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A Guide to the Use of Stochastic Models in Analyzing Social Security

Social Security's long-term financial problems, and the various proposals to correct these problems, have aroused intense interest throughout the United States. One consequence is that the process for measuring Social Security's financial condition has come under detailed public scrutiny.

Each year, the trustees of the Social Security trust funds report on the financial condition of the system. The principal results from the trustees' report are generated by a deterministic model of the annual income and expenses of the system over a 75-year projection period. The report also includes, in Appendix D, a sensitivity analysis, which describes how the results from the deterministic model would change if individual assumptions were changed in specified ways. Since 2003, the report has also included, in Appendix E, the results from a stochastic model of the system. The stochastic model differs from the deterministic model in that the inputs to the model are not fixed, instead taking on many values based on the probability those values will occur. These appendices add considerably to the analysis of future prospects for continuing solvency of the Social Security program.

This guide provides an overview of what deterministic modeling, sensitivity analysis, and stochastic modeling mean, and how these methods are applied to the problem of evaluating the financial status of Social Security. It also discusses some of the benefits of sensitivity analysis and stochastic modeling, as well as some of their limitations. In addition, for those wishing to see an example of how stochastic modeling works in practice, an Appendix describes a simulation of the investment returns from a savings or 401(k) plan, together with some of the inferences that might be drawn from such an analysis.

Approaches to Modeling Social Security: Deterministic Modeling, Sensitivity Analysis, and Stochastic Modeling

The Social Security System is very complex, and it should come as no surprise that mathematical models of the system are also very complex. This complexity, however, need not pose an impenetrable barrier to understanding the basic concepts of mathematic modeling as they apply to Social Security.

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Independent and Dependent Variables

Mathematic models include variables, which are intended to represent measurable values of real world phenomena and relationships among the variables. For example, a mathematic model of a car trip might include three variables, the speed of the car and the time and distance traveled. These variables are related by the mathematical expression: distance equals speed multiplied by time. The speed and time can be set to any values within the physical constraints of the car — its maximum speed and the capacity of its fuel tank, for example, are called *independent variables*. The distance traveled, which depends on the speed and time, is called a *dependent variable*.

This model is obviously simplistic in that it can only describe a car traveling at constant speed. We could make the model more realistic by adding periods of acceleration and deceleration. Each acceleration and deceleration would add new independent variables and new mathematical relationships. In addition, we might add new dependent variables, such as the distance traveled during various segments of the trip. These are called *intermediate results*, and the total distance traveled is the *final result*.

Deterministic Modeling

In deterministic modeling, all inputs to the model — the independent variables — are assigned definite values for each period of time between the beginning and the end of the projection period. Actuaries usually call these values the *assumptions*. Further, the relationships among the independent variables and any values derived in the course of running the model — the dependent variables — are fixed in advance. The term *deterministic* is used to distinguish this type of modeling from *stochastic* (or *probabilistic*) modeling, in which some or all of the independent variables take values that are assigned based on the probabilities those values will be realized and in which the relationships between the independent and dependent variables may not be fixed in advance. Stochastic modeling is described later in this issue brief.

The Social Security actuaries use a deterministic approach to obtain three estimates of Social Security's long-term financial situation: Alternative I, a low-cost or optimistic forecast; Alternative II, the intermediate or "best estimate" forecast, and Alternative III, a high-cost or pessimistic forecast.

In order to project trust fund balances, three types of independent variables require assumptions: demographic variables (e.g., mortality, fertility), economic variables (e.g., average earnings, GDP and future interest rates), and program-specific variables (e.g., taxable payroll, insured population, and factors underlying automatic adjustments). For more information on assumptions, see the Academy's issue brief *Assumptions Used to Project Social Security's Financial Condition.*

Each of the three alternatives requires 75 years' worth of assumptions for each of these variables. For example, for the mortality assumption in the 2005 valuation, the age-sex-adjusted death rate per 100,000 lives for the 65 and over age group in the intermediate assumption set starts at 5,266 in the year 2005 and declines to 3,153 by the year 2080. In the low-cost set of assumptions the rate declines from 5,299 to 4,359 (implying shorter life expectancy), and in the high-cost set of assumptions the rate declines from 5,232 to 2,111 (implying longer life expectancy). Each of the other key independent variables has three sets of assumptions for each year from 2005 through 2080.

