its attention and that may have some value to practicing actuaries. The Work Group does not endorse or recommend any particular method, nor should the exclusion of any method or research paper from this Bibliography be considered significant.

The investigation and use of some methods may have secondary legal and financial considerations such as patents restricting their use without the purchase of an appropriate license from the patent holder. This issue has arisen specifically with respect to the use of low discrepancy sequences as part of Quasi Monte Carlo methods, for which certain patents are held by Columbia University in both Canada and United States. The Work Group has not attempted to address such issues.

## **Using this bibliography**

References in the Bibliography are organized into seven types or categories of solutions as defined and described in Appendix 1. These categories are subdivided into two groupings: Actuarial modeling techniques and Hardware/software solutions. This results in the following structure:

1. Actuarial Modeling Techniques
  - 1a Scenario Design and Selection
  - 1b Mathematical and/or Model Design
  - 1c Model Data Building Techniques
  - 1d Miscellaneous Actuarial/Mathematical Techniques
  
2. Technology Solutions
  - 2a Hardware Design
  - 2b Software Design
  - 2c Miscellaneous Technology Techniques

Under each heading, published papers, conference transcripts and Internet references are noted in reverse order by date of publishing, along with a brief description of the reference. More detailed synopses of articles reviewed by the Work Group have been included in Appendix 2, as noted in the Bibliography listing.

## **Modeling Efficiency Bibliography**

### **1. Actuarial Modeling Techniques**

#### **1a. Scenario Design and Selection**

Craighead, Steve 2006, "Use of Cluster Analysis for Scenario Reduction," *2006 SOA Annual Meeting, Session 101*

Available at: <http://www.soa.org/files/pdf/101bk-annual06.pdf>

This PowerPoint presentation outlines how the speaker used a specific cluster analysis algorithm to conduct scenario selection. The presentation also discusses the effectiveness of this reduction regarding a VA Hedging Application.

'C3 Phase II Pre-packaged Asset Scenarios' (Internet page), *American Academy of Actuaries* [Online]

Available at: [http://www.actuary.org/life/phase2\\_2.asp](http://www.actuary.org/life/phase2_2.asp)

This Academy web page includes a link to the Scenario Picking tool developed by Geoff Hancock as part of the work supporting the Academy's Life Capital Adequacy Subcommittee C3 Phase II project for Variable Annuities. The Scenario Picking tool enables the selection of a number of representative scenarios out of the prepackaged set of 10,000 developed by the Subcommittee, using a stratification approach and a specific distance measure. More information on this tool and its use can be found on page 19 of the March 2005 report of the LCAS which is also available from this web page.

Chueh, Yvonne 2005, "Efficient Stochastic Modeling: Scenario Sampling Enhanced by Parametric Model Outcome Fitting," *Contingencies*, Jan/Feb 2005, pp. 39-43

Available at: <http://www.contingencies.org/janfeb05/0105workshop.pdf>

This article examines sampling effectiveness by fitting the sample run distribution to a parametric probability density and then compares the resulting density with the full-run distribution. Using the Accumulation Stochastic Economic Model (ASEM) as an example, it shows that the sampling may be enhanced through the parametric fitting.

Chueh, Yvonne 2003, "Insurance Modeling and Stochastic Cash Flow Scenario Testing: Effective Sampling Algorithms to Reduce Number of Runs," *Contingencies*, Nov/Dec 2003

Available at: <http://www.contingencies.org/novdec03/workshop.pdf>

This article provides strategies to reduce the number of runs in stochastic insurance modeling. Three interest rate sampling algorithms and a computer software program SALMS that performs sampling are introduced.

Longley-Cook, Alistair 2003, "Efficient Stochastic Modeling Utilizing Representative Scenarios: Application to Equity Risks," *2003 Stochastic Modeling Symposium*

Available at:

<http://www.actuaries.ca/members/resources/meetings/pdf/modeling/Longley-cook.pdf>

This paper applies the representative scenario method to equity risk and insurance products with complex, path-dependent guarantees that emerge only in the extreme tail of the outcome distribution.

