Economic Scenario Work Group Status Report

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NAIC Spring National Meeting
LHATF and LRBC Working Group
March 2007

AMERICAN ACADEMY of ACTUARIES

Objectives

- Reparameterize the existing C-3 Phase 1 Interest Rate Model
- Develop Calibration Criteria

Reparameterization Phase

Significant changes from current parameterization

- Target (mean reversion) long rate – 6.55% vs updated rate of 5.5%
- Time step of the long rate volatility
- Formula to update target long rate
- Formula to update starting value for long rate volatility process
- Volatility of long spread process is dynamic
- Change process to complete the yield curve

Stochastic Log Volatility Model Equations

$$SLV(1):$$

$${}_1i_t = Min\left[{}_1\lambda_U, (1-\beta_1)\cdot {}_1i_{t-1} + \beta_1 \cdot \ln\tau_1 + \psi\cdot({}_2\tau_t - \alpha_{t-1})\right] + {}_1\sigma_t \cdot {}_1Z_t$$

$$\alpha_t = (1-\beta_2)\cdot\alpha_{t-1} + \beta_2 \cdot {}_2\tau_t + \phi\cdot({}_1i_{t-1} - \ln\tau_1) + \sigma_2 \cdot {}_2Z_t \cdot ({}_1r_{t-1})^{\theta}$$

$$\nu_t = (1-\beta_3)\cdot\nu_{t-1} + \beta_3 \cdot \ln\tau_3 + \sigma_3 \cdot {}_3Z_t$$

where

$${}_1i_t = \ln({}_1r_t)$$

$${}_1\lambda_U = \ln({}_1r_{Max})$$

$${}_2r_t = \exp({}_1i_t) - \alpha_t$$

If ${}_2r_t < {}_2r_{Min}$, then ${}_2r_t = \kappa \cdot {}_1r_t$

$${}_1\sigma_t = \exp(\nu_t)$$

${}_1Z_t, {}_2Z_t, {}_3Z_t \sim N(0,1)$ with constant correlation matrix ρ

$SLV(1.1)$: same as $SLV(1)$, except

$${}_2r_t = \exp({}_1i_t - \alpha_t)$$

$SLV(2)$: same as $SLV(1)$, except

$${}_1i_t = Min\left[{}_1\lambda_U, (1-\beta_1)\cdot {}_1i_{t-1} + \beta_1 \cdot \ln\tau_1 + \psi\cdot\left\langle {}_2\tau_t - ({}_1i_{t-1} - {}_2i_{t-1})\right\rangle\right] + {}_1\sigma_t \cdot {}_1Z_t$$

$${}_2i_t = (1-\beta_2)\cdot {}_2i_{t-1} + \beta_2 \cdot ({}_1i_{t-1} - {}_2\tau_t) + \sigma_2 \cdot {}_2Z_t \cdot ({}_1r_{t-1})^{\theta}$$

$${}_2i_t = \ln({}_2r_t)$$

If $\tau_2^{\#} = 0$, then ${}_2\tau_t = \tau_2$.

Otherwise, ${}_2\tau_t$ is indirectly defined by χ_t (target "short/long" ratio)

If $\tau_2^{\#} = 1$, then $\chi_t = 1.479\cdot({}_1r_{t-1})^{0.2215}$

If $\tau_2^{\#} = 2$, then $\chi_t = 0.1442\cdot {}_1i_{t-1} + 1.2342$

Summary - 20,000 Scenarios
30 Year Time Horizon
Starting 1 Yr Rate = 4.5% - 20 Yr Rate = 5.5%

Stochastic Log Volatility (1)

2/2/2007

Param Set: **25** 27
Parameter Set Name: **DynSprd_1**
Model Type: 1

Inversion Frequency	Simulated	History 1953-2006
Threshold	**15 bps**	
Median	8.9%	16.2%
Average	11.8%	

Stdev Diff Log Rates	Short (1y)	Long (20y)
History	6.9%	3.1%
Median	7.9%	3.3%
Average	8.3%	3.4%

Short/Long Correlation	Simulated	History 1953-2006
Diff Rates	0.70	0.73
Diff LogR	0.67	0.64
Chg Sprd/LR	-0.17	-0.23

Starting Rates:
Horizon (yrs):