Once all the assumptions are set, each of the three sets of assumptions is run through a mathematical model (programmed on a computer), incorporating relationships among the variables that simulate the behavior of the entire Social Security system over a 75-year time period. The three estimates of the status of the trust fund are part of the output of the model.

These three estimates vary over a fairly wide range. While the projected actuarial balance — cumulative income less expenses expressed as a percent of covered payroll — at the end of 75 years is −1.92 percent of covered payroll using the best-estimate assumptions, the range of variation is 5.34 percent, from a high under Alternative I of +0.38 percent to a low under Alternative III of −4.96 percent (See Table IV.B4). The wide range of results raises some obvious questions — how reliable is the best-estimate result and what factors could cause the actual outcome to be closer to the low-cost or high-cost result? The discussion in the next section describes one way to obtain information on the relative importance of the various inputs to the final estimates.

Sensitivity Analysis – A Closer Look at Deterministic Modeling

As is evident from the discussion of deterministic modeling, an enormous number of inputs into the model are required to simulate the behavior of the Social Security system. Not only is the number of variables large, but each variable requires 75 assumed values for each of the three estimates. Deterministic modeling alone cannot tell us which of the input variables are the most important to the final result. We cannot determine whether, for example, a small change in the rate at which mortality improves has a greater or lesser effect on trust fund balances than a small change in the rate of price inflation. Further, we cannot determine the degree to which a change in one direction for a given variable is offset by a change in the other direction for a different variable. This shows the importance of trying to determine the *sensitivity* of the final results to specified changes to model inputs.

Appendix D of the trustees' report illustrates how sensitivity analysis can be used to better understand the long-range prospects of the trust fund. The appendix uses the intermediate-cost projection as the baseline for the analysis, and shows how the actuarial balance changes when a single assumption is changed from its value under the intermediate-cost projection to its value under the low-cost or high-cost projection. The assumptions studied in this way include the total fertility rate, death rates, net immigration, real-wage differentials, the consumer price index, the real interest rate, disability incidence rates, and disability termination rates. The three projections in the body of the report allow us to see how the results vary when we change all the assumptions at once to their low-cost or high-cost values; sensitivity analysis allows us to measure how sensitive the projection results are to each of these eight assumptions individually.

For example, if we set the ultimate total fertility rate at its low-cost value, the actuarial balance changes from –1.92 percent, its value under the intermediate-cost assumptions, to −1.64 percent; and if we set this assumption to its high-cost value, the actuarial balance changes to −2.22 percent (Table VI.D2). Similarly, if we change the rate at which the annual probability of death declines over the 75-year projection period to its low-cost value, the actuarial value changes to −1.33 percent; and if we change the rate to its high-cost value, the actuarial balance changes to −2.63 percent (Table VI.D2). This illustrates that changing the mortality improvement assumption to its low-cost or high-cost value has a greater effect on the actuarial balance than changing the fertility assumption to its low-cost or high-cost value. We can infer that mortality improvement has a greater effect than fertility in producing the total change in the actuarial balance from the intermediate to the low-cost or high-cost assumption set.

Although we will gain considerable insight into the sensitivity of the ending trust fund balance to changes in each of the underlying assumptions, as well as how the assumptions may interact, this approach does not provide any information about the probability that a given scenario will be realized. Stochastic modeling can provide this additional insight.

Stochastic Modeling

The user of a deterministic model generally runs the model for only a small number of scenarios. For example, the Social Security actuaries run their model once for each of the three assumption sets and twice for each of the eight assumptions studied in the sensitivity analysis, 19 times in all for each report. In contrast, the user of a stochastic model may run the model hundreds or thousands of times.

Of course, it would not be possible to compose by hand hundreds or thousands of assumption sets for a model as complex as that required for Social Security. Instead, the model includes a routine to choose the assumption set each time the model is run from the universe of all possible assumption sets based on the probability the situation described by the assumption set will occur in the real world. If the model has been properly designed and the probabilities of the assumptions correctly determined, the distribution of the results of the model after a large number of runs can provide important information about the probabilities that various end results will be realized. Using the probabilities of the independent variables to determine the probabilities of the dependent variables is the essence of stochastic modeling.

An important step in developing any stochastic model is determining the range of reasonably possible values for each independent variable and assigning a probability to each value. The result is called the *probability distribution* of the independent variable. There are several ways to go about this. One approach bases the probability distributions of the independent variables on empirical studies of the real-world phenomena they represent. Under this approach, the value chosen for each year over the projection period is independent of the values chosen for the other years.