Chueh, Yvonne 2002, "Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms," *North American Actuarial Journal*, Vol. 6, No. 3 (July, 2002)

Available at: [http://www.soa.org/library/journals/north-american-actuarial-journal/2002/july/naaj0207\\_8.pdf](http://www.soa.org/library/journals/north-american-actuarial-journal/2002/july/naaj0207_8.pdf)

This paper focuses on the strategies that enable efficient stochastic modeling for large and consolidated insurance business blocks, such as by applying effective interest rate sampling algorithms. See Appendix 2 for a synopsis.

Christiansen, Sarah 1998, "Representative Interest Rate Scenarios," *North American Actuarial Journal*, Vol. 2, No. 3 (July, 1998)

Available at: [http://www.soa.org/library/journals/north-american-actuarial-journal/1998/july/naaj9807\\_3.pdf](http://www.soa.org/library/journals/north-american-actuarial-journal/1998/july/naaj9807_3.pdf)

This paper introduces the idea of using an unweighted subset of representative interest rate scenarios to approximate the distribution of a generated set of stochastic scenarios.

Tan, Ken Seng 1997, "Efficient Algorithm for High-dimensional Simulation," *Actuarial Research Clearing House*, Vol.1

Available at: <http://www.soa.org/library/research/actuarial-research-clearing-house/1990-99/1997/arch-1/arch97v115.pdf>

This paper deals with the quasi random Monte Carlo method. This approach uses low discrepancy sequences, which have the property that they tend to be evenly dispersed throughout the unit cube, rather than random sequences as in Monte Carlo. The issue of computing the standard error is addressed. The method is tested using the example of a derivative security.

Longley-Cook, Alistair 1996, "Probabilities of Required 7 Scenarios (and a few more)", *The Financial Reporter*, Issue No. 29 (July, 1996), pp 1-6

This paper describes a scenario reduction technique based on the New York 7 scenarios. See Appendix 2 for a synopsis.

## 1b. Mathematical and/or Model Design

Imai, J. & Tan, K.S. 2006, "A General Dimension Reduction Technique for Derivative Pricing," *Journal of Computational Finance*, Vol. 10, No. 2 (Winter 2006/2007)

May be purchased from:

[http://www.thejournalofcomputationalfinance.com/public/showPage.html?page=jcf\\_v10#v10n2](http://www.thejournalofcomputationalfinance.com/public/showPage.html?page=jcf_v10#v10n2)

This paper proposes improving on Quasi Monte Carlo methods by minimizing the effective dimension of the underlying function being modeled. The technique is illustrated by applying it to two high-dimensional applications: pricing Asian basket options and European call options using a stochastic volatility model.

Papageorgio, A. & Paskov, S. 1999, "Deterministic Simulation for Risk Management," *Journal of Portfolio Management*, May 1999

May be purchased from:

<http://www.ijournals.com/JPM/default.asp?Page=2&ISS=8261&SID=319698>

The authors show how deterministic simulation methods (quasi-Monte Carlo) can be applied to calculate value at risk. The approach is tested using a portfolio of collateralized mortgage obligation tranches.

Zhang, H. 1998, "Economic Scenario Generators (ESG) and Actuarial Practice," *1998 Valuation Actuary Symposium Proceedings*, Session 42D, pp 874-886

Available at: <http://www.soa.org/library/proceedings/valuation-actuary-symposium-proceedings/1985-99/1998/january/vasp9829.pdf>

Zhang described the use of low discrepancy sequences and Browning Bridge techniques applied to cash flow testing using multi-factor economic models.

Boyle, P., Tan, K.S. 1997 "Quasi Monte Carlo Methods," *AFIR Conference*, August, 1997

Available at: [http://www.actuaries.org/AFIR/colloquia/Cairns/Boyle\\_Tan.pdf](http://www.actuaries.org/AFIR/colloquia/Cairns/Boyle_Tan.pdf)

The paper describes the Quasi-Monte Carlo approach to evaluating high dimensional integrals and summarizes some of the applications to finance problems.

Joy, C., Boyle, P., Tan, K.S. 1996 "Quasi-Monte Carlo Methods in Numerical Finance," *Management Science*, Vol. 42, No.6 (June 1996), pp 926-938

May be purchased from INFORMS (Institute for Operational Research and Management Science) or accessed through <http://www.jstor.org/pss/2634604>

This paper explains and illustrates the application of the Quasi-Monte Carlo method, and describes some advantages over traditional Monte Carlo.

