LIFE PRACTICE NOTE 1995–4
December 1995

Interest Rate Models

Introduction

This practice note was prepared by a work group organized by the Committee on Life Insurance Financial Reporting of the American Academy of Actuaries. The work group was charged with developing a description of some of the current practices used by valuation actuaries in the United States. This work group was originally formed in 1992 and issued the first set of Life Practice Notes that year; changes have been made to this set of practice notes on an annual basis to reflect additional information on current practices.

The practice notes represent a description of practices believed by the work group to be commonly employed by actuaries in the United States in 1995. The purpose of the practice notes is to assist actuaries who are faced with the requirement of adequacy testing by supplying examples of some of the common approaches to this work. However, no representation of completeness is made; other approaches may also be in common use. It should be recognized that the information contained in the practice notes provides guidance, but is not a definitive statement as to what constitutes generally accepted practice in this area. Moreover, these practice notes are based upon the model Standard Valuation Law of the National Association of Insurance Commissioners (NAIC). To the extent that the laws of a particular state differ from the NAIC model, practices described in these practice notes may not be appropriate for actuarial practice in that state. This practice note has not been promulgated by the Actuarial Standards Board, nor is it binding on any actuary.

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Comments are welcome as to the appropriateness of the practice notes, desirability of annual updating, validity of substantive disagreements, etc. Comments should be sent to Donna R. Claire at her Directory address.

Q. What approaches to modeling interest rates are currently included in appointed actuaries' practice when doing asset adequacy testing?

A. Approaches currently used to represent interest rates in actuarial models may be broadly categorized as deterministic and stochastic. The most familiar deterministic approach is a single interest rate model, in which projections are made and present values are calculated using a single interest rate. A slight generalization of this approach is the single scenario method, in which a series of interest rates are used for future years, such as one rate for 15 years and another rate thereafter. A second deterministic approach is the multiple deterministic scenario method. In this approach, several scenarios of future interest rates are used. Examples of these are the 7 Required Interest Rate Scenarios (hereafter the Basic 7 scenarios) stated in the NAIC model *Actuarial Opinion and Memorandum Regulation* (hereafter the *Model Regulation*). (*Note:* These basic scenarios were first specified in New York Regulation 126, so some actuaries refer to them as the *New York 7* scenarios.)

The multiple deterministic scenario method can be further generalized by constructing yield curve scenarios (i.e., a series of future yield curves).

Stochastic methods generally fall into two categories: random scenario models and option-pricing models. Random scenario models generate scenarios of future interest rates or yield curves by applying a random number generator to one or more probability distributions. The interest rate or yield curve for each period is generated from the probability distribution and based on the interest rates that apply to the previous period. In this way, a full set of interest rates for all future periods can be developed. This interest rate scenario can be used to determine the magnitude of cash flows (if interest sensitive) and to discount them to a valuation date. A number of such scenarios and the corresponding present values of the cash flows are developed. Option-pricing models use a somewhat different approach. They too are based on an interest rate model, but the model is typically applied to asset and/or liability cash flows to produce an option-adjusted present value. The behavior of this value with respect to incremental changes in the initial yield curve is then studied. Option-pricing models do not necessarily produce values for individual underlying scenarios.

Q. Which of the above approaches are appropriate if cash flow testing is required?

A. A *cash flow test* is an adequacy test that involves sensitivity testing using the results of the interest rate model. Since stochastic models are based on a range of values for interest rates, this test is usually met if a sufficient number of scenarios are used. Sensitivity testing involves the examination of variations in the results of a test as one or more of the assumptions are varied. In order to test sensitivity to the interest rate model, the results of individual scenarios may be examined. Thus, option-pricing models are not typically used for cash flow testing unless the models produce values for each distinct underlying scenario. The multiple deterministic scenario method is in effect a sensitivity test for the single scenario method, and so is appropriate if the business tested is sensitive to interest rate changes. Application of a single scenario deterministic model would generally *not* be considered to constitute adequate cash flow testing.

Q. Is there any time when a single interest rate scenario path may be appropriate?

A. If interest rate involvement is not a critical variable, such as for short-term health insurance backed by short-term assets, then a single interest rate scenario with multiple *other assumption* scenarios is the approach used by some actuaries.