Another approach, the one used by the Social Security actuaries in their stochastic projections, bases the value of an independent variable each year on the values in prior years, together with some random yearly fluctuation. In the analysis by the Social Security actuaries, these relationships are defined in such a way that, in the absence of the random fluctuation, values for a given year would equal those under the intermediate set of assumptions. These two approaches can be combined in various ways; for example, the yearly fluctuations in the second approach can be assigned values based on empirically determined probability distributions characteristic of the first approach.

To make this more concrete, consider the fertility rate. In the first approach our analysis regarding the fertility rate assumption might proceed as follows: the total fertility rate has varied between 1940 and 2004 from a low of 1.74 (average children per woman) in 1976 to a high of 3.68 in 1957. Values over the last 15 years have been very close to 2.0, and the projected range from the high-cost assumption to the low-cost assumption is 1.7 to 2.2. So we might assume for the purposes of the stochastic model that fertility will be distributed like a bell-shaped curve ranging from 1.7 to 2.2 with its peak at 1.95, and assign values each year on that basis.

The second approach described above would lead to the following type of formulation (oversimplified from what is in fact done by the Social Security actuaries): this year's fertility rate is equal to 10 percent of the fertility rate from 10 years ago, plus 20 percent of the fertility rate from 7 years ago, plus 30 percent of the fertility rate from 5 years ago, plus 40 percent of the fertility rate from 2 years ago, plus a random number between -0.05 and +0.05.

Two complicating factors in this process are correlation and covariance. Although the independent variables can be chosen with a high degree of freedom, they may not be entirely independent of each other. For example, productivity growth and GDP growth are strongly correlated in a positive manner, in the sense that they often move in the same direction — when productivity growth is high, GDP growth is usually high as well; and when productivity growth is low, GDP growth tends to be low. Similarly, GDP growth and unemployment are somewhat negatively correlated — when unemployment rises, GDP growth may slow, and vice versa. In this context, *correlation* refers to the tendency of two or more independent variables to move in the same (positive correlation) or opposite (negative correlation) directions, and *covariance* refers to the strength of these relationships. It is important to remember that correlation only measures a tendency and not a certainty. For example, in recent years we have at times experienced productivity increases along with a decrease in the rate of GDP growth.

When two or more independent variables are correlated, the selection of a particular value for one will constrain to some extent the possibilities for what may be chosen for the second. In other words, once we randomly choose a value for one independent variable, the probability distribution used to choose a value for any other independent variable that is correlated with the first must be adjusted to take into account the correlation.

Determining probability distributions, correlations, and covariances for the independent variables is the most difficult and crucial aspect of a correctly designed stochastic analysis. It can be far more complicated than choosing assumptions for a deterministic model. The value of a stochastic model depends on how well this process is carried out.

As noted above, a stochastic model can be run hundreds or thousands of times. Each run, or trial, entails choosing the values to be used for each independent variable, running the model, and saving the result. Once the process is complete, the results of all the trials are tabulated and ordered so that statistical inferences can be made. If, for example, 10,000 trials of the Social Security model were run to the year 2030 and the ending trust fund balance was positive in 9,750 of these trials, we would interpret that result to mean that there is a 97.5 percent probability of having a positive balance in the year 2030. The example in the brief's appendix describes the entire process in more detail.

In the stochastic models discussed up to now, the relationships by which the dependent variables are derived from the independent variables have been fixed, as they are in deterministic models. In some situations, the exact relationships between the independent and dependent variables are not known in advance; only the general form of the model can be specified. In these situations, *regression analysis* can be used to help define more completely the relationships among the independent and dependent variables in the model. The techniques of regression analysis allow the user to derive the best possible definition of these relationships from observed past values of the independent and dependent variables. The newly specified relationships can then be used to refine the model for forecasting future values of the dependent variables.

Time series analysis is often used in situations where the value of a dependent variable in the current period is a function of its value in the prior period or periods. A simple example of this would be the size of a pension fund balance, which will depend not only on market returns during the current period, but also on its balance at the end of the prior period. It is often relatively easy to derive relationships among variables over short periods of time through empirical studies. Time series analysis can be used to see how these short-term relationships develop over longer time periods. Time series have the advantage that relatively little knowledge of the underlying causal relationships is needed to begin to draw conclusions.