### 1c. Model Data Building Techniques

Fitch, J., Sobel, U., Welander, R. 2005, "Liability Modeling Concepts," 2005 *Valuation Actuary Symposium*, Session 8TS

Available at: [http://www.soa.org/library/proceedings/valuation-actuary-symposium-proceedings/2005/september/va05\\_8ts.pdf](http://www.soa.org/library/proceedings/valuation-actuary-symposium-proceedings/2005/september/va05_8ts.pdf)

Panelists discuss the basics of modeling, including: model point selection; appropriate model granularity; static and dynamic model validation; efficient model refresh processes; assumption development; and reconciliation to actual financials.

Abdella, J., Doll, D., Elam, P. 1994 "Life and Deferred Annuity Liability Models," 1994 *Valuation Actuary Symposium*, Session 8

Available at: <http://www.soa.org/library/proceedings/valuation-actuary-symposium-proceedings/1985-99/1994/january/vasp946.pdf>

Panelists discuss techniques for modeling separate account products, characteristics of a good liability model, and dynamic behavior models.

Freedman, A., Reynolds, C. 2008 "Cluster analysis: A spatial approach to actuarial modeling."

Available at: <http://publications.milliman.com/research/life-rr/pdfs/cluster-analysis-a-spatial-rr08-01-08.pdf>

This paper describes a technique, using cluster analysis, to create highly compressed cell models for liability projections using an automated process. Projection results for three case studies using larger models are compared to results using models compressed with the technique. While not addressed in detail in this paper, the same technique can be used for asset cell compression as well.

Freedman, A., Reynolds, C. 2009 "Cluster Modeling: A New Technique To Improve Model Efficiency."

Available at: <http://soa.org/library/newsletters/compact/2009/july/com-2009-iss32.pdf>

This paper provides a basic implementation of a cluster analysis technique in R, and discusses decisions that companies may be faced with in implementing the technique for production use.

Reynolds, C. 2010 "Model Compression and Stochastic Modeling."

Available at <http://www.soa.org/library/newsletters/financial-reporter/2010/june/frn-2010-iss81.pdf>

This article documents a case study for application of cluster modeling to calculation of AG-43 reserves for a block of variable annuities.

Wininger, M. 2011 "Inforce Data Compression Methods for Actuarial Modeling"  
Available at:  
<http://www.math.uconn.edu/Conferences/arc2011/paper/Wininger,%20Matt%20presentation>

This presentation provides a practitioner's viewpoint on current inforce compression methods, optimization techniques, and a qualitative validation approach.

#### **1d. Miscellaneous Actuarial/Mathematical Techniques**

Craighead, S. 2008, "PBA Reserves and Capital Modeling Efficiency: Representative Scenarios and Predictive Modeling," *The Financial Reporter*, Issue No. 73.  
Available at: <http://www.soa.org/library/newsletters/financial-reporter/2008/june/frn-2008-iss73.pdf>

This paper describes a hybrid approach based on and comparing the use of two different representative scenario techniques to create training data for a predictive model. The author tests these methods compared to known Monte Carlo model results, and evaluates the effectiveness of the methods. An Appendix provides details on the Projection Pursuit Regression predictive model used in the study.

Manistre, J., & Hancock, G. 2005, "Variance of the CTE Estimator," *North American Actuarial Journal*, Vol. 9, No. 2  
Available at: <http://www.soa.org/library/journals/north-american-actuarial-journal/2005/april/naaj0502-7.pdf>

This paper looks at some statistical properties of the methods that are commonly used to estimate the Conditional Tail Expectation (*CTE*) and develops a simple formula for the variance of the *CTE* estimator. It also explores importance sampling, a form of variance reduction, as a way to improve the quality of the estimators for a given sample size. See Appendix 2 for a synopsis.



Glasserman, Paul 2004, *Monte Carlo Methods in Financial Engineering*, Springer Science + Business Media, Inc., 233 Spring Street, New York 10013.

This book discusses principles underlying the use and analysis of Monte Carlo methods. Chapters are devoted to variance reduction techniques, Quasi-Monte Carlo, discretization methods, and estimating sensitivities.