Q. What considerations guide the use of the multiple deterministic scenario method?

A. The usefulness of the multiple deterministic scenario method depends on the range of scenarios used. Normally, practitioners utilize scenarios representing a number of significantly different future interest rate environments. These environments typically differ by level of interest rate and by rate and direction of change of interest rates. Also, yield curve inversions are frequently represented. The range of scenarios includes moderately adverse interest rate environments.

Q. Are any scenario sets in common use?

A. The most commonly used set of deterministic scenarios is the Basic 7 scenarios required under the *Model Regulation.* These scenarios are actually redetermined each year so that the initial values can be set to equal current interest rates. A common practice is to extend the Basic 7 approach to yield curves, and to add scenarios in which inversions are assumed to occur.

The minimum interest rate in the Basic 7 scenarios is floored at one-half of the starting 5-year Treasury rate. For determining the interest rates for other than the 5-year Treasury rate, some actuaries use parallel yield curves, reducing all interest rates used by the same amount that the 5-year Treasury curve would be reduced. Others reduce other points along the yield curve proportionately to the reduction in the 5-year Treasury rate.

New York Regulation 126 also sets a maximum interest rate of 25%. This maximum interest rate is commonly used by actuaries, although some actuaries also feel that some scenarios which reach interest rates higher than the Regulation 126 maximum should be considered, especially when the current interest rates are high. The *Model Regulation* does not specify any maximum interest rates.

In the survey of what actuaries did for year-end 1992 asset adequacy testing, 29% tested only the Basic 7 scenarios. Another 29% tested the Basic 7 scenarios plus 1–3 inverted yield curve scenarios. Approximately 40% of the actuaries tested under more than 10 scenarios; some of those used stochastically generated scenarios in addition to the Basic 7 scenarios.

In the 1993 survey of appointed actuaries, the most commonly cited additional scenarios were those with inverted yield curves. Testing changes in yield curves can provide useful information for assets such as structured notes, where the investment income and market values will change with changes in the shape of the yield curve.

Some actuaries develop their own scenarios for use in forming their opinions regarding adequacy, and look to the Basic 7 scenarios as part of their sensitivity testing.

Q. Is testing under the Basic 7 scenarios always adequate?

A. Actuarial Standard of Practice (ASOP) No. 22 states the following: “. . . [T]he actuary should be satisfied that the number and types of scenarios tested are adequate. Limiting such scenarios to those contained in the *Model Regulation* is not necessarily adequate.”

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Q. What meaning can be attached to the mean of the results under multiple deterministic scenarios?

A. Mean results under scenarios chosen in such a manner generally have no meaning since, in most cases, the probability that each scenario will occur cannot be assumed to be equal. For the same reason, statements related to the probability of satisfactory results generally cannot be reliably made when using such sets.

Q. What types of random scenario models are included in current actuarial practice?

A. There are several types of random scenario models commonly used. One type of model uses a binomial lattice to predict future rates. Another method is to use a Monte Carlo approach to calculate period-to-period changes in interest rates. Sometimes, changes in long- and short-term rates are calculated separately (i.e., using distinct distribution functions), and an interpolation procedure is used to approximate a yield curve. The standard deviation of the distribution is called the *volatility*.

Q. What distribution functions are commonly used by actuaries in determining a specific model of the term structure?

A. The lognormal distribution currently is commonly used for models which assume that long- and short-term interest rates are the random variables. (Other interest rate models, such as the Cox-Ingersoll-Ross and the Brennan-Schwartz models, use distribution functions for the error terms or residuals of the model equations.) Such a distribution, with quarterly volatility of around 16% for short-term rates and 8% for longer-term rates, is believed by many actuaries to validate reasonably to recent Treasury yield curves.

However, some recent research indicates other distributions may be preferable. A source of information on this subject is found in a report by Dr. David Becker, entitled “Statistical Tests of the Lognormal Distribution as a Basis for Interest Rate Changes,” in the *Transactions of the Society of Actuaries*, v. XLIII (1991), p. 7. A discussion of various stochastic models is given in the article “An Actuarial Layman's Guide to Building Stochastic Interest Rate Generators,” by Dr. James Tilley, found in the *Transactions of the Society of Actuaries*, v. XLIV (1992), p. 509. Another reference is Gordon E. Klein's “The Sensitivity of Cash-Flow Analysis to the Choice of Statistical Model for Interest Rate Changes,” which can be found in the *Transactions of the Society of Actuaries*, v. XLV (1993), p. 79.