It is important to understand that deterministic and stochastic modeling are not mutually exclusive. Few models of complex systems are entirely one or the other. For example, when the annual Social Security cost-of-living adjustment was deferred in years when the cost-of-living increase fell below a fixed threshold, the Social Security actuaries' deterministic model used stochastic methods to calculate the cost effect. Under the deterministic model there would have been no effect, because the fixed COLA assumption was higher than the threshold, so the adjustment would never be deferred.

An important potential use for stochastic modeling of the Social Security system is the analysis of various reform options debated in the U.S. Congress. For example, in the case of recent proposals to partially privatize Social Security through the use of individual investment accounts, stochastic modeling could be used to try to answer the following question: Find an age "X" so that if a worker invests "w" percent of his wages into an individual investment account from age "X" to the normal retirement age, there is a 95 percent probability that at the worker's normal retirement age, the worker will have combined benefits (from a redefined Social Security reduced benefit and from the annuity purchased from the individual investment account) at least as great as under current law. Clearly, there are a plethora of other policy questions that could also be addressed by stochastic modeling.

Benefits and Limitations of Specific Approaches to Modeling Social Security

Before listing some of the benefits and limitations of the specific approaches to modeling Social Security, it will be useful to make some general observations about models. All models, from the simplest to the most complicated, involve varying degrees of abstraction, both explicit and implicit. That is to say, no model is a perfect representation of its subject. Models are more or less useful in direct proportion to the degree to which they capture the most significant dynamics of the program or phenomenon being studied.

Deterministic Modeling

Benefits
- Relatively easy to understand and explain.
- Results can be independently duplicated and verified.

Limitations

- Every assumption needs to be explicitly chosen before model is run.
- No insight into relative importance of the major assumptions.
- No assessment of the likelihood any particular scenario will actually occur.

Sensitivity Analysis

Benefits

- Provides insight into the relative importance of major assumptions.
- Can be used to analyze how assumptions interact.
- Well suited to dealing with specific "what if" types of questions.
- Since all assumptions are still explicit, results can still be independently duplicated and verified.

Limitations

- No assessment of the likelihood any particular scenario will actually occur.
- Choice of additional scenarios is somewhat subjective.

Stochastic Modeling

Benefits

- Allows for an analysis that in principle can cover a very significant range of possible future outcomes.
- Permits assessments of the likelihood of results.
- Can uncover "unexpected" interplay among the independent variables. Unexpected is in quotes because, at some level, any result could be anticipated by detailed analysis. However, it might not be easy to determine the outcome of particular combinations of assumptions, and stochastic analyses involving large numbers of trials can help uncover unanticipated relationships.
- A stochastic model, though complicated, is much more flexible than the typical deterministic model. For example, the initial inadequacies of a model may be corrected over time. This is a significant consideration when we ponder the rapid development of computing power. Techniques that are too computationally complex to be implemented now might be easily executed within a few years.
- When the low, intermediate, and high deterministic assumptions are judiciously chosen, stochastic modeling can provide assurance that a large proportion (typically 95 percent) of possible outcomes falls within the range defined by the low and high assumptions.

Limitations

- Model risk is a significant issue. For each independent variable we allow to vary stochastically (i.e., which will be sampled from a probability distribution), we are assuming that the probability distribution used in the model correctly reflects the actual distribution for that variable and its correlations with the other independent variables. The risk increases rapidly with the number of independent variables, because the correlation relationships among the independent variables proliferate as the number of variables increases.
- Even if correlation relationships are correctly characterized in the early years of the projection period, potential changes in these relationships over time are nearly impossible to predict accurately.
- By stressing technique rather than assumptions, complex stochastic modeling may distract us from where the true effort needs to be applied. With all the uncertainty surrounding economic assumptions, stochastic models may provide us with a false sense of exactitude.
- Since the number of possible scenarios is infinite, and because randomness has explicitly been added to the assumptions, the results are difficult to replicate, although summary statistics should be fairly similar in two properly constructed simulations of sufficiently many trials.

Summary and Recommendations:

This brief has discussed several approaches to modeling Social Security: deterministic modeling, sensitivity analysis, and stochastic modeling. Each has its strengths and weaknesses.

For example, deterministic models are the easiest to analyze and explain, but they provide no insight regarding the relative importance of the independent variables to the final result.