Abbott, M., Chalke, S., George, D., Reese, T., Vanderhoof, I. 1996, "Technology and the Actuarial Pricing Model – What Comes Next?" *Record of the Society of Actuaries*, Vol. 22, No. 1, 1996 Spring Meeting, Session 38 OF

Available at: <http://www.soa.org/library/proceedings/record-of-the-society-of-actuaries/1990-99/1996/january/r96v22n138of.pdf>

This panel discussion offers a retrospective and prospective look at the development of actuarial pricing models

## 2. Technology Solutions

### 2a. Hardware Design

'Computer Architecture' (Internet article), *Wikipedia*, December, 2007 [Online]  
Available at: [http://en.wikipedia.org/wiki/Computer\\_architecture](http://en.wikipedia.org/wiki/Computer_architecture)

This Wikipedia article offers useful background information on hardware design issues. Note section on Performance in particular.

'CPU Design' (Internet article), *Wikipedia*, December, 2007 [Online]  
Available at: [http://en.wikipedia.org/wiki/CPU\\_design](http://en.wikipedia.org/wiki/CPU_design)

This Wikipedia article offers additional background information on performance issues related to CPU Design.

Kimin, Mark 'BIOS Settings' (Internet article), *About.com: PC Hardware/Reviews*, [Online]  
Available at:

<http://compreviews.about.com/cs/pchardwarebasics/a/aaBIOS.htm>

This article provides an overview to BIOS settings and how they can affect computer performance.

'Secret Tweaks' (Internet article), *About.com:PCWorld Computing Centre* [Online]  
Available at: <http://pcworld.about.com/magazine/2303p074id119267.htm>

How to tweak the hardware to improve its performance.

'Tweaking Tips' (Internet article), *Your Basic Hardware Page*, [Online]  
Available at: [http://www.basichardware.com/tweaking\\_tips.html](http://www.basichardware.com/tweaking_tips.html)

How to tweak the hardware to improve its performance.

McClellan, Keith (2000), 'Simple Quicker Speed Picker Uppers' (Internet article), *Tweak 3D*, October, 2000 [Online]  
Available at: <http://www.tweak3d.net/tweak/quickspeed/>

This article provides tips on how to tweak a computer's performance; for example, updating the drivers associated with the various peripherals.

'How-to Boost Your Computer's Performance Guide' (Internet article), *Gil's Method*, March, 2006 [Online]  
Available at: <http://www.gilsmethod.com/boostcomputerguide.>

This article provides a how-to guide to boosting a computer's performance.

## **2b. Software Design**

Michael J. Meyer 2003, "Monte Carlo Simulation with Java and C++," January 15, 2003

Available at: <http://prdownload.berlios.de/martingale/MCBook-1.2.pdf.gz>

## **2c. Miscellaneous Technology Techniques**

Gold, P, Korogodsky, A., Sabatini, F. 2005, "The Need for Speed: Achieving Maximum Run Time Performance," *Society of Actuaries 2005 Life Spring Meeting*, Session 53 OF

Available at: <http://www.soa.org/library/proceedings/record-of-the-society-of-actuaries/2000-09/2005/may/rsa05v31n153of.pdf>

This session has practitioners discussing the ways they have addressed run time issues by leveraging technology, using thoughtful approaches to coding and model construction and techniques such as software and coding optimization, hardware optimization, distributed processing and parallel and grid computing techniques.

## **Appendix 1: Categories of modeling efficiency techniques**

These categories of modeling efficiency techniques recognize that a key consequence of a principle-based approach for many types of business is the processing of multiple scenarios over a long projection period, so that one must carefully craft each of the scenarios, models, data, software and hardware to be as efficient as possible.

### **1. Actuarial Modeling Solutions**

#### **1a. Scenario Design and Selection**

How we choose or design our scenarios can reduce the total number of scenarios required in stochastic runs. For example, instead of using 1,000 scenarios for a model run, there may be a way to reduce this to say 100 scenarios while keeping the error of our metric within an acceptable level of risk.

#### **1b. Mathematical and/or Model Design**

The design and mathematical approach to a model can simplify calculations and/or the time to conduct these calculations. For example, for runs that require an estimate of future market prices, instead of having to generate a set of market consistent scenarios at each future point in time, we may be able to use a closed form mathematical solution, such as Black-Scholes.

#### **1c. Model Data Building Techniques**

Here we consider how to group or cluster business data representing liabilities or assets so as to reduce the number of model points required to be processed within an actuarial model. This category covers concepts such as grouping contracts by issue age, duration or by contract features. Another alternative is to use cluster analysis and survey analysis to determine an optimal subpopulation of contracts that represent the overall population.