Q. What is *reversion to the mean?*

A. *Reversion to the mean* is a tendency, built into a model, for random values to move toward a target value (mean) as the number of trials increases. For random scenario models, this is accomplished by modifying the output of the sampling procedure, perhaps by multiplying by a reversion factor that, in turn, is a function of a parameter called the *strength* of mean reversion. If the strength is zero, no mean reversion occurs; if it is unity, the interest rate is set to the target value. The reversion factor may be a function of the difference between the random value and the target value. At this time, there does not appear to be a substantial amount of research into choosing the proper value for a target value.

Q. How can an interest rate model be validated?

A. Normally, an interest rate model will revolve around the current yield curve. Moreover, for random scenario models, the volatility typically will fall within the range observed in recent history. The frequency of inversions usually is also considered in validating the interest rate model in most instances (compare with Dr. David Becker's "The Frequency of Inversions of the Yield Curve and Historical Data on the Volatility and Level of Interest Rates," found in *Risks and Rewards: The Newsletter of the Society of Actuaries Investment Section* (October, 1991), p. 6).

Q. Are models ever used that violate the validation requirements?

A. Yes. Such models may be used for sensitivity tests and other purposes. For example, some practitioners set the mean of the *change* random variable to a level that will cause a large number of scenarios to fall in the regions that are expected to produce less acceptable results.

Q. What does *yield curve normalization* mean?

A. A number of actuaries surveyed said that the yield curve was abnormally steep at the end of 1992–1994, i.e., the short-term interest rates were abnormally lower than long-term rates. Therefore, a number of actuaries changed the yield curves tested so that the yield curve would be *normal* (i.e., less steep) after a period of years, typically 2 years. Many actuaries would agree that normalization makes sense if the starting yield curve has an abnormal shape—either flat, inverted, or unnaturally steep.

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Q. How many random scenarios are sufficient?

A. Given the complexity of interest rates, a definitive answer cannot be given. Currently, some practitioners use from forty to several thousand scenarios. The accuracy of the estimate of the possible range of surplus for the business being tested can be expected to increase with the square root of the number of scenarios. Testing the improvement gained from additional scenarios in a given situation may be useful. Some practitioners examine the scenario set to assure the presence of a significant number of scenarios of the kind thought most likely to produce less acceptable results.

Q. If some elements of a set of random scenarios are clearly unreasonable, can these be ignored or replaced?

A. Practitioners generally resist this practice. First, throwing out selected scenarios in a random sample destroys the randomness of the sample. In addition, recent history is not a safe guide to what is *reasonable*; most actuaries in the 1970s never expected the high interest rates of the early 1980s, and most actuaries in the 1980s did not expect the low interest rates we see today. However, if the set as a whole seems to be *too wild, too tame*, or *too sparse* in inversions, then many actuaries would consider modifying the parameters and generating another set.

Q. Does current actuarial practice include the use of option-pricing models for reserve adequacy testing?

A. Option-pricing models have been used for pricing and profitability testing of insurance and annuity products. Research is currently being conducted by the Society of Actuaries in the hope of extending the applicability of such models to reserve adequacy testing. Normally, option-pricing models are constructed to estimate market values of options. In order to use option-pricing models for reserve testing, it is necessary to take account of the book-value orientation of statutory accounting.

Q. What considerations govern the use of option-pricing models for reserve adequacy testing?

A. It is usually prudent to check option-pricing models for internal consistency—for example, to avoid the possibility of risk-free arbitrage. Dr. Tilley's article, “An Actuarial Layman's Guide to Building Stochastic Interest Rate Generators,” referenced above, discusses the possibility that interest rate models do not necessarily have to be completely arbitrage free. If several different option-pricing models are used (e.g., for certain assets and liabilities), many actuaries feel the consistency of the calculations must be checked. Also, the inability to study the variation of results as a function of the underlying interest rate scenarios leads some actuaries to adopt a higher degree of conservatism in using option-pricing models than other methods.

One limitation of option-pricing models is that they generally focus on C-3 risk and usually ignore the C-1 and C-2 risks.

At this time, there are regulators who will not accept asset adequacy testing done solely on the basis of option-pricing as described above, since the method has not yet been proved to their satisfaction to be adequate to test reserve adequacy.