Sensitivity analysis provides insight into how the final result can vary as independent variables are systematically changed but offers no convenient mechanism to assess the likelihood of the scenario being examined.

Finally, stochastic modeling allows us to make probabilistic statements regarding the likelihood of various outcomes, but at the cost of a fairly elaborate mathematical and statistical infrastructure, and, even more importantly, with the additional risk of model misspecification.

Sensitivity analysis will always be a useful "what if" methodology in assessing the impact of a change in one or more underlying assumptions. We believe stochastic modeling is useful in performing additional analysis, especially the type of analysis required to evaluate the likelihood of particular scenarios.

Appendix – Simulating a Savings Plan Account Balance

Any time that we talk about mathematical modeling, we are talking about taking some type of system whose behavior can be abstracted in a way that allows for meaningful analysis. Let's take the example of a thrift or 401(k) savings plan that currently has $1,000 in it, and assume we are interested in determining what the balance would be five years from now, assuming no additional contributions.

Before we proceed, it is important to note that this example is not intended to demonstrate how the actuaries in the Social Security Administration do their modeling. Rather, it is intended to provide the reader with an example, simplified for illustrative purposes.

If we were modeling the balance in a purely deterministic way, we would be given (or we would assume) some annual rate of growth, say 6 percent. We would then take the $1,000 and multiply it by 1.06 five times in order to obtain an answer of $1,338.23.

If we were interested in performing what is called sensitivity analysis, which is basically the methodology being used when the term "scenario testing" is employed, we might want to determine what the five-year ending balance would be if the growth rate were –2 percent or 12 percent (instead of the assumed 6 percent). In the first case we would calculate an ending balance of $903.92; in the second case it would be $1,762.34.

Now let's take a look at what would be involved in performing a stochastic simulation of the five-year ending balance. "Stochastic" is simply a synonym for "probabilistic." In stochastic simulations, we choose inputs into the model (in this case the rate of growth) according to some mechanical rules that reflect the probability of that input actually being observed.

To restate our problem, let's suppose that we wanted to determine the range of ending account balances in a thrift savings plan five years from now, assuming that we start with $1,000 and that we have only one asset class (investment category or mutual fund) in which to invest. We also assume further that the only possible annual returns are –4 percent, 0 percent, 4 percent, 8 percent, 12 percent and 16 percent, each equally likely. We could then simulate each five-year period with five consecutive rolls of a standard die, with faces numbered 1 to 6. We will interpret a 1 as indicating a –4 percent return, a 2 as a 0 percent return, a 3 as a 4 percent return, and so on.

We can begin the first simulation by rolling the die five times. Suppose the rolls are 3, 5, 1, 1, and 4. Then, at the end of the first year, the $1,000 has grown by 4 percent, to $1,040. During the second year, the fund grows by 12 percent, to $1,164.80. During the third year the fund grows by –4 percent (or, what is the same, it loses 4 percent) to end the year at $1,118.21. During the fourth year the fund loses 4 percent again, to end the year at $1,073.48. Finally, during the fifth year, the fund grows by 8 percent to end the five-year period at $1,159.36.

We would then start the second simulation by rolling the die five more times, but this time we roll 1, 6, 2, 5, and 1. Working through what each roll means produces a five-year ending balance of $1,197.34.

We would then perform these simulations over and over again (perhaps using a spreadsheet or some other electronic aid), maybe a hundred or a thousand times. After performing all these simulations, we would then have a range of ending values, from $815.37 (rolling five 1's) to $2,100.34 (rolling five 6's), with most values in the vicinity of $1,300 (this is because the way possible returns were set up implies an average return of 6 percent per year).

How would the results be interpreted? Let's suppose that 100 simulations were performed and that the results were distributed as follows:

Ending 5 year balance	Number of simulations
Less than or equal to $1,000.00	12
$1,000.01 - $1,200.00	18
$1,200.01 - $1,400.00	32
$1,400.01 - $1,600.00	15
$1,600.01 - $1,800.00	10
$1,800.01 - $2,000.00	9
Equal to or more than $2,000.01	4

Then, using these results, we could make the following assertions:

- There is a 12 percent chance of ending the five-year period with less than or no more than the starting balance of $1,000 (since in 12 simulations out of the 100 performed, the ending balance was less than or equal to $1,000).

- There is a 4 percent chance of the account doubling or better by the end of five years.