Spread Defn: R1-R2

Simulated Rates	Short (1y)	Long (20y)	Spread
Min	0.33%	1.10%	-11.19%
0.01	0.95%	2.45%	-1.97%
0.05	1.70%	3.12%	-0.78%
0.1	2.14%	3.52%	-0.31%
Median	4.30%	5.40%	0.99%
0.9	8.29%	8.85%	2.34%
0.95	10.15%	10.35%	2.80%
0.99	15.30%	15.42%	3.85%
Max	100.06%	94.98%	7.83%
Avg	4.95%	5.95%	1.00%
Stdev	3.26%	2.85%	1.14%
Skew	5.799	6.509	-0.218
Kurt	103.075	128.678	3.026
Dispersion	1.964	1.341	3.603

History 1953-2006	Short (1y)	Long (20y)	Spread
Min	0.82%	2.57%	-3.33%
0.01	1.07%	2.64%	-1.85%
0.05	1.45%	3.07%	-1.01%
0.1	2.35%	3.75%	-0.36%
Median	5.43%	6.28%	0.91%
0.9	9.39%	10.62%	2.70%
0.95	11.66%	12.06%	3.40%
0.99	14.87%	13.88%	3.80%
Max	16.72%	15.13%	4.08%
Avg	5.70%	6.66%	0.96%
Stdev	2.99%	2.65%	1.24%
Skew	1.011	0.828	0.169
Kurt	1.294	0.412	0.221
	1.880	1.432	4.846

Parameters	
$\tau 1$	5.50%
$\beta 1$	0.00509
θ	1
$\tau 2$	0.01
$\beta 2$	0.02685
$\sigma 2$	0.04148
$\tau 3$	0.0287
$\beta 3$	0.04001
$\sigma 3$	0.11489
$\rho(1,2)$	-0.19197
$\rho(1,3)$	0
$\rho(2,3)$	0
ψ	0.25164
ϕ	0.0002
$\tau 2\#$	0

Summary - 20,000 Scenarios
30 Year Time Horizon
Starting 1 Yr Rate = 10% - 20 Yr Rate = 12%

Stochastic Log Volatility (1)

2/2/2007

Param Set: **25** 27

Parameter Set Name: **DynSprd_1**

Model Type: 1

	Inversion Frequency	
	Simulated	**History 1953-2006**
Threshold	**15 bps**	
Median	10.2%	16.2%
Average	13.3%	

	Stdev Diff Log Rates	
	Short (1y)	**Long (20y)**
History	6.9%	3.1%
Median	8.1%	3.3%
Average	8.7%	3.4%

	Short/Long Correlation	
	Simulated	**History 1953-2006**
Diff Rates	0.70	0.73
Diff LogR	0.67	0.64
Chg Sprd/LR	-0.16	-0.23

Starting Rates:

Horizon (yrs):

	Simulated Rates			History 1953-2006		
	Short (1y)	**Long (20y)**	**Spread**	**Short (1y)**	**Long (20y)**	**Spread**
Min	0.33%	1.13%	-13.67%	0.82%	2.57%	-3.33%
0.01	0.93%	2.52%	-2.19%	1.07%	2.64%	-1.85%
0.05	1.72%	3.20%	-0.86%	1.45%	3.07%	-1.01%
0.1	2.17%	3.62%	-0.34%	2.35%	3.75%	-0.36%
Median	4.47%	5.61%	1.04%	5.43%	6.28%	0.91%
0.9	8.89%	9.48%	2.47%	9.39%	10.62%	2.70%
0.95	11.07%	11.31%	2.98%	11.66%	12.06%	3.40%
0.99	17.91%	17.70%	4.15%	14.87%	13.88%	3.80%
Max	113.27%	100.12%	8.78%	16.72%	15.13%	4.08%
Avg	5.27%	6.32%	1.05%	5.70%	6.66%	0.96%
Stdev	4.12%	3.72%	1.23%	2.99%	2.65%	1.24%
Skew	8.363	9.240	-0.273	1.011	0.828	0.169
Kurt	151.193	175.702	4.230	1.294	0.412	0.221
Dispersion	2.094	1.446	3.694	1.880	1.432	4.846

Spread Defn: R1-R2

Parameters	
$\tau 1$	5.50%
$\beta 1$	0.00509
θ	1
$\tau 2$	0.01
$\beta 2$	0.02685
$\sigma 2$	0.04148
$\tau 3$	0.0287
$\beta 3$	0.04001
$\sigma 3$	0.11489
$\rho(1,2)$	-0.19197
$\rho(1,3)$	0
$\rho(2,3)$	0
ψ	0.25164
ϕ	0.0002
$\tau 2\#$	0

Calibration Criteria

- SLV(1)-Dynamic Spread Model statistics+ Tolerances
- Which statistics
- Which time horizons