#### **1d. Miscellaneous Actuarial/Mathematical Techniques**

This category includes actuarial or mathematical modeling techniques which are general in nature, or do not fit into the preceding categories.

## 2. Technology Solutions

### 2a. Hardware Design

Efficiency can be addressed by considering how software is implemented on specific hardware. If one has a good knowledge of the hardware or software-to-hardware interface (the firmware), one may do small enhancements either to the hardware settings or the software settings and dramatically improve the run time. For example, if one formats the hard drive that is used to collect the data such that the swap space (this is the area of the hard drive that takes the contents of the CPU and moves it back and forth between the hard drive and the CPU to process different tasks) is in the center of the hard drive, the time it takes to move data in and out is reduced because the average travel time of the write heads to read and write data is reduced.

### 2b. Software Design

Efficiency can be addressed by considering how models are implemented in software, including programming techniques. If one is able to influence the design of the modeling system, there are various system optimization techniques that will aid in run time. One example is that the system is originally written in Visual Basic. If the reserve calculation routines are rewritten in C++ and then compiled as a Dynamic Linked Library (DLL), one can link VB to the DLL and the run time for the reserve calculations may be improved up to 40 times.

### 2c. Miscellaneous Technology Techniques

This category includes technology based techniques that are general in nature, or do not fit into the preceding categories.

## **Appendix 2: Overview of selected papers**

**Manistre, J., & Hancock, G. 2005, "Variance of the CTE Estimator," *North American Actuarial Journal*, Vol. 9, No. 2**

*Synopsis:*

The paper gives an example of biased sampling. The approach is as follows:

- 1) Develop a "model" portfolio of risks (e.g., a small but representative sample of the risks you are going to value).
- 2) Run the model portfolio through a large number  $N$  of scenarios (e.g.  $N = 1000$ ).
- 3) To compute  $CTE(90)$  take the 200 worst scenarios from (2) and 1 out of every 10 scenarios from the remaining 800.
- 4) Run the full portfolio through the 280 scenarios. Attach a weight  $W = 1/1000$  to the first set and  $W = 10/1000$  to second set.
- 5) Apply math as presented in the paper to estimate the CTE and its variance. If the model is good, you get 1000-scenario precision at a cost of running 280 scenarios. If the model is not good, you could get a precision comparable to running only 100.

*Applications beyond the principle-based approach to reserves and capital:*  
Economic capital.

*References/related articles:*

None.

*Special terms used in the article:*

None

**Christiansen, Sarah 1998, "Representative Interest Rate Scenarios," *North American Actuarial Journal* ,Vol. 2, No. 3 (July, 1998)**

*Synopsis:*

The paper describes a scenario reduction technique that can be used to develop a set of representative scenarios, and analyzes the application of this technique to some specific test data.

A general algorithm is described for picking representative scenarios from a stochastically generated set. The representative scenarios are then used with equal or no weighting. (Contrast this with other scenario reduction techniques that assign weights to each scenario in the reduced subset). The paper selects a reduction target of 4% (400 scenarios to represent 10,000).

Christiansen's algorithm first generates multiple subsets of the whole set (a "candidate list" of scenario subsets) for consideration, then tests each subset using a customized choice function, which measures the fit of the subset to the whole set using multiple criteria (e.g., mean of each term rate in scenario, extremes, standard deviations, etc.). The subset with the best fit is used. The criteria in the choice function can and should be selected and weighted according to the intended purpose of the scenarios.

The author also used "partitioning" to improve manageability. She ran the algorithm multiple times on independently generated sets combining the results.

The candidate list creation is also an arbitrary process that is intended to make the application of the choice function manageable while ensuring that all reasonable candidates for testing are included. Each subset on the candidate list will have 10 scenarios (because of the partitioning decision) and the 10 are selected for each candidate subset by the following three step algorithm:

- (a) Two are selected that respectively contain the lowest and highest interest rate found in the whole scenario set at any term and in any projection year (thus ensuring the extremes are included).
- (b) Four are selected such that the mean interest rate at term  $t$  is closest to 4 calculated target values above and below the corresponding means for the whole set (thus ensuring some values between the means and the extremes); the four targets are the means plus and minus 65% and 85% of the standard deviation.
- (c) Four more are selected to ensure that the mean rate at term  $t$  for the subset replicates the mean rate of the whole set for term  $t$ .

The candidate list can be quite large because every valid combination of scenarios satisfying step (a) will generate a new subset for each interest term  $t$ . Each of these candidate subsets is then completed (the remaining 8 scenarios selected) using steps (b) and (c).

Christiansen applied this technique to 10,000 scenarios using 40 partitions to select 400 representative scenarios. Steve Craighead of the Work Group tested these representative scenarios against the whole set using an actual ruin study on 31 annuity lines of business, and also tested the representative scenarios against

similar sized subsets randomly extracted from the whole set. Statistical tests were done on the scenarios themselves and on the distribution of the ruin study results.

Christiansen concludes her method provides a much better fit of the mean rates in the representative set to the whole set, while providing greater standard deviations and variance. Tests performed on the truncated left tail gave less satisfactory fits than on the whole distribution. Thus, this method is effective for purposes where a range of tests is important and a true mean is required. The implication is that it may not be as good for capital testing. She comments that the form of the choice function is important as are the weights used for each measure.

*Applications beyond the principle-based approach to reserves and capital:*  
Cash flow testing, planning or pricing models

*References/related articles:*  
None

*Special terms used in the article:*  
Scenario reduction



**Yvonne Chueh, “Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms,” NAAJ, Vol. 6, No. 3 (July, 2002)**

*Synopsis:*

Cheuh uses three separate measurements of distances (metrics) regarding interest rate scenarios. The first two reflect the distance from one scenario to another and the third is an absolute measurement associated with a scenario.

What is innovative here is that each of her metrics takes the time value of money into consideration. Once she has created distance measurements, she then introduces separate sampling processes that will select a representative scenario. Her paper then goes on to examine how well these separate distance and sampling techniques work.

The first metric is:

$$D_1 = \sqrt{\sum_{t=1}^{30} V^t (i_t - i_t^P)^2}.$$

Here  $i_t$  is a one-year rate at time  $t$ .  $i_t^P$  is the pivot interest rate at time  $t$ .  $V$  is some value between 0 and 1 and is a weighting factor to place significance on the interest rate for each projection year. This effectively is a weighted average least squares metric. On samples of 50 scenarios out of 1500, the associated sampling algorithm found the extremes of the distribution, but the central portion of the distribution was poor.

The second metric is:

$$D_2 = \sqrt{\sum_{t=1}^{30} \left( \prod_{k=1}^t \frac{1}{1+i_k} - \prod_{k=1}^t \frac{1}{1+i_k^P} \right)^2}.$$

In this metric one compares the least squares metric of the compound discount at each time. On samples of 50 scenarios out 1500, the associated sampling algorithm found the extremes as well as covered the overall support of the distribution well.

The final metric is:

$$S = \sqrt{\sum_{t=1}^{30} \left( \prod_{k=1}^5 \frac{1}{1+i_k} \right)^2}$$

This metric (actually a norm) creates the absolute value associated with each scenario. The associated sampling algorithm, did not find the extremes, but did give a very good representation of the central portion of the distribution.

*Applications beyond the principle-based approach to reserves and capital:*  
Cash flow testing, planning or pricing models

*References/related articles:*  
None

*Special terms used in the article:*  
None

**Alistair Longley-Cook, "Probabilities of Required 7 Scenarios (and a few more)," July 1996, Financial Reporter 29, 1-6 (July 1996)**

*Synopsis:*

Alistair Longley-Cook argues that a more probabilistic analysis of New York 7 outcomes could be achieved without running projections using thousands of stochastic scenarios. Using least squares, he fitted sets of 1000 stochastic scenarios to the New York 7. The probability assigned to each NY7 scenario was the percentage of stochastic scenarios with "best fit" to that scenario. He notes that the relative fit was quite sensitive to the interest rate generator assumptions chosen. Adding six deterministic scenarios to the NY7, he reduced the total error.

*Applications beyond the principle-based approach to reserves and capital:*  
Cash flow testing, planning or pricing models

*References/related articles:*

Cited by Christiansen & Chueh, and specifically built upon by Chueh.

*Special terms used in the article:*

None