- The likeliest five-year ending balance is between $1,200 and $1,400.

It is important to keep in mind that if another set of 100 simulations were performed, the distribution of five-year ending balances would likely be slightly different. Nonetheless, as more and more simulations were performed, the final results would tend to stabilize.

Often, simulations are performed in models where there is more than one independent variable affecting the system. For example, most people have their savings invested in more than one type of asset. It is usually the case that the various returns on the different asset classes are not entirely independent of each other. Because all asset classes represent available choices in the investment universe, the returns on some classes are influenced by the available returns on others. This phenomenon is what is being described when the term "correlation" is being used — it indicates the tendency to move in a similar direction (positive correlation) or the opposite direction (negative correlation).

In order to acquire a feel for how correlation can enter into simulations, let's suppose that we now have two asset classes in which our initial $1,000 account balance is invested, with $500 in each. Suppose that the first asset class is the same as above and that the second is negatively correlated with the first. In other words, if the first asset class does well, the second tends to do poorly, and vice-versa. It is possible for both to do well or both to do poorly, although it is less likely.

The following table will show what we will be assuming for possible annual returns on the second asset class, given a return on the first:

If the return on the first asset class is:	The possible returns on the second will be:
-4%	2%, 4%, 6%, 8%, 10%, 12%
0%	0%, 2%, 4%, 6%, 8%, 10%
4%	-2%, 0%, 2%, 4%, 6%, 8%
8%	-4%, -2%, 0%, 2%, 4%, 6%
12%	-6%, -4%, -2%, 0%, 2%, 4%
16%	-8%, -6%, -4%, -2%, 0%, 2%

In other words, we will be assuming that if the return during the year on the first asset class is 8 percent, then there is a 1/6 chance of a –4 percent return on the second asset class, a 1/6 chance of a –2 percent return, a 1/6 chance of a 0 percent return, and so on.

So now when we perform a simulation, we will need a pair of dice. The first die (possibly of a different color than the second so we can tell them apart) will be interpreted as before. The interpretation of the second die, however, will depend on the number showing on the first die.

For the first simulation, suppose that we first roll a 4 on the first die and a 2 on the second die to simulate the first year. The 4 on the first die means that the $500 invested in the first asset class has grown by 8 percent to $540, while the 2 on the second die indicates that, given an 8 percent return on asset class one, there has been a –2 percent return on the $500 invested in the second asset class, resulting in a drop in value to $490.

For the second year suppose we roll a 1 on the first die and a 5 on the second die. The $540 invested in asset class one will lose 4 percent and drop to $518.40, while asset class two will gain 10 percent and increase to $536. For years three, four and five, suppose that the pairs of tosses are (3,3), (1,4) and (4,2). Using the information we have about returns of the two asset classes, we would be able to determine that at the end of the five-year period, the amount in asset class one will have grown to $558.98, while the amount in asset class two will have grown to $581.89, for a total ending account balance of $1,140.87.

As before, we will perform a second simulation by tossing the dice five more times and interpreting the results accordingly. After performing simulations numerous times, a range of possible outcomes, together with a chart showing the distribution of results (similar to the one for the one asset class example) could be developed. From this chart we would be able to infer regarding the likelihood of various outcomes.

Here are a few items to keep in mind. First, it does not need to be the case that each of the returns we used in the previous examples had to have equal likelihood. For example, suppose that a –4 percent return occurred 1/6 of the time, a 0 percent return occurred 1/3 of the time, a 4 percent return occurred 1/3 of the time and an 8 percent return occurred 1/6 of the time. We would be able to simulate this situation by using a special die whose six faces were labeled 1, 2, 2, 3, 3, 4 and interpreting a 1 as a –4 percent return, a 2 as a 0 percent return, a 3 as a 4 percent return and a 4 as an 8 percent return.

Second, all the possible returns don't have to "fit" on a standard six-sided die. If there are "N" possible outcomes, we will be able to simulate them by envisioning an N-sided die, appropriately marked. (Here is where it begins to be useful to have a spreadsheet or some other electronic aid.)

Furthermore, the number of possible outcomes can even be infinite, as they would be if our annual returns were chosen randomly from an interval ranging from –4 percent to 16 percent. The important thing to keep in mind is that what is needed is a way to sample (simulate) from the "sample space" (the range of possible "rolls" or random numbers) and a way to interpret, in the context of the system being modeled, the value which was chosen